Coordinating multiple model predictive controllers for the management of large-scale water systems

Abhay Anand, Stefano Galelli, Lakshminarayanan Samavedham and Sitanandam Sundaramoorthy

ABSTRACT

The optimal management of multi-purpose water reservoir networks is a challenging control problem, because of the simultaneous presence of multiple objectives, the uncertainties associated with the inflow processes and the several interactions between the subsystems. For such systems, model predictive control (MPC) is an attractive control strategy that can be implemented in both centralized and decentralized configurations. The latter is easy to implement and is characterized by reduced computational requirements, but its performance is sub-optimum. However, individual decentralized controllers can be coordinated and driven towards the performance of a centralized configuration. Coordination can be achieved through the communication of information between the subsystems, and the modification of the local control problems to ensure cooperation between the controllers. In this work the applicability of coordination algorithms for the operation of water reservoir networks is evaluated. The performance of the algorithms is evaluated through numerical simulation experiments on a quadruple tank system and a two reservoir water network. The analysis also includes a numerical study of the trade-off between the algorithms' computational burden and the different levels of cooperation. The results show the potential of the proposed approach, which could provide a viable alternative to traditional control methods in real-world applications.

Key words | aggregation-decomposition methods, coordination algorithms, model predictive control, quadruple tank system, water resources management

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INTRODUCTION

The growing world population is leading to a greater demand for depleting natural resources, including fresh water, and the current climate change scenario is making this worse. Droughts, storms and mismanagement of available water supplies are adversely affecting the drinking and irrigation water supply. Moreover, with sharply increasing oil prices, hydroelectricity is becoming a very lucrative alternative (Brown 2001). Water reservoirs are still being constructed worldwide to form integrated networks that can provide water for irrigation and drinking consumption and also be used to generate energy. These large multi-purpose reservoirs are generally spread across vast areas and are developed as systems of connected reservoirs. The management of these networks is a challenging task, because of their large dimensionality, the simultaneous presence of multiple and often conflicting water users, the nonlinearities in the model of the system and the uncertainties associated with the inflow processes (Castelletti *et al.* 2008).

The optimal operation of a reservoir network can be obtained by formulating an optimal control problem, whose resolution yields a control policy, namely a finite (periodic) set of control laws expressing the control actions as a function of the system's state. Among the different solution strategies developed in the last decades, dynamic programming (DP) and its stochastic version (i.e. SDP) are the most studied and adopted, as they can, in principle,

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provide the exact solution of the optimal control problem (for a review, see Labadie 2004 and references therein). However, the practical implementation of DP and SDP is limited by their computational complexity, which grows exponentially with the number of state, control and disturbance variables in the controlled system (the so-called 'curse of dimensionality'; Bellman 1957). As a consequence, DP and SDP can be adopted only for the optimal operation of small networks composed of few reservoirs. Moreover, the presence of different regulation authorities, each one governing one or a few reservoirs, can provide a strong resistance towards adopting a single, centralized policy, even when it is technically possible (Pianosi & Galelli 2010).

A viable approach to overcome this computational lock is based on the idea of simplifying the model of the controlled system by means of aggregation or decomposition techniques. In the former case, the subsystems are aggregated until a computationally feasible configuration is obtained, while in the latter case the system is decomposed into a tractable number of subsystems with a specific iterative procedure employed to solve the optimal control problem. One of the first contributions can be traced back to Turgeon (1981), who proposed an algorithm to decompose a *N*-reservoirs problem into *N* sub-problems, each one considering two reservoirs (i.e. one of the actual reservoirs plus an equivalent one accounting for the remaining storages), resulting in a total computing time that grows linearly with *N*. Archibald *et al.* (1997) proposed a similar aggregation-decomposition technique, with each subproblem presenting an actual reservoir and two equivalent storages (i.e. one for the upstream and one for the downstream part of the network), and the total computational burden reduced to a quadratic function of the state vector. Further developments of this approach can be found in Archibald *et al.* (1999, 2001). Other contributions in this field can be found in Saad *et al.* (1994), Ponnambalam & Adams (1996), and Mahootchi *et al.* (2010).

