Calibration of Hazen–Williams coefficients in pipe networks using tracers

Abbas S. Al-Omari and Ahmad I. Jamrah

ABSTRACT

A model that directly estimates HW coefficients for pipe networks by making tracer concentration measurements at selected nodes is developed utilizing the inverse method. The model considers even-, over- and under-determined cases, depending on the number of measurements and estimated parameters. For the purpose of model verification, tracer concentrations obtained by forward simulation are considered as measurements. Results showed that the model made accurate determinations of the unknown HW coefficients for both even- and over-determined cases for some scenarios and less accurate determinations for other scenarios. The under-determined case made reasonable estimates of the unknown HW coefficient for most of the scenarios considered. Error analysis showed that errors are introduced into the determined HW coefficient as a result of measurement errors and as a result of making the measurements at low sensitive nodes. Error analysis also showed that the determined HW coefficient is more sensitive to the measurement location (sensitivity) than to the measurement error. Both conservative and non-conservative tracers may be used. However, conservative tracers are expected to make better estimation of HW coefficients as they produce larger sensitivities.

Key words | inverse modeling, network calibration, water distribution, water quality modeling

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ABBREVIATIONS AND NOTATION

The following symbols are used in this paper

$a_1, a_2, a_3, a_4 =$ constants
c = cross-sectional average tracer concentration
$c_{\mathrm{d}i}$ = downstream concentration of pipe i
c_i^n = concentration at node i at the start of the time step
c_i^{n+1} = concentration at node i at the end of the time step
c_{i+1}^n = concentration at node $i+1$ at the start of the time step
c_{i+1}^{n+1} = concentration at node $i+1$ at the end of the time step
c_{nj} = concentration at node j
c_{nj}^{c} = calculated concentration at node j
c_{nj}^{m} = measured concentration at node j
$c_{\mathrm{u}i}$ = upstream concentration of pipe i
D_x = coefficient of diffusion in the x direction
H_{wi} = Hazen–Williams coefficient for pipe i
k_1 = first-order chlorine decay coefficient
N_k = number of known nodal concentrations
N_n = number of nodes
N_p = number of pipes
N_{pn} = number of pipes that meet at a node
N_{u} = number of unknown Hazen–Williams coefficients
Q_i = flow at pipe i
T = simulation period
t = time
u = flow velocity
x = distance
ΔH_{wi} = incremental change in the Hazen–Williams coefficient
Δt = time increment
Δx = spatial increment
$\Delta\lambda$ = incremental change in Lagrange multiplier
α = Courant number
λ = Lagrange multiplier
θ = weighting coefficient

INTRODUCTION

Several network models that simulate flow rates, pressures, and constituent transport in networks, were developed in the previous decades by, among others are Epp & Fowler (1970), Wood & Charles (1972), Rossman (1993), Islam & Chaudhry (1998) and Al-Omari & Chaudhry (2001). The accuracy with which these models predict flow rates, pressures and constituent concentration throughout networks depends on the accuracy of the input data. Uncertainties in input data include uncertainties in tank elevations, node elevations, pipe roughness, nodal demands and, in the case of the constituent transport simulation, decay or growth rate coefficient of the constituent.

There is a consensus in the literature that pipe friction factors and nodal demands, and their variation with time, are among data pieces that may create the greatest discrepancies between calculated and observed pressures and flow rates due to difficulties encountered in their estimation. However, there is controversy about which is more responsible for these discrepancies: pipe friction factor or estimated nodal demands and their variation with time. The American Water Works Association (AWWA) Research Committee on Water Distribution Systems (1974) believes that nodal demand and demand variation with time is a major source of error in simulation "… The major source of error in simulation of contemporary performance will be in the assumed loading distributions and their variations". In contrast, Eggener & Polkowski (1975) believe that the pipe friction factor is the weakest piece of input: "the weakest piece of input information is not the assumed loadings condition, but the pipe friction factor". Despite the controversy about which piece of information is responsible for the greatest discrepancies between observed and calculated pressures and flow rates, there is a consensus that network models have to be calibrated so that they can be reliable tools in predicting flow rates and pressures in networks.

When the constituent concentration is simulated, another weak piece of input in addition to Hazen–Williams (HW) coefficients and nodal demands responsible for discrepancies between calculated and measured constituent concentrations is the wall decay coefficient which can only be determined by model calibration.

Calibration as defined by Shamir and Howard (see Walski 1983) "consists of determining the physical and operational characteristics of an existing system and determining the data [that] when input to the computer model will yield realistic results". The AWWA Research Committee on Water Distribution Systems (Walski 1983) described the process of calibration by "system simulation is considered verified during preliminary analysis for design when calculated pressures are satisfactorily close to observed field gage readings for given field source send – out and storage conditions. If simulation is not satisfactory, the possibility of local aberrations, such as open boundary valves, is investigated. In the absence of other expected causative factors, the assumed local arterial network loads are adjusted until computed and observed field pressures are within reasonable agreement for various levels and extremes of demand pumping and storage". From the two previous definitions, a model is considered calibrated when it can predict flow rates and pressures within a reasonable agreement, over a wide range of boundary conditions, i.e. demand variation and pump operation.

Calibrating network models can be a very complicated task and may become frustrating due to the large size of networks in the real world which results in a huge number of model parameters to be calibrated (i.e. pipe roughness) as it changes with pipe age, uncertainty in the collected data (i.e. nodal pressures and pipe flow rates) and uncertainty in model parameters. Ormsbee & Lingireddy (1997) described calibrating network models as being as difficult as climbing Mount Everest: "to a novice, careful calibration of a hydraulic network model may be as daunting a task as climbing Mt. Everest".

Due to these difficulties encountered in network model calibration, several models that calibrate networks for either pipe roughness, or nodal demand, or both, have been developed by different researchers, amongst who are Walski (1983), Ormsbee & Wood (1986), Ormsbee (1989), Boulos & Wood (1990), Lansey & Basnet (1991), Ligget & Chen (1994), Greco & Giudice (1999), Covas *et al.* (2002) and Araujo *et al.* (2003). Boulos & Wood (1990) presented an algorithm that determines explicitly design variables and model parameters to meet measured flow rates and pressures at critical nodes. Greco & Giudice (1999) presented a new approach for calibrating the pipe friction factor. Their procedure uses a nonlinear optimisation algorithm along with a network solver. The objective function in this approach is the minimisation of the squared difference between initially assumed pipe friction factors and calibrated ones subject to a set of constraints determined from the sensitivity matrix. Pipe roughness is adjusted until simulation results agree with the observed values. Lansey & Basnet (1991) developed a nonlinear algorithm that incorporates a network simulation model that minimises the sum of the squares or absolute values of pipe flows or nodal pressure heads to estimate pipe roughness coefficients, valve settings and nodal demands. Ligget & Chen (1994) and Covas *et al.* (2002) developed an algorithm that uses inverse transient analysis for leak detection as well as network calibration.