Unlike DP and SDP, less research effort has been spent by the water resources community on the development and application of aggregation-decomposition techniques for real-time control methods. With analogous intent this development was pursued in the process engineering community, where severe computational problems plague the control of large-scale process plants. In this case, real-time controllers (generally in the form of model predictive control, MPC) can be implemented in either a centralized or a decentralized fashion (Camacho & Bordons 2003; Allgower & Zheng 2000). In the former case, a single monolithic controller is employed to manage the entire network of interconnected subsystems. While centralized MPC leads to system wide optimality, it is computationally intensive, and relatively difficult to implement, tune and maintain. In the latter case, the MPC is implemented in a decentralized fashion with individual controllers defined for every subsystem (or a smaller network of subsystems). While this makes the controllers more flexible, reliable and easy to implement and maintain, it also leads to solutions that are not system wide optimal (Kariwala 2007). Over the past few years, coordination (or distributed) techniques were developed to address the shortcomings of both centralized and decentralized control methods (Camponogara *et al.* 2002), while combining their advantages (Rawlings & Stewart 2008): the decentralized structure of the system is maintained, but the performance is driven towards that of a centralized scheme. Coordination can be achieved through different approaches, such as game theory, sensitivity-based mechanisms and decomposition techniques (see, for example, Cheng *et al.* 2007; Alvarado *et al.* 2011; Maestre *et al.* 2011, and references therein).

Coordination techniques are developed and adopted in process engineering problems (see Rantzer (2009) and Scattolini (2009) for a theoretical survey, and Alvarado *et al.* (2011) for an overview of the performance of various coordination strategies using an experimental setup), but they have been poorly adopted in the water resources community. The use of coordination techniques is typically limited to the management of irrigation canals (Cardona *et al.* 1997; Negenborn *et al.* 2009a, b), while, to the authors' knowledge, only Niewiadomska-Szynkiewicz *et al.* (1996) have considered the problem of coordinating multiple real-time controllers for water reservoir networks operation.

With the purpose of exploring the potential of coordination techniques for the operation of reservoir networks, this paper analyses the performance of a communication-based and a cooperation-based coordination algorithm first introduced by Venkat *et al.* (2006). The algorithm coordinates the actions of the individual real-time controllers relying on the information exchange between the various individual controllers to account for the interaction effects existing between the different subsystems. The coordinator uses information such as states, predicted output and control trajectories to decide the best set of control actions for each individual controller. At the same time, the cost function and process model of each individual controller are modified to a common structure to enable communication and cooperation between the individual controllers. The algorithm is tested on two numerical case studies: a quadruple tank system often adopted in the process-engineering community (Johansson 2000) and a relatively simple network composed of two multi-purpose reservoirs in cascade. The rationale behind the selection of these case studies is that they allow evaluating the effectiveness of the proposed coordination algorithm in approximating the true solutions (obtained with a centralized approach), and this is key to performing a comparative evaluation of the adopted method. The algorithm performance is evaluated via numerical simulation experiments, and compared against the results obtained with the centralized and decentralized approaches, which represent the two extremes of cooperation. The analysis also comprises a numerical study of the trade-off between the cooperation algorithm computational burden and the different levels of cooperation that can be achieved.

The rest of the manuscript is organized as follows. In the next section, centralized and decentralized strategies are first formulated, and the coordination algorithm is then introduced. This forms the methodological framework that is employed in the subsequent case studies, and then the key conclusions and empirical evaluations derived from this work are summarized.

COORDINATING MULTIPLE MODEL PREDICTIVE CONTROLLERS

Problem formulation

MPC is a form of advanced process control widely adopted for the control of large-scale systems. At each decision time-step t, a real-time control problem is formulated on the basis of a prediction of the future disturbances and an internal process model describing the dynamics of the controlled system to optimize a set of control decisions over a finite horizon $[t, t+h]$ (with h being the length of the prediction horizon). Though a trajectory of control decisions over the entire prediction horizon is calculated, only the first control $\boldsymbol{u}_t$ is implemented. At the next time-step $t+1$, the optimization problem is re-formulated over the horizon $[t+1, t+h+1]$ based on the current states and a new set of available predictions (receding horizon principle). The MPC controllers can be implemented in either a centralized or a decentralized fashion. In a centralized strategy, a single monolithic controller is employed to manage the entire network of interconnected subsystems (Figure 1). On the other hand, the MPC can also be implemented in a completely decentralized fashion (Sandell *et al.* 1978) with individual controllers defined for every subsystem or a smaller network of subsystems in a flexible architecture (Figure 2). Centralized and decentralized controllers define the limiting extremes of a controller design.

For the present work we consider a centralized deterministic MPC problem of the following form over the finite

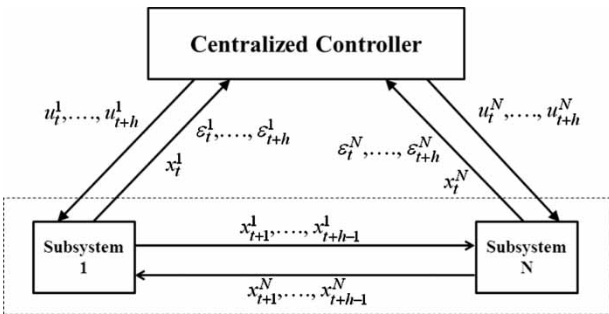

Figure 1 | Schematic of a centralized control configuration.

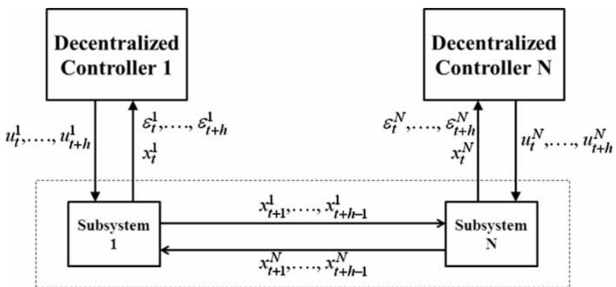

Figure 2 | Schematic of a decentralized control configuration.

horizon $[t, t + h]$:

$$\min_{\boldsymbol{u}_t,\ldots,\boldsymbol{u}_{t+h-1}} \left[\sum_{\tau=t}^{t+h-1} g_\tau(\boldsymbol{x}_\tau, \boldsymbol{u}_\tau, \varepsilon_{\tau+1}) + \bar{g}_{t+h}(\boldsymbol{x}_{t+h}) \right] \quad (1a)$$

subject to

$$\boldsymbol{x}_{\tau+1} = f(\boldsymbol{x}_\tau, \boldsymbol{u}_\tau, \varepsilon_{\tau+1}) \qquad \tau = t, \ldots, t+h-1 \quad (1b)$$

$$0 \le \boldsymbol{u}_\tau \le \boldsymbol{u}^{\max} \qquad \tau = t, \ldots, t+h-1 \quad (1c)$$

$$\Delta \boldsymbol{u}^{\min} \le \Delta \boldsymbol{u}_\tau \le \Delta \boldsymbol{u}^{\max} \qquad \tau = t, \ldots, t+h-1 \quad (1d)$$

$$\boldsymbol{x}^{\min} \le \boldsymbol{x}_\tau \le \boldsymbol{x}^{\max} \qquad \tau = t, \ldots, t+h-1 \quad (1e)$$

where $\boldsymbol{x}_\tau$ is the given state of the complete system and $f_\tau(\cdot)$ the corresponding state transition function (i.e. the process model), $\boldsymbol{u}_\tau$ the vector of control variables belonging to the finite set $[0, \boldsymbol{u}^{\max}]$, $\varepsilon_{\tau+1}$ is a given vector of the system disturbances, whose temporal evolution on the finite horizon $[t+1, t+h]$ is provided by a suitable dynamic predictor, $g_\tau(\cdot)$ the step-cost expressing the cost associated to the state transition from τ to $\tau+1$, and $\bar{g}_{t+h}(\cdot)$ a penalty function associated to the final state $\boldsymbol{x}_{t+h}$.

In most large-scale systems such a monolithic centralized controller cannot be implemented due to the issues described previously. In such cases, a decentralized control structure is adopted where a local MPC is designed for each individual subsystem (or a small group of subsystems). The individual MPC problem for the j-th subsystem is formulated as follows:

$$\min_{u^j_t,\ldots,u^j_{t+h-1}} \left[\sum_{\tau=t}^{t+h-1} g^j_\tau\left(x^j_\tau, u^j_\tau, \varepsilon^j_{\tau+1}\right) + \bar{g}^j_{t+h}\left(x^j_{t+h}\right) \right] \quad (2a)$$

subject to

$$x^j_{\tau+1} = f(x^j_\tau, u^j_\tau, \varepsilon^j_{\tau+1}) \qquad \tau = t, \ldots, t+h-1 \quad (2b)$$

$$0 \le u^j_\tau \le u^{\max,j} \qquad \tau = t, \ldots, t+h-1 \quad (2c)$$

$$\Delta u^{\min,j} \le \Delta u^j_\tau \le \Delta u^{\max,j} \qquad \tau = t, \ldots, t+h-1 \quad (2d)$$

$$x^{\min,j} \le x^j_\tau \le x^{\max,j} \qquad \tau = t, \ldots, t+h-1 \quad (2e)$$

where x^j_t is the given state of the j-th subsystem and $f_\tau(\cdot)$ the corresponding state transition function (i.e. the process model), u^j_τ the control variable belonging to the finite set $\left[0, u^{j,\max}\right]$, $\varepsilon^j_{\tau+1}$ a given vector of the system disturbances, whose temporal evolution on the finite horizon $[t+1, t+h]$ is provided by a suitable dynamic predictor, $g^j_\tau(\cdot)$ the step-cost expressing the cost associated to the state transition from τ to $\tau+1$, and $\bar{g}^j_{t+h}(\cdot)$ a penalty function associated to the final state x^j_{t+h}. The resolution of the optimization problem solved at each local controller is less computationally intensive than the centralized optimization problem, Equations (1a)–(1e). Also, such an architecture is more flexible, reliable and easy to implement and maintain. However, the decentralized controller results in solutions that are not optimum system-wide (Kariwala 2007).