Ormsbee & Wood (1986) solved the network equations explicitly for head loss adjustment to meet one or more measured conditions of flow or pressure for a given loading condition. The adjustments determined are then used to revise pipe roughness or defined local head (minor) losses to meet the measured conditions. Ormsbee (1989) developed a mathematical model that uses a nonlinear optimisation algorithm along with a network solver to adjust pipe roughness, hydraulic grades at source nodes and nodal demands so that measured pressure values at specified nodes are met. The technique presented by Walski (1983) helps the user determine whether to adjust the pipe friction factor or nodal demands and by how much. He also developed formulae that require the user to observe pressures in the system for at least two significantly different use rates. Recently, other researchers developed methodologies or procedures for calibrating hydraulic network models that aim at achieving good model calibration (Ormsbee & Lingireddy 1997; Lansey *et al.* 2001; Araujo *et al.* 2003).

All prescribed models and algorithms use pressure and/or flow measurements to calibrate networks. The model presented in this paper uses the measurement of a tracer concentration to calibrate network models for HW coefficients and the number of measured tracer concentrations may be equal to, greater than, or less than the number of calibrated model parameters. The model presented in this paper utilises an inverse algorithm that minimises the difference between the measured tracer concentrations and calculated ones to solve for unknown HW coefficients.

Even-, over- and under-determined cases are considered. In the even-determined case, the number of measurements made equals the number of calibrated model parameters. In the over-determined case, the number of measurements made is greater than the number of calibrated model parameters. In the under-determined case, the number of measurements made is less than the number of calibrated model parameters. In the even-determined case, the model minimises the absolute difference between calculated and measured concentrations subject to the equations that determine tracer concentrations in networks. For the over-determined case, the model minimises the squared difference between measured tracer concentrations and calculated ones subject to the equations that govern tracer concentrations in networks. For the under-determined case, the model minimises the length of the solution vector subject to the equations that determine tracer concentrations in networks.

The model developed in this paper assumes that nodal demands, reservoir elevations, nodal elevations, valve settings and overall constituent decay coefficient are known to a high degree of certainty. The model also assumes that leaks are either negligible or known and taken into consideration.

In the following paragraphs, the equations that govern the even-determined, over-determined and under-determined cases and their solutions are presented. The model is then verified by applying it to two hypothetical example networks. Error analysis is then performed to show how errors are introduced into the determined HW coefficients.

THE ALGORITHM

The model developed by Al-Omari & Chaudhry (2001) is further developed in this paper for the determination of unknown HW coefficients so that the difference between measured tracer concentrations and calculated ones is minimised. In order to solve for the unknown HW coefficients, the tracer concentration has to be solved for first, as nodal concentrations are needed for the solution of the inverse problem (the determination of HW coefficients). The equations that determine the unsteady-state concentration of a tracer that moves with the water, such as chlorine, in networks are described in detail in Al-Omari & Chaudhry (2001). These equations are described here briefly as they appear in the algorithm developed in this paper. Unsteady-state tracer concentration in pipe networks is governed by the following equations:

(a) The one-dimensional advection–diffusion equation with a decay term (James 1993):

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + D_x\frac{\partial^2 c}{\partial x^2} + k_1 c = 0 \quad (1)$$

where c is the cross-sectional average tracer concentration, u is the cross-sectional average flow velocity, t is the time, D_x is the coefficient of diffusion in the x direction and k_1 is the first-order tracer decay rate coefficient. A four-point, implicit, second-order accurate finite-difference scheme known as the Preissmann scheme is used for the integration of (1). Islam (1995) found, through detailed numerical simulations, that diffusion is insignificant in pipe networks, so by neglecting the diffusion term and by making some mathematical manipulations, the resulting equation (2) is given below. For more details about the Presseiman scheme and details of the derivation of (2), the reader is referred to Chaudhry (1993) and Al-Omari & Chaudhry (2001):

$$a_1 c_{i+1}^{n+1} + a_2 c_i^{n+1} + a_3 c_{i+1}^n + a_4 c_i^n = 0 \quad (2)$$

where

$$a_1 = 0.5 + \alpha\theta + \frac{k_1\theta\Delta t}{2}$$

$$a_2 = 0.5 - \alpha\theta + \frac{k_1\theta\Delta t}{2}$$

$$a_3 = \alpha(1-\theta) - 0.5 + \frac{(1-\theta)k_1\Delta t}{2}$$

$$a_4 = \frac{k_1\Delta t}{2}(1-\theta)\alpha - (1-\theta) - 0.5$$

$$\alpha = \frac{u\Delta t}{\Delta x}$$

where i is the node number, n is the start of the time step, $n+1$ is the end of the time step, Δt is the time interval, Δx is the spatial interval and θ is a weighting coefficient. Depending on the value of θ, the scheme is explicit or implicit. For θ equal to 1, the scheme is implicit and for θ equal to 0, the scheme is explicit.

(b) The upstream concentration of a pipe equals the concentration at the upstream node, i.e.

$$c_{ui} - c_{nj} = 0 \quad (i = 1, 2, \ldots, N_{pn};\ j = 1, \ldots, N_n) \quad (3)$$

where c_{ui} is the upstream concentration of pipe i, c_{nj} is the concentration at node j, N_n is the number of nodes and N_{pn} is the number of pipes that join at node j.

(c) Concentration at a branching node equals the weighted average of the downstream concentrations of all the pipes that join at that node, i.e.

$$c_{nj} = \frac{\sum_{i=1}^{N_{pn}} Q_j c_{di}}{\sum_{i=1}^{N_{pn}} Q_i} = 0 \quad (j = 1, \ldots, N_n) \quad (4)$$

where Q_i is the flow rate for pipe i.

RELATIONSHIP BETWEEN NODAL CONCENTRATIONS AND HW COEFFICIENT

The model developed in this paper is based on making tracer concentration measurements to determine HW coefficients. Thus, before going into details of the even- and over-determined cases, it is important to understand how changes in the HW coefficient are translated into changes in nodal concentrations. As the HW coefficient (coefficients) of a pipe or a group of pipes changes (change), the flow rate in the network changes. Consequently flow velocities and travel times change, and hence tracer concentrations at the nodes also change. The amount by which the tracer concentration at a specific node changes in response to a change in HW coefficient for a specific pipe is determined by the sensitivity of the tracer concentration at that node to the HW coefficient of that specific pipe $\partial c_{nj}/\partial H_{wj}$. The sensitivity of the tracer concentration at a specific node to the HW coefficient of a specific pipe is a matter of how much flow reaching that node has passed through that pipe as well as the tracer decay coefficient for that pipe.

EVEN-DETERMINED CASE

As mentioned previously, the number of calibrated model parameters equals the number of measurements in the even-determined case. The solution for model parameters is obtained by minimising the absolute difference between measured and calculated concentrations subject to equations (2), (3) and (4), which is a constrained minimisation problem that may be solved by utilising a Lagrange multiplier as follows:

$$F_1 = \text{Min}|c^{c}_{n1} - c^{m}_{n1}| + \lambda_{11}\sum_{t=0}^{T}\sum_{j=1}^{N_n}\sum_{i=1}^{N_{pn}}(c_{u_i} - c_{nj}) + \lambda_{12}\sum_{t=0}^{T}\sum_{m=1}^{N_p}\left(a_1c^{n+1}_{i+1} + a_2c^{n+1}_{i} + a_3c^{n}_{i+1} + a_4c^{n}_{i}\right) + \lambda_{13}\left(c_{nj} - \frac{\sum_{i=1}^{N_{pn}}Q_ic_{di}}{\sum_{i=1}^{N_{pn}}Q_i}\right) \quad (5)$$

$$F_2 = \text{Min}|c^{c}_{n2} - c^{m}_{n2}| + \lambda_{21}\sum_{t=0}^{T}\sum_{j=1}^{N_n}\sum_{i=1}^{N_{pn}}(c_{u_i} - c_{nj}) + \lambda_{22}\sum_{t=0}^{T}\sum_{m=1}^{N_p}\left(a_1c^{n+1}_{i+1} + a_2c^{n+1}_{i} + a_3c^{n}_{i+1} + a_4c^{n}_{i}\right) + \lambda_{23}\left(c_{nj} - \frac{\sum_{i=1}^{N_{pn}}Q_ic_{di}}{\sum_{i=1}^{N_{pn}}Q_i}\right) \quad (6)$$

$$F_{nk} = \text{Min}\left|c^{c}_{n_{nk}} - c^{m}_{n_{nk}}\right| + \lambda_{nk1}\sum_{t=0}^{T}\sum_{j=1}^{N_n}\sum_{i=1}^{N_{pn}}(c_{u_i} - c_{n_{nk}}) + \lambda_{nk2}\sum_{t=0}^{T}\sum_{m=1}^{N_p}\left(a_1c^{n+1}_{i+1} + a_2c^{n+1}_{i} + a_3c^{n}_{i+1} + a_4c^{n}_{i}\right) + \lambda_{nk3}\left(c_{nj} - \frac{\sum_{i=1}^{N_{pn}}Q_ic_{di}}{\sum_{i=1}^{N_{pn}}Q_i}\right) \quad (7)$$

where c^{c}_{ni} stands for the ith calculated nodal concentration, c^{m}_{ni} stands for the ith measured nodal concentration, λ are Lagrange multipliers and nk is the number of measured nodal concentrations.

The second and third terms of the right-hand side of (5), (6) and (7) are zero [(3) and (2)]. The fourth term is also zero (4) except for measurement nodes. Therefore, (5), (6) and (7) reduce to

$$F_1 = \text{Min}|c^{c}_{n1} - c^{m}_{n1}| + \lambda_1(c^{c}_{n1} - c^{m}_{n1}) \quad (8)$$

$$F_2 = \text{Min}|c^{c}_{n2} - c^{m}_{n2}| + \lambda_2(c^{c}_{n2} - c^{m}_{n2}) \quad (9)$$

$$F_{nk} = \text{Min}|c^{c}_{nnk} - c^{m}_{nnk}| + \lambda_{nk}(c^{c}_{nnk} - c^{m}_{nnk}) \quad (10)$$

By taking the derivative of (8) with respect to H_{w1} and λ_1, (9) with respect to H_{w2} and λ_2 and (10) with respect to H_{wnu} and λ_{nu} the following equations are obtained:

$$\frac{\partial F_1}{\partial H_{w1}} = \frac{\partial c_{n1}}{\partial H_{w1}} + \lambda_1\frac{\partial c_{n1}}{\partial H_{w1}} \quad (11)$$

$$\frac{\partial F_1}{\partial \lambda_1} = c^{c}_{n1} - c^{m}_{n1} \quad (12)$$

$$\frac{\partial F_2}{\partial H_{w2}} = \frac{\partial c_{n2}}{\partial H_{w2}} + \lambda_2\frac{\partial c_{n2}}{\partial H_{w2}} \quad (13)$$

$$\frac{\partial F_2}{\partial \lambda_2} = c^{c}_{n2} - c^{m}_{n2} \quad (14)$$

$$\frac{\partial F_{nk}}{\partial H_{wnu}} = \frac{\partial c_{nnk}}{\partial H_{wnu}} + \lambda_3\frac{\partial c_{nnk}}{\partial H_{wnu}} \quad (15)$$

$$\frac{\partial F_{nk}}{\partial \lambda_{nu}} = c^{c}_{nnk} - c^{m}_{nnk} \quad (16)$$

where H_{wi} is the ith unknown HW coefficient and nu is the number of unknown HW coefficients. Equations (11)–(16) are the governing equations of the even-determined case. The number of unknowns in these equations is $2nu$, which are unknown model parameters and one λ value for every unknown model parameter. These equations may be solved by the Newton–Raphson method. Initially assumed values of the unknowns are corrected iteratively until the absolute difference between assumed and calculated values is less than a pre-specified tolerance. Solving for tracer concentrations throughout the network is necessary before solving for unknown model parameters. So equations (2), (3) and (4) are first solved for tracer concentrations. The step-by-step method of resolution is described below:

(a) Estimate all unknown HW coefficients and λ values;
(b) Solve equations (2), (3) and (4) for nodal concentrations;
(c) Compute $(\partial C_{nj}/\partial H_{wi})$ and $(\partial/\partial H_{wk})(\partial C_{nj}/\partial H_{wi})$;
(d) Solve equations (11)–(16) for ΔH_{wi} and $\Delta\lambda_i$ by the Newton–Raphson method;
(e) If $\Sigma(|\Delta H_{wi}| + |\Delta\lambda_i|)$ is less than a pre-specified tolerance, then the estimated HW coefficients are correct; if not,

then set $H_{wi} = H_{wi} + \Delta H_{wi}$ and $\lambda_i = \lambda_i + \Delta\lambda_i$, and repeat steps b) – e).

OVER-DETERMINED CASE

In the over-determined case, the number of measurements made is greater than the number of unknown model parameters. The solution to this problem may be obtained by minimising the squared difference between the calculated and measured tracer concentrations, i.e.

$$\mathrm{Min}\sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{m}\right)^2 \quad (17)$$

where c_{nj}^{c} is the calculated concentration at node j, c_{nj}^{m} is the measured concentration at node j, N_k is the number of measured nodal concentrations and T is the simulation period. The problem now is the minimisation of (17) subject to (2), (3) and (4), which is a constrained minimisation problem that may be solved by utilising the Lagrange multiplier as follows. Let

$$F = \mathrm{Min}\sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{\mathrm{m}}\right)^2 + \lambda_1\sum_{t=0}^{T}\sum_{j=1}^{N_n}\sum_{i=1}^{N_{pn}}\left(c_{ui} - c_{nj}\right)$$
$$+ \lambda_2\sum_{t=0}^{T}\sum_{m=1}^{N_p}\left(a_1c_{i+1}^{n+1} + a_2c_i^{n+1} + a_3c_{i+1}^{n} + a_4c_i^{n}\right) \quad (18)$$
$$+ \lambda_3\sum_{t=0}^{T}\sum_{j=1}^{N_n}\left(c_{nj} - \frac{\sum_{i=1}^{N_{pn}}Q_ic_{di}}{\sum_{i=1}^{N_{pn}}Q_i}\right)$$

The second and third terms of the right-hand side of (18) are always zero [(3) and (2)] and the fourth term is always zero (5) except for measurement nodes. So, (18) reduces to

$$F = \mathrm{Min}\sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{\mathrm{m}}\right)^2 + \lambda\sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{\mathrm{m}}\right). \quad (19)$$

By taking the derivative of (19) with respect to every unknown HW coefficient and with respect to λ, the following N_{u+1} equations are obtained:

$$\frac{\partial F}{\partial H_{wi}} = 2\sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{\mathrm{m}}\right)\frac{\partial c_{nj}}{\partial H_{wi}} + \lambda\sum_{t=0}^{T}\sum_{j=1}^{N_k}\frac{\partial c_{nj}}{\partial H_{wj}} \quad (20)$$

$(i = 1, \ldots, Nu)$

$$\frac{\partial F}{\partial \lambda} = \sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{\mathrm{m}}\right) \quad (21)$$

where H_{wi} is the HW coefficient for pipe i and Nu is the number of unknown HW coefficients.