With the need to control large-scale systems, a coordination algorithm driving the local controllers to the global optima is desired. This algorithm is implemented hierarchically in a layer lying above the real-time control one and working towards integrating the local real-time controllers at the lower level. The coordinator ensures that the goals of the higher level are attained and also manages the information flow within the immediate lower layer. It incorporates the goals derived by the layers above in its objective function and uses the information from the individual real-time controllers at the lower level to drive the whole system performance towards the overall optimum.

Communication and cooperation-based coordination

Early formulations of coordinated MPC are based on the assumption that the communication of information in the form of predicted trajectories and states is sufficient to account for the effects of the interactions between subsystems. It was then demonstrated by Venkat *et al.* (2006) that only the communication of interaction information among the subsystem controllers is not adequate to guarantee closed-loop stability. This instability arises due to the contest between the local controllers working with independent local objectives. To overcome this contest, the controllers need to cooperate with each other.

The cooperation between controllers can be achieved by modifying the objective functions of the local optimization problems, incorporating the interaction models into the local subsystem model. These principles form the basis of coordination strategies (see Venkat 2006). Hence, the main task of the coordinator is to provide information (such as state and predicted output trajectories, and calculated control actions at each time-step) to the local controllers, enabling them to derive interaction factors, and also to modify the local optimization problem such that the coordinated performance of the local optimization problems is driven towards the performance of the centralized global optimization problem (i.e. goal coordination; Mesarovic *et al.* 1970). This is achieved through the interaction prediction and interaction balance principles as postulated by Sadati (2005a, b). The former is based on the prediction and modification of the control variable values after accounting for the effect of subsystems' interaction effects, while the latter is based on the prediction of the correct values for the interaction variables, with the coordination outputs based on the error between the predicted and actual values of the interaction variables. The interaction balance principle includes the interaction variables in addition to the input variables in the manipulated variable set of the local controllers, with the coordinator working towards balancing the error between the desired (calculated) and real interaction variables. On the other hand, the interaction prediction principle considers only the input variables in the manipulated variable set, and then the coordinator works towards calculating the correct input variables after predicting and accounting for the effects of the interactions.

These two principles form the basis for introducing the communication-based and cooperation-based algorithms (Venkat *et al.* 2006). In the communication-based coordination strategy, subsystem controllers exchange interaction information at every time instant. Since an MPC optimization scheme is being employed, trajectories for the input variables as well as state evolutions are available at each time instant and this information is exchanged between the subsystem controllers through the coordinator. Each communication-based MPC transmits the current state and input trajectory information to all interconnected subsystems' MPCs through the coordinator as indicated by the following modified state transition equation:

$$x^j_{\tau+1} = f_\tau\left(\boldsymbol{x}_t, \boldsymbol{u}_t, \varepsilon^j_{\tau+1}\right) \tag{3}$$

It is seen that, though we have a decentralized control structure, each controller utilizes a model that incorporates the state and control vector $\boldsymbol{x}_t$ and $\boldsymbol{u}_t$ of every other subsystem. This modified state transition equation is used to explicitly account for the interaction effects. This means that, though the controllers communicate, each individual controller has no knowledge of the cost functions being utilized at each of the other local controllers. Since the objectives of each subsystem's MPC controller are frequently in conflict with the objectives of the other controllers, the equilibrium of such a control strategy is driven to a non-cooperative equilibrium or Nash equilibrium. Due to the non-cooperative and competing effect, such a strategy is usually suboptimal and when the interactions are strong, closed-loop stability is not guaranteed (Venkat 2006).

To overcome the drawbacks associated with communication-based coordination strategies, the cooperation-based coordination strategy works towards enabling the local controllers to support each other in driving the performance towards global optima. To achieve this, the local objective functions of each subsystem MPC controller are converted to a common global objective function. This is achieved by using a weighted convex sum of the individual local objective functions, as described below:

$$\min_{u^j_t,\ldots,u^j_{t+h-1}} \sum_{j=1}^{N} w_j \left[\sum_{\tau=t}^{t+h-1} g^j_\tau\left(x^j_\tau, u^j_\tau, \varepsilon^j_{\tau+1}\right) + \bar{g}^j_{t+h}\left(x^j_{t+h}\right)\right] \tag{4a}$$

where,

$$\sum_{j=1}^{N} w_j = 1, \quad \text{with} \quad w_j > 0 \tag{4b}$$

with w_j being the weight assigned to the j-th objective function (assigned heuristically based on process knowledge and system-dependent operating conditions). Since all the local MPC controllers are solving an optimization problem with the same objective function, the optimal control profile generated at all iterates of the cooperative based coordination is closed-loop stable or Pareto Optimal (see Venkat 2006).