The number of unknowns for the over-determined case is $N_u + 1$ which is the number of unknown HW coefficients plus the unknown λ value. The equations that govern the over-determined case are (20) and (21). Equation (20) provides N_u equations and (21) provides 1 equation, so the number of equations available equals the number of unknown model parameters. These equations are nonlinear, so they are solved iteratively by the Newton – Raphson method. As mentioned before, tracer concentrations at the nodes are needed before HW coefficients can be determined by (20) and (21), so (2), (3) and (4) are first solved for tracer concentrations at the nodes based on assumed HW coefficient values. The method of solution for the over-determined case is summarised below in a step-by-step manner based on the following methodology:

(a) Estimate all unknown HW coefficients and λ;
(b) Solve equations (2), (3) and (4) for nodal concentrations;
(c) Compute $(\partial c_{nj}/\partial H_{wi})$ and $(\partial/\partial H_{wk})(\partial c_{nj}/\partial H_{wi})$;
(d) Solve equations (20) and (21) for ΔH_{wi} and for $\Delta\lambda$ by the Newton – Raphson method;
(e) If $\Sigma|\Delta H_{wi}| + |\Delta\lambda|$ is less than a pre-specified tolerance, then the estimated HW coefficients are correct; if not, set $H_{wi} = H_{wi} + \Delta H_{wi}$ and $\lambda = \lambda + \Delta\lambda$, and repeat steps b) – e).

UNDER-DETERMINED CASE

In the under-determined case, the number of unknown model parameters is greater than the number of measurements made. In this case, there is more than one solution that satisfies the governing equations. A unique solution to this problem may be obtained by minimising the Euclidean length of the solution vector (Menke 1989), i.e.

$$\mathrm{Min}\sum_{i=1}^{N_u}((H_{wi})^2)^{1/2}. \quad (22)$$

The problem now is the minimisation of (22) subject to (2), (3) and (4), which is a constrained minimisation problem that can be solved by the utilisation of the Lagrange multiplier as follows:

$$F = \mathrm{Min}\sum_{k=1}^{N_u}\left((H_{wk})^2\right)^{1/2} + \lambda_1\sum_{t=0}^{T}\sum_{j=1}^{N_p}\sum_{i=1}^{N_{pn}}(c_{u_i} - c_{n_j})$$
$$+ \lambda_2\sum_{t=0}^{T}\sum_{m=1}^{N_p}\left(a_1c_{i+1}^{n+1} + a_2c_i^{n+1} + a_3c_{i+1}^{n} + a_4c_i^{n}\right) \quad (23)$$
$$+ \lambda_3\sum_{t=0}^{T}\sum_{j=1}^{N_n}\left(c_{nj} - \frac{\sum_{i=1}^{N_{pn}}Q_ic_{d_i}}{\sum_{i=1}^{N_{pn}}Q_i}\right).$$

The second and third terms in (23) are always zero [(3) and (2)]. The fourth term is also zero (4) except for the nodes where measurements are made. Therefore, equation (23) reduces to

$$F = \mathrm{Min}\sum_{k=1}^{N_u}\left((H_{wk})^2\right)^{1/2} + \lambda\sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{\mathrm{m}}\right). \quad (24)$$

By taking the derivative of (24) with respect to every unknown HW coefficient and with respect to λ, the following $N_u + 1$ equations are obtained:

$$\frac{\partial F}{\partial H_{wk}} = \sum_{k=1}^{N_u}\left((H_{wk})^2\right)^{-1/2}H_{wk} + \lambda\sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(\frac{\partial c_{nj}}{\partial H_{wk}}\right) \quad (25)$$
$$(k = 1, \ldots, N_u)$$

$$\frac{\partial F}{\partial \lambda} = \sum_{t=0}^{T}\sum_{j=1}^{N_k}\left(c_{nj}^{\mathrm{c}} - c_{nj}^{\mathrm{s}}\right) \quad (26)$$

Equations (2), (3), (4), (25) and (26) are solved for the unknowns as follows:

(a) Estimate values for all unknown HW coefficients and for λ;
(b) Solve equations (2), (3) and (4) for the unknown nodal concentrations;
(c) Compute $(\partial C_{nj}/\partial CH_{wk})$ and $(\partial/\partial H_{wk})(\partial C_{nj}/\partial H_{wk})$;
(d) Solve equations (25) and (26) by the Newton-Raphson method for ΔH_{wk} and $\Delta\lambda$;
(e) Calculate $\Sigma|\Delta H_{wk}| + |\Delta\lambda|$;
(f) If the summation obtained in step e) is less than a pre-specified tolerance, then the estimated H_{wk} values are correct; if not, then $H_{wk} = H_{wk(i-1)} + \Delta H_{wk}$, $\lambda = \lambda_{i-1} + \Delta\lambda$, $H_{wk} = 0.5(H_{wk}) + 0.5H_{wk},_{(i-1)}$ and $\lambda = 0.5\lambda + 0.5\lambda_{i-1}$ and then repeat steps b) through f).

The previously outlined procedures for the solution of the even-, over- and under-determined cases show that $(\partial c_{nj}/\partial H_{wi})$ and $(\partial/\partial H_{wk})(\partial c_{nj}/\partial H_{wi})$ need to be evaluated first. For the evaluation of $(\partial c_{nj}/\partial H_{wi})$, equations (2), (3) and (4) are derived with respect to H_{wi} which results in a set of linear equations, the solution of which yields $(\partial c_{nj}/\partial H_{wi})$. $(\partial/\partial H_{wk})(\partial c_{nj}/\partial H_{wi})$ is evaluated by deriving the set of linear equations obtained in the previous step with respect to H_{wk} and solving the resulting set of linear equations. The resulting set of linear equations for the determination of $(\partial c_{nj}/\partial H_{wi})$ and $(\partial/\partial H_{wk})(\partial c_{nj}/\partial H_{wi})$ are ill-determined, so a specially designed sub-program that utilises singular value decomposition is used for their solution.

MODEL VERIFICATION

For the purpose of verifying the model, it was applied to two example networks as follows:

(a) The model was first run to obtain tracer concentrations at the nodes for known HW coefficients.
(b) HW coefficients for some pipes are assumed unknowns.
(c) Tracer concentrations obtained in step a) were input to the model as measurements to calculate the unknown HW coefficients.
(d) The model is considered verified when the HW coefficients used in step a) are obtained in step c). The model was run for even-, over- and under-determined cases for both networks.

NETWORK 1

Network 1, shown in Figure 1, consists of sixteen pipes, ten junction nodes and two constant head reservoirs. Pipe characteristics are given in Table 1, node elevations and nodal demands at time zero are given in Table 2 and demand factors for the simulation period are given in Table 3. The water surface elevation for reservoir 1 is 219.5 m and for reservoir 2 is 210.3 m. Initial tracer concentration at reservoir 1 is 3.00 mg/l and at reservoir 2

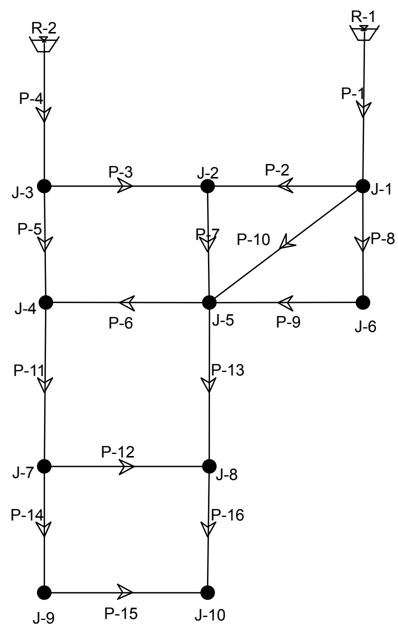

Figure 1 | Example network 1.

Table 1 | Pipe characteristics for network 1

Pipe no.	Pipe length (m)	HW coefficient	Diameter (cm)
1	76.2	100.0	25.4
2	91.4	100.0	25.4
3	91.4	100.0	25.4
4	76.2	100.0	25.4
5	91.4	100.0	25.4
6	91.4	100.0	25.4
7	91.4	100.0	20.3
8	91.4	100.0	20.3
9	91.4	100.0	20.3
10	106.7	100.0	20.3
11	91.4	100.0	20.3
12	91.4	100.0	20.3
13	91.4	100.0	20.3
14	91.4	100.0	25.4
15	91.4	100.0	25.4
16	91.4	100.0	25.4

is 2.00 mg/l. The tracer decay rate coefficient for the two reservoirs is 0.10/h and for all pipes is 0.15/h. The model was first run to obtain tracer concentrations at the nodes and these concentrations are given in Table 4. The model was then run to determine HW coefficients for different scenarios of even-, over- and under-determined cases. Below is a description of these scenarios.

Even-determined case

Five different scenarios of measurement locations and times to estimate the HW coefficients of different pipes are considered. It is important to note that the number of unknown HW coefficients and the number of measurements are equal for the even-determined case. The scenarios for the even-determined case considered here are:

Scenario 1: one measurement is assumed at node number 10 at time 2 h for the estimation of the HW coefficient of pipe 16.

Scenario 2: two measurements are assumed at node 6 at time 1 h and at node 8 at time 2 for the estimation of the HW coefficients of pipes 8 and 13.

The remaining three scenarios and the estimated HW coefficients for all scenarios are summarised in Table 5.

Over-determined case

Nine different scenarios of measurement locations and times to estimate HW coefficients of different pipes for the over-determined case are considered. It is important to note that the number of unknown HW coefficients is always less than the number of measurements for the over-determined case. The scenarios for the over-determined case considered here are:

Scenario 1: three measurements are assumed at nodes 6, 7, and 8 at the following times, respectively 2, 2 and 1 h, for the estimation of HW coefficients for pipes 1 and 6.

Table 2 | Node elevations and nodal demands at time 0 for network 1

Node number	Node elevation (m)	Nodal demand (l/s)
1	158.5	14.16
2	167.6	0.00
3	176.8	28.32
4	155.4	70.79
5	173.7	42.48
6	175.3	56.63
7	157.0	28.32
8	161.5	42.48
9	158.5	0.00
10	160.0	70.79

Table 3 | Demand factors for networks 1 and 2 for the simulation period

Time (h)	Demand factor
0	1.00
1	1.10
2	1.15
3	1.20
4	1.25
5	1.30
6	1.35
7	1.25
8	1.15
9	1.05
10	1.25
11	1.30
12	1.00

Scenario 2: same measurement locations and times as for scenario 1 for the estimation of HW coefficients for pipes 5 and 7. The remaining seven scenarios and the estimated HW coefficients for all scenarios are summarised in Table 5.

Under-determined case

Three different scenarios of measurement locations and times to estimate the HW coefficients for different pipes for the under-determined case are considered. It is important to note that the number of unknown HW coefficients is always more than the number of measurements for the under-determined case. The scenarios for the under-determined case considered here are:

Scenario 1: one measurement is assumed at node 5 at time 3 h for the estimation of the HW coefficients of pipes 4 and 5.

Scenario 2: one measurement is assumed at node 6 at time 2 h for the estimation of the HW coefficient of pipes 2 and 4.

Scenario 3: one measurement is assumed at node 7 at time 1 h for the estimation of the HW coefficient of pipes 8 and 10.

Scenarios 1, 2 and 3 and the estimated HW coefficients for these scenarios are summarised in Table 5.

NETWORK 2

Network 2, shown in Figure 2, consists of twenty four pipes, fourteen nodes and two constant head reservoirs. Pipe characteristics are given in Table 6. Node elevations and nodal demands at time zero are given in Table 7. Demand factors for the simulation period are given in Table 3. The water surface elevation for reservoir 1 is 320 m and for reservoir 2 is 326 m. Initial tracer concentrations at reservoirs 1 and 2 are 3.00 and 1.00 mg/l, respectively. The tracer decay rate coefficient for all pipes and for the two reservoirs is 0.15/h. The model was first run to determine tracer concentrations throughout the network for the simulation period. It was then run to determine HW coefficients for different scenarios for even-, over- and under-determined cases. Tracer concentrations as obtained by the model for the simulation period are given in Table 8. Seven even-

Table 4 | Unsteady state tracer concentrations (mg/l) for network 1

	Node number									
Time (h)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
1	2.714	2.712	1.986	2.152	2.711	2.712	2.151	2.642	2.147	2.423
2	2.456	2.454	1.746	1.896	2.453	2.454	1.895	2.384	1.891	2.167
3	2.222	2.220	1.561	1.696	2.219	2.221	1.695	2.154	1.692	1.951
4	2.011	2.009	1.397	1.519	2.008	2.010	1.519	1.947	1.516	1.757
5	1.819	1.818	1.252	1.362	1.817	1.818	1.362	1.760	1.359	1.584
6	1.646	1.645	1.123	1.222	1.644	1.645	1.222	1.592	1.220	1.428
7	1.489	1.488	1.007	1.098	1.488	1.489	1.097	1.439	1.095	1.288
8	1.348	1.347	0.927	1.009	1.346	1.347	1.009	1.304	1.007	1.173
9	1.219	1.219	0.857	0.931	1.218	1.219	0.930	1.182	0.929	1.071
10	1.103	1.103	0.795	0.863	1.102	1.103	0.862	1.072	0.860	0.979
11	0.998	0.998	0.687	0.748	0.997	0.998	0.747	0.966	0.746	0.869
12	0.903	0.903	0.616	0.671	0.902	0.903	0.671	0.874	0.669	0.784

determined scenarios, nine over-determined scenarios and five under-determined scenarios of measurement locations and times were considered for the determination of HW coefficients of the different pipes. These scenarios and the estimated HW coefficients calculated by the model are summarised in Table 9.