To make the different controllers converge to the globally optimal centralized control policy $u_t^j, \ldots, u_{t+h-1}^j$, the coordinator employs a direct substitution algorithm, whose iterations proceed as follows:

1. At each time instance τ, the iteration begins by assuming that there is no interaction between the subsystems. Each j-th controller calculates its individual control trajectory $u_t^j, \ldots, u_{t+h-1}^j$;
2. Through the coordinator, the j-th subsystem receives the calculated control and state trajectories from all the other subsystems;
3. Based on the interaction information (effect of predicted trajectories calculated using Equation (3)), the j-th controller recalculates its individual control trajectory $u_t^j, \ldots, u_{t+h-1}^j$;

(a) Communication-based Coordination: the local MPC controllers solve the optimization problem defined by Equations (2a)–(2e) with the modified state transition equation defined by Equation (3);

(b) Cooperation-based Coordination: the local MPC controllers solve the optimization problem defined by Equations (4a)–(4b) with the modified state transition equation defined by Equation (3) and the constraints defined in Equations (2c)–(2e).

Steps 2 and 3 are repeated till convergence (convergence is guaranteed only if there are no linking constraints or the linking constraints set is inactive at convergence) or limited to a predefined maximum number of iterations.

A schematic view of the coordination structure, with the underlying layer of MPC controllers and information exchange, is given in Figure 3. The subsystem block has a disturbance predictor embedded in it and transmits this prediction information along with the states to the controller block. The communication-based and cooperation-based algorithm requires iterating the information exchange process, which, as demonstrated in Venkat (2006) and

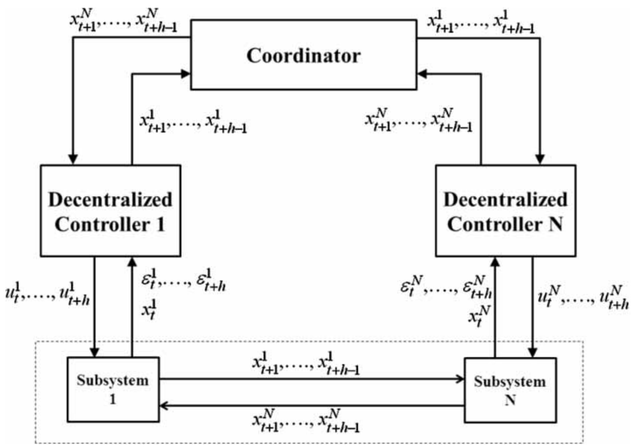

Figure 3 | Schematic of the communication- and coordination-based MPC configurations.

Venkat et al. (2006) and Stewart et al. (2011), is applicable to both linear and nonlinear systems.

QUADRUPLE TANK SYSTEM

System description

The quadruple tank system is a common benchmark employed in the process engineering community for evaluating the capabilities of control algorithms. The configuration adopted here is based on the work described by Mercangöz & Doyle (2007): the system is composed of four tanks and it is desired to control the water level in the two bottom tanks. The water level can be controlled by manipulating two pumps that are used to transport water from a water storage reservoir into the four overhead tanks. A bypass valve system is used to distribute the water between the lower level tanks (tanks 1 and 2) and the upper level tanks (tanks 3 and 4). The water in the upper level tanks drains into the lower level tanks through an orifice, and this setup is designed to introduce an interaction between the liquid level of both the tanks in the lower level. By adjusting the bypass valves, the proportion of water distributed between the tanks can be changed, and this in turn has a significant effect on the level of interaction between the lower level tanks. A schematic representation of the complete system is given in Figure 4.

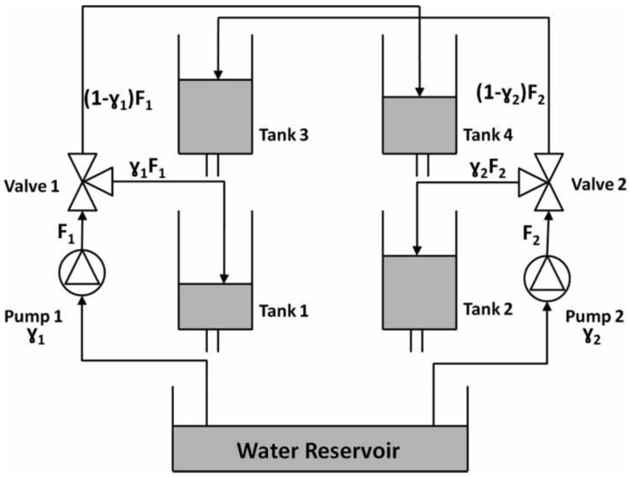

Figure 4 | Quadruple tank system.

Mass balance equations and Bernoulli's Law in the form of nonlinear differential equations are used to model the water levels in the tanks (Johansson 2000). The cross-sectional areas of the tanks and outlets are denoted with A_i and a_i, respectively, while the state vector contains the water level h in each tank. The position γ_i of the bypass valve represents the distribution of water between the tanks, while the control variables are voltages u_1 and u_2 supplied to the pumps that manipulate the pump flow rates (with $F_i = k_i u_i$) where k_i are the pump gains. The state transition function of each tank is thus represented as

$$\frac{\mathrm{d}h_1}{\mathrm{d}t} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_3}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1 \tag{5a}$$

$$\frac{\mathrm{d}h_2}{\mathrm{d}t} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_4}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2 \tag{5b}$$

$$\frac{\mathrm{d}h_3}{\mathrm{d}t} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2 \tag{5c}$$

$$\frac{\mathrm{d}h_4}{\mathrm{d}t} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \tag{5d}$$

The nonlinear differential Equations (5a)–(5d) are subsequently linearized about the set points of the two level control tanks and the state-space representation is derived as follows (with the parameters described in Table 1):

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ x_3(t+1) \\ x_4(t+1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{1}{T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{1}{T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{6a}$$

$$y_1 = x_1 \tag{6b}$$

$$y_2 = x_2 \tag{6c}$$

$$\frac{1}{T_i} = \frac{a_i}{A_i}\sqrt{\frac{g}{2h_i}} \tag{6d}$$

where x_1 and x_2 are the states (water levels) in the two controlled tanks and u_1 and u_2 are the control variables (pump voltages). The step costs are defined in terms of the deviations from setpoint and the utilization of pumps and are normalized. The constraints on the system are

Table 1 | Parameter values for the quadruple tank case study

Parameter	Value
A_1, A_3	28 cm^2
A_2, A_4	32 cm^2
a_1, a_3	0.071 cm^2
a_2, a_4	0.057 cm^2
g	981 $\mathrm{cm\,s^{-2}}$
k_1, k_2	3.33 $\mathrm{cm^3\,V^{-1}\,s^{-1}}$
h_1°, h_2°	12.5 cm
h_3°, h_4°	1.5 cm
u_1°, u_2°	3 V

as follows:

$$\left.\begin{array}{l} 0 \leq u_i \leq 5 \\ -1 \leq \Delta u_i \leq 1 \\ -5 \leq x_i \leq 5 \end{array}\right\} \quad i = 1, 2$$

The system is simulated under two different valve settings $\gamma_1 = \gamma_2 = 0.8$ and $\gamma_1 = \gamma_2 = 0.3$. These two settings are chosen because the level of interaction between the individual tanks significantly changes, having a severe effect on the system dynamics. Under the first configuration the interactions are not very severe (minimum phase behavior), but under the second configuration the interactions increase markedly (non-minimum phase behavior), making the system harder to control. The system was simulated under both these configurations by manipulating the position of the external bypass valve and the different controller coordination algorithms are compared. The set points of tank 1 and tank 2 were changed by 1 cm at sample times 10 and 100, respectively. Also, flow disturbances were introduced to the two upper level tanks at sample times 150 and 200, respectively.

Application results

With the purpose of exemplifying the nature of the coordination strategies and to bring out their uniqueness, the quadruple tank system was simulated in closed-loop with the different control strategies. The setpoint tracking and disturbance rejection performances of the algorithms are quantified as the total deviations from the setpoint. Equal weights were adopted for both the control objectives. The individual MPC controllers were tuned for the best performance and the prediction and control horizon was set to 24 and six time steps, respectively.

The performance of the control algorithms is compared by their ability to track the setpoint. The sum of squared errors (SSE) is used to quantify the deviations from the setpoint. Under minimum phase behavior, both communication-based and cooperation-based coordination algorithms are able to provide a closed-loop stable solution with performances better than a decentralized controller and also close to the centralized controller performance (Table 2 and Figure 5). Also, by increasing the number of iterations, the cooperation-based strategy asymptotically converges to the centralized controller performance. This is not the case when the valve positions are modified, and the system exhibits a non-minimum phase behavior: indeed, the controller performance deteriorates and the system is then harder to control (Table 3). A non-minimum phase behavior results in severe interactions between the subsystems, and only the cooperation-based coordination results in a closed-loop stable solution, while the communication-based one fails. The communication-based coordinator failure is due to the competition between the controllers that leads to an unstable Nash optimality. This result shows one of the major advantages of employing a coordinated control strategy: even in systems where a stable decentralized configuration is not feasible, a coordinated control strategy not only enables the utilization of a decentralized configuration, but also results in a performance similar to the centralized scheme.