ERROR ANALYSIS

Tables 5 and 9 show that the model predicted the unknown HW coefficients accurately for some scenarios, while errors were produced for other scenarios for both networks. This section illustrates how errors are introduced into the estimated HW coefficients and an application is then given for networks 1 and 2. Let

$$c_n = c_n(H_{w1}, H_{w2}, \ldots, H_{wnuk}) \quad (27)$$

where *nuk* is the number of unknown HW coefficients. From Taylor series expansion

$$c_n(H_w + \Delta H_w) = c_n(H_w) + \frac{\partial c_n}{\partial H_w}\Delta H_w + \frac{\partial^2 c_n}{\partial H_w^2}\frac{\Delta H_w^2}{2!} + \ldots \quad (28)$$

By neglecting second- and higher-order terms, equation (28) reduces to

$$c_n(H_w + \Delta H_w) = c_n(H_w) + \frac{\partial c_n}{\partial H_w}\Delta H. \quad (29)$$

Solving (29) for ΔH_w yields

$$\Delta H_w = \frac{c_n(H_w + \Delta H_w) - c_n(H_w)}{\partial c_n / \partial H_w} \quad (30)$$

where ΔH_w is the error in the HW coefficient. The first term in the numerator of equation (30), $c_n(H_w + \Delta H_w)$, represents the tracer concentration at a specific node assuming an error

Table 5 | HW coefficients as determined by the model for even-, over- and under-determined cases for network 1

Case	Scenario no.	Measurement Node no.	Time (h)	Estimated HW coefficient
Even-determined	1	10	2	$H_{W16} = 100.0$
	2	6	1	$H_{W8} = 100.0$, $H_{W13} = 100.0$
		8	2	
	3	3	1	$H_{W3} = 100.0$, $H_{W4} = 100.0$, $H_{W11} = 100.0$
		4	1	
		7	2	
	4	8	1	$H_{W5} = 100.0$, $H_{W16} = 100.0$
		10	2	
	5	7	1	$H_{W11} = 100.0$, $H_{W14} = 100.0$
		9	2	
Over-determined	1	8	1	$H_{W1} = 100.0$, $H_{W6} = 100.0$
		7	2	
		6	2	
	2	8	1	$H_{W5} = 100.0$, $H_{W7} = 100.0$
		7	2	
		6	2	
	3	8	1	$H_{W5} = 100.0$, $H_{W8} = 100.0$
		9	2	
		10	3	
	4	5	2	$H_{W3} = 80.8$, $H_{W4} = 99.7$
		6	2	
		7	3	
		8	3	
	5	5	2	$H_{W4} = 91.6$, $H_{W8} = 82.3$
		6	2	
		8	3	

Table 5 | (*continued*)

		Measurement		
Case	**Scenario no.**	**Node no.**	**Time (h)**	**Estimated HW coefficient**
	6	4	1	$H_{W8} = 100.0$
		5	1	
		6	2	
		7	2	
	7	4	1	$H_{W7} = 66.5$, $H_{W8} = 105.0$
		5	1	
		6	2	
		7	2	
	8	5	1	$H_{W8} = 100.0$, $H_{W13} = 100.0$
		6	1	
		8	2	
		10	2	
	9	4	1	$H_{W4} = 100.0$, $H_{W8} = 100.0$
		5	1	
		6	2	
		7	2	
Under-determined	1	5	3	$H_{W4} = 106.0$, $H_{W5} = 92.5$
	2	6	2	$H_{W2} = 88.4$, $H_{W4} = 108.3$
	3	7	1	$H_{W8} = 92.6$, $H_{W10} = 105.4$

in the HW coefficient of ΔH_w. The second term in the numerator represents the tracer concentration assuming the HW coefficient free of error. So, the difference between the two terms represents the measurement error. Equation (30) shows the error in the estimated HW coefficient is directly proportional to the measurement error and inversely proportional to the sensitivity of the measurement node to the unknown HW coefficient.

As mentioned before, tracer concentrations obtained by the forward simulation are input to the model as measurements to determine the unknown HW coefficients. As no real measurements are made, measurement errors are simulated as follows:

(a) The tracer concentration obtained by forward simulation is considered error free when input to the model with up to six digits.

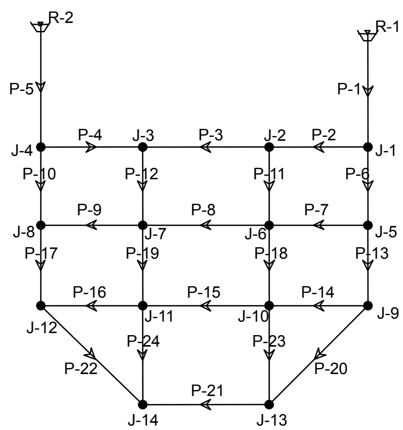

Figure 2 | Example network 2.

(b) When the tracer concentration is input to the model with less than six digits, the difference between the six-digit concentration and the input concentration is considered the measurement error.

Errors in the determined HW coefficients are calculated as follows:

(a) The HW coefficient is determined by inputting the tracer concentration with up to six digits.
(b) The HW coefficient is determined by inputting the tracer concentration with less than six digits, depending on the desired error to be simulated.
(c) The difference between the HW coefficient determined in steps a) and b) is the error in the calculated HW coefficient.

Figure 3 shows the relationship between measurement error/sensitivity and error in the estimated HW coefficient. This figure was generated by estimating the HW coefficient for different pipes (network 2) by assuming measurements at different nodes. This figure shows that the relationship between measurement error/sensitivity and error in the estimated HW coefficient is linear and the higher measurement error/sensitivity is also the higher the error in the estimated HW coefficient.