The computational efforts were also compared and have been summarized in Tables 2 and 3. The computational effort is quantified as the average computational time required by the controller per decision step. It is seen that, as expected, the centralized control scheme, which solves a higher dimensionality optimization problem, requires significantly greater computational resources (evaluated using MATLAB 7.8.0.347 on an Intel Core 2 Duo 2.80 GHz processor). At the other end of the spectrum, the decentralized control scheme, where each individual controller solves an N-times smaller optimization problem, the computational demand is lower. The additional computational resources required to enable communication and cooperation among the individual controllers is not significantly higher and the overall computational demands of such a scheme are still significantly less than the centralized control scheme (Anand et al. 2012).

Table 2 | Performance indices for the quadruple tank case study

Control algorithm	Total SSE	Average computational time per controller (ms)
Centralized MPC	1.346	9.1
Decentralized MPC	1.460	3.68
Comm. based MPC	1.437	4.26
Coop. based MPC (2 iterations)	1.457	4.64
Coop. based MPC (10 iterations)	1.348	5.34

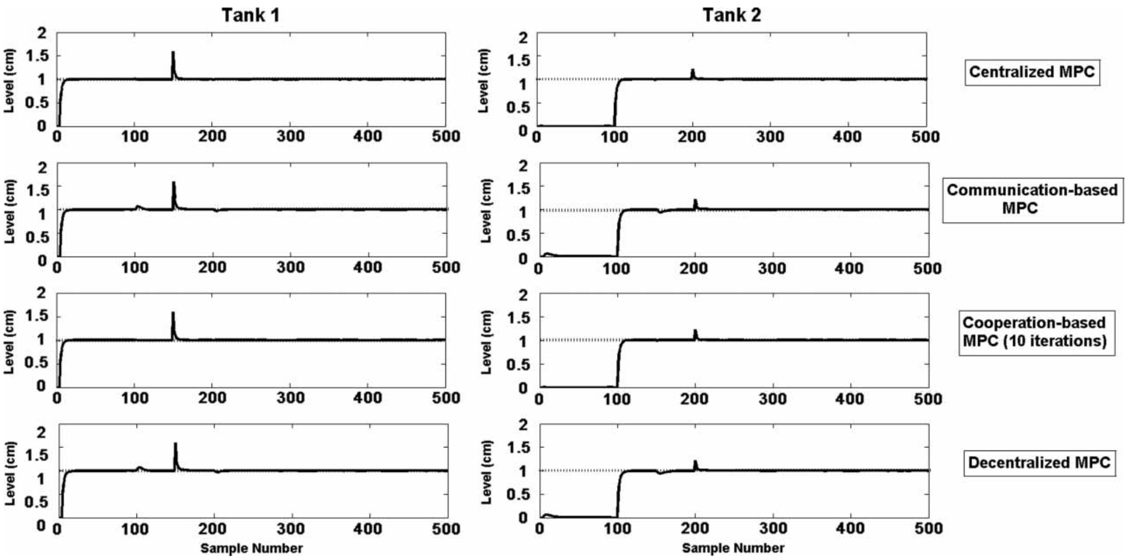

Figure 5 | Normalized level comparison (Quadruple tank system with $\gamma_1 = \gamma_2 = 0.8$). Setpoint (dotted line) initially 0 cm, is changed to 1 cm at sample number 10 (tank 1) and sample number 100 (tank 2).

TWO MULTI-PURPOSE RESERVOIRS NETWORK

System description

The water system considered is composed of two multi-purpose reservoirs in cascade, and it is developed from a single-reservoir system first presented in Castelletti *et al.* (2011). The dynamics of the upstream and downstream storage s_t^1 and s_t^2 [m^3] is modeled by means of the following mass balance equations:

$$s_{t+1}^1 = s_t^1 + (a_{t+1}^1 - u_t^1).\Delta \quad (7a)$$

$$s_{t+1}^2 = s_t^2 + (q_{t+1} - u_t^2).\Delta \quad (7b)$$

Table 3 | Performance indices for the quadruple tank case study

Control algorithm	Total SSE	Average computational time per controller (ms)
Centralized MPC	8.328	17.88
Decentralized MPC	Unstable	n/a
Comm. based MPC	Unstable	n/a
Coop. based MPC (2 iterations)	9.088	9.04
Coop. based MPC (10 iterations)	8.461	11.44

where u_t^1 and u_t^2 [m^3/s] are the release decisions (controls), both belonging to the interval [0, 60] m^3/s, and Δ is the integration time-step. In this particular formulation of the case study, there are no constraints on Δu and x. The reservoirs are assumed to be cylindrical with unit surface area.