For the purpose of identifying what produces larger errors, measurement errors or nodal sensitivity, the model was run for three different scenarios (network 1). The first scenario is the calculation of the HW coefficient for pipe number 1 by assuming measurements at node number 4, which is a highly sensitive node to the HW coefficient of pipe number 1, since $(\partial c_{n4}/\partial H_{w1}) = 5667 \times 10^{-6}$; the second scenario is for pipe number 6 by assuming

Table 6 | Pipe characteristics for network 2

Pipe no.	Pipe length (m)	HW coefficient	Diameter (cm)
1	61.0	110.0	30.5
2	121.9	110.0	20.3
3	121.9	110.0	20.3
4	182.9	110.0	20.3
5	45.7	110.0	25.4
6	76.2	110.0	25.4
7	137.2	110.0	20.3
8	106.7	110.0	20.3
9	167.6	110.0	20.3
10	91.4	110.0	25.4
11	61.0	110.0	25.4
12	61.0	110.0	25.4
13	106.7	110.0	25.4
14	61.0	110.0	20.3
15	106.7	110.0	20.3
16	137.2	110.0	20.3
17	106.7	110.0	25.4
18	112.8	110.0	25.4
19	97.5	110.0	25.4
20	121.9	110.0	25.4
21	121.9	110.0	20.3
22	137.2	110.0	25.4
23	76.2	110.0	25.4
24	85.3	110.0	25.4

Table 7 | Node elevations and nodal demands at time 0 for network 2

Node number	Node elevation (m)	Nodal demand (l/s)
1	310.9	42.48
2	310.3	28.32
3	312.4	0.00
4	317.0	28.32
5	307.8	56.63
6	309.4	0.00
7	304.8	70.79
8	306.3	84.95
9	309.4	0.00
10	310.9	56.63
11	312.4	0.00
12	310.9	42.48
13	309.4	84.95
14	307.8	70.79

measurements at node number 4, which is moderately sensitive to the HW coefficient of pipe number 6, given that $(\partial c_{n4}/\partial H_{w6}) = 145 \times 10^{-6}$; and the third scenario is for pipe number 1 by assuming measurements at node number 1 which has low sensitivity to the HW coefficient of pipe number 1, because $(\partial c_{n1}/\partial H_{w1}) = 3 \times 10^{-6}$. The relationship between measurement errors and errors in the estimated HW coefficients for pipes number 1 and 6 in node 4, and 1 in node 1 are shown in Figures 4, 5 and 6, respectively. These figures show that the slope of the correlation line which represents the sensitivity of the error in the estimated HW coefficient to measurement errors is very high when measurements are made at very low sensitivity nodes to the unknown HW coefficient, inducing high errors in the estimated HW coefficient, even if measurement errors are as low as 0.000 01 mg/l (Figure 6). However, relatively accurate HW coefficients are obtained when measurements are made at highly sensitive nodes to the unknown HW coefficient, even when measurement errors are as high as 0.05 mg/l (Figure 4).

Figure 7 shows the sensitivity of node 2 (network 2) to the HW coefficient for different pipes (24 pipes) at $t = 1$ h. This figure shows that there are big differences in node 2 sensitivity to the HW coefficients of different pipes. This means that the HW coefficient can be accurately estimated for some pipes, such as pipes 4 and 5, by making the measurement at node 2 as the tracer concentration at node 2 is highly sensitive to the HW coefficients of pipes 4 and 5, while large errors are expected in the estimated HW coefficient for other pipes, such as pipes 23 and 24, by making the measurement at node 2 as the tracer concentration at node 2 is of very low sensitivity to the HW coefficients of pipes 23 and 24, as Figure 7 shows.

CONSERVATIVE VERSUS NON CONSERVATIVE TRACERS

An important question to answer is: which produces better HW coefficient estimates, the use of a conservative tracer or a non-conservative tracer? Figure 8 shows the sensitivity of tracer concentration at node 2 (network 2) to the HW coefficient of different pipes for conservative and non-conservative tracers. It is observed from this figure that conservative tracers produce larger sensitivity values. Thus it is expected that more accurate HW coefficients are estimated when conservative tracers are used. Another advantage for using conservative tracers is that the error that may result from the uncertainty in the tracer decay rate coefficient is eliminated.

SUMMARY AND CONCLUSIONS

A model to estimate the HW coefficients for pipe networks by measuring tracer concentrations at selected nodes is developed by using the inverse method. The model can

Table 8 | Unsteady state tracer concentrations (mg/l) for network 2

	Time (h)												
Node number	0	1	2	3	4	5	6	7	8	9	10	11	12
1	0.000	2.852	2.222	1.913	1.646	1.417	1.220	1.050	0.904	0.778	0.669	0.576	0.496
2	0.000	2.461	2.167	1.884	1.636	1.417	1.220	1.050	0.904	0.766	0.646	0.576	0.496
3	0.000	0.861	0.741	0.638	0.549	0.460	0.419	0.367	0.303	0.259	0.223	0.187	0.170
4	0.000	0.861	0.741	0.638	0.549	0.472	0.407	0.350	0.301	0.259	0.223	0.192	0.165
5	0.000	2.582	2.222	1.913	1.646	1.417	1.220	1.050	0.904	0.778	0.669	0.576	0.496
6	0.000	2.504	2.188	1.895	1.640	1.417	1.220	1.050	0.904	0.770	0.654	0.576	0.496
7	0.000	1.120	0.982	0.851	0.737	0.630	0.558	0.486	0.408	0.346	0.293	0.256	0.227
8	0.000	0.861	0.741	0.638	0.549	0.472	0.407	0.350	0.301	0.259	0.223	0.192	0.165
9	0.000	2.582	2.223	1.913	1.646	1.417	1.220	1.050	0.904	0.778	0.669	0.576	0.496
10	0.000	2.389	2.094	1.815	1.572	1.360	1.174	1.013	0.868	0.738	0.625	0.553	0.477
11	0.000	1.099	0.969	0.843	0.732	0.627	0.557	0.486	0.406	0.343	0.289	0.255	0.227
12	0.000	0.861	0.741	0.638	0.549	0.472	0.407	0.350	0.301	0.259	0.223	0.192	0.165
13	0.000	2.462	2.160	1.870	1.614	1.392	1.200	1.034	0.888	0.760	0.645	0.566	0.488
14	0.000	0.993	0.869	0.741	0.663	0.575	0.508	0.443	0.370	0.308	0.260	0.234	0.207

consider even-, over- and under-determined cases: the even-determined case is solved by minimising the absolute difference between measured and calculated tracer concentrations subject to the equations that govern the tracer concentration in the network; the over-determined case is solved by minimising the sum of the squared differences between measured and calculated concentrations subject to the equations that govern the tracer concentration in the network; while the under-determined case is solved by minimising the Euclidean length of the solution vector subject to the equations that govern the tracer concentration in the network. For the purpose of model verification, the model was applied to two example networks for even-, over- and under-determined cases. Different scenarios of unknown HW coefficient, measurement location and measurement time were considered. Results showed that the model made accurate estimates of the HW coefficient for both even- and over-determined cases. However, for some scenarios errors were introduced into the estimated HW coefficient. Results also showed that the under-determined case produced reasonable estimates of the unknown HW coefficients for most of the scenarios considered here. Error analysis showed that errors in the estimated HW coefficient depend on measurement error as well as on measurement location, which represent the sensitivity of the measurement node to the unknown HW coefficient. Results also showed that errors in the estimated HW coefficient are more sensitive to measurement location than to measurement errors. Errors are produced in the calculated HW coefficient even

Table 9 | HW coefficients as determined by the model for even-, over- and under-determined cases for network 2

		Measurement		
Case	**Scenario no.**	**Node no.**	**Time (h)**	**Estimated HW coefficient**
Even-determined	1	11	2	$H_{W12} = 110.0$, $H_{W19} = 110.0$
		7	2	
	2	14	1	$H_{W12} = 110.0$
	3	9	2	$H_{W6} = 110.2$
	4	10	2	$H_{W14} = 110.0$, $H_{W16} = 110.0$,
		11	1	
	5	6	2	$H_{W7} = 110.0$, $H_{W8} = 110.0$
		7	1	
	6	7	1	$H_{W9} = 110.0$
	7	2	1	$H_{W2} = 110.0$, $H_{W13} = 110.0$, $H_{W6} = 110.0$
		6	2	
		10	2	
Over-determined	1	5	1	$H_{W6} = 110.0$
		13	1	
	2	9	1	$H_{W13} = 110.0$, $H_{W14} = 110.0$
		13	1	
		10	2	
	3	2	1	$H_{W1} = 110.0$, $H_{W2} = 110.0$
		6	1	
		10	2	
		13	2	
	4	7	1	$H_{W4} = 92.0$, $H_{W19} = 91.7$
		11	2	
		14	2	
	5	8	1	$H_{W5} = 101.3$, $H_{W17} = 124.8$
		12	2	

Table 9 | (*continued*)

Case	Scenario no.	Measurement Node no.	Time (h)	Estimated HW coefficient
		14	2	
	6	6	1	$H_{W6} = 110.0$, $H_{W11} = 110$
		7	1	
		11	2	
		14	2	
	7	12	1	$H_{W17} = 84.4$, $H_{W10} = 115.5$
		11	1	
		10	2	
		13	2	
	8	14	1	$H_{W24} = 87.9$, $H_{W19} = 104.6$
		11	2	
		7	2	
	9	1	1	$H_{W13} = 117.9$, $H_{W20} = 101.3$,
		5	1	
		9	2	
		13	2	
Under-determined	1	13	1	$H_{W6} = 121.6$, $H_{W23} = 83.5$
	2	10	2	$H_{W1} = 118.1$, $H_{W6} = 96.8$
	3	12	1	$H_{W5} = 98.0$, $H_{W17} = 97.8$
	4	9	2	$H_{W6} = 118.5$, $H_{W1} = 98$, $H_{W13} = 108.1$
	5	11	1	$H_{W10} = 122.1$, $H_{W12} = 100.6$, $H_{W16} = 31$
		12	1	

when measurement errors are as low as 1×10^{-6} mg/l when they are obtained at very low sensitivity nodes. Relatively accurate HW coefficients are obtained when measurements are made at highly sensitive nodes, even when measurement errors are as high as 0.05 mg/l. Better results are expected when conservative tracers are used because these produce larger nodal sensitivities, in addition to the fact that the use of conservative tracers eliminates errors expected from the uncertainty in the decay rate coefficient of the tracer.

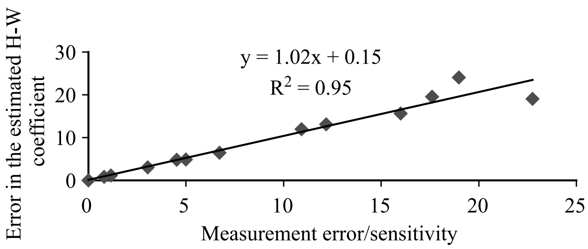

Figure 3 | Error in the estimated HW coefficient as a function of measurement error divided by sensitivity, network 2.

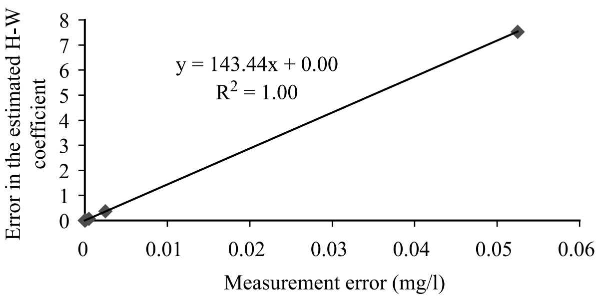

Figure 4 | Relationship between measurement error and error in the estimated HW coefficient for pipe number 1, network 1, with measurements made at node 4.

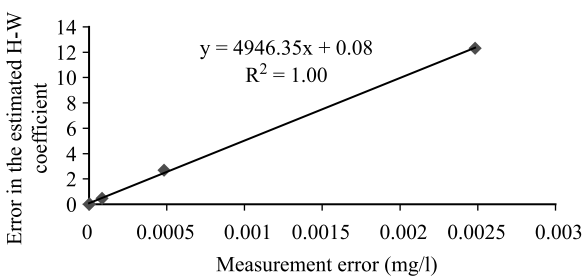

Figure 5 | Relationship between measurement error and error in the estimated HW coefficient for pipe number 6, network 1, with measurements made at node 4.

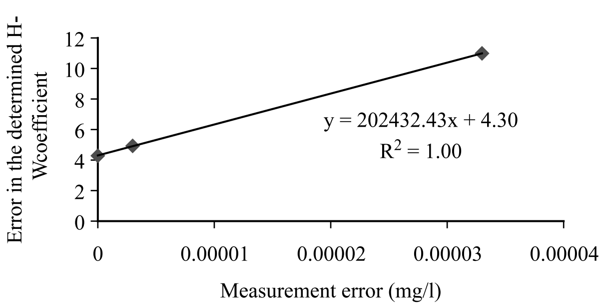

Figure 6 | Relationship between measurement error and error in the estimated HW coefficient for pipe number 1, network 1, with measurements made at node 1.

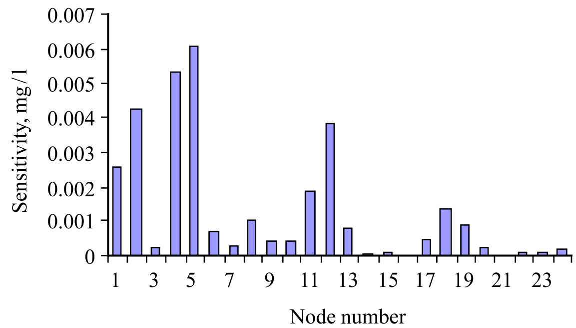

Figure 7 | Sensitivity of node 2 (network 2) to the HW coefficient for different pipes at $t = 1$ h.

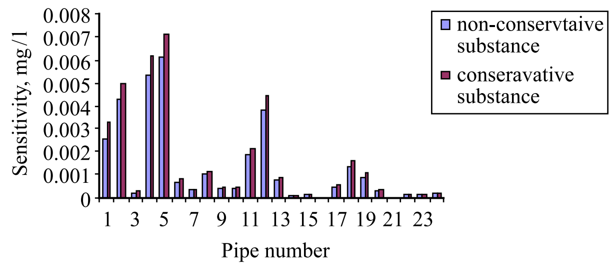

Figure 8 | Sensitivity of node 2 (network 2) to the HW coefficient of different pipes for conservative and non-conservative substances at $t = 1$ h.

The model presented here needs to be validated by applying it to real life networks where actual measurements are made at sensitive nodes and HW coefficients are estimated for a wide range of boundary and operational conditions.

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