The inflow a_{t+1}^1 [m^3/s] in the interval $[t, t+1]$ to the upstream reservoir is produced by an uncontrolled catchment whose behavior is modeled with a simplified Thomas-Fiering model (Loucks *et al.* 1981), namely

$$a_{t+1}^1 = \mu^1 + \rho_{\text{flow}}.(a_t^1 - \mu^1) + \sqrt{1 - \rho_{\text{flow}}^2}.(\mu^1 C_v \delta) \quad (8)$$

where the parameters are the mean μ^1, the coefficient of variation C_v and the correlation coefficient ρ_{flow}, respectively, equal to 40, 0.10 and 0.40, while δ is a standard normal random number. As for the downstream reservoir, the total inflow q_{t+1} [m^3/s] in the interval $[t, t+1]$ is given by the contribution a_{t+1}^2 of an uncontrolled catchment (generated with the same model of Equation (8), with the mean μ^2 equal to 20) and the release from the upstream reservoir. Notice that the Thomas-Fiering model allows accounting implicitly for the spatial correlation between the inflow processes in the whole system: in the present work this is obtained by adopting the same random number sequence (at each Monte Carlo generation; see next section) for both the upstream and downstream process.

The reservoirs are controlled with the purpose of satisfying two objectives each: hydropower generation and flood protection upstream, and irrigation supply and flood protection downstream. The step-costs associated to the upstream reservoir operation are the deficit of hydropower generation, i.e.

$$g_t^{1,u} = \max(4.36 - P_t, 0) \tag{9a}$$

where 4.36 kWh is the maximum energy generated in the time interval Δ and P_t is the energy production, which depends on the release u_t^1 and on the reservoir level h_t^1; and the squared deviation from the flooding threshold $\bar{h}^1 = 50$ m, i.e.

$$g_t^{2,u} = \max(h_t^1 - \bar{h}^1, 0)^2 \tag{9b}$$

The step-costs associated to the downstream reservoir are the squared deficit of irrigation supply, i.e.

$$g_t^{1,d} = \max(\bar{i} - u_t^2, 0)^2 \tag{10a}$$

where the demand $\bar{i}$ corresponds to 60 m^3/s; and the squared deviation from the flooding threshold $\bar{h}^2 = 50$ m, i.e.

$$g_t^{2,d} = \max(h_t^2 - \bar{h}^2, 0)^2 \tag{10b}$$

where h_t^2 is the reservoir level.

Application results

With the purpose of evaluating the algorithm performance under different, synthetic hydrological conditions, a Monte Carlo approach is adopted to generate 100 different combinations of initial storage conditions and inflow realizations, over a horizon of 100 time intervals. The value of the objective function is computed as the average, normalized value of the four step-costs (Equations (9)–(10)) over the simulation horizon, with the same weight adopted for all the control objectives. In the cooperation based coordinator, the weight w_i adopted on each of the subsystems is equal to 0.50, while a weight of 0.50 is internally adopted for each of the two local objectives at each reservoir.

In the first experiment we assume that a perfect prediction of the inflow process is available over a three-step prediction horizon; this means that the inflow predictions employed by the local MPC controllers are assumed to be known without any error. As shown in Table 4, the decentralized MPC performance is sub-optimal as compared with the centralized strategy, while the coordination algorithm is able to improve the performance of the controllers, driving them closer to the global optimum. The performance of the coordinated control algorithm improves with an increase in the number of iterations, which are limited to 50 in the present application. This was seen to significantly improve the decentralized controller performance, resulting in a performance very close to that of a centralized controller with an acceptable increase in computational cost.

With the purpose of assessing the robustness of the different control algorithms with respect to the system's disturbances (i.e. the uncontrolled inflow), the second experiment utilizes a mismatch between the actual disturbance realization and the one implemented in the MPC controllers. This mismatch (or non-perfect forecast) is obtained by means of a randomly generated noise (Sivapragasam *et al.* 2007). The performance, though degraded from the previous experiment, is seen to follow the same trends, with coordinated MPC improving the existing decentralized controller performance as shown in Table 5. It can be noticed that the centralized control strategy provides a worse performance than the decentralized

Table 4 | Performance indices for the two reservoirs network with a perfect inflow prediction

Control algorithm	Upstream cost	Downstream cost	Total cost
Centralized MPC	0.091	0.121	0.212
Comm Based MPC	Unstable	Unstable	Unstable
Coop. based MPC (5 iterations)	0.102	0.135	0.237
Coop. based MPC (50 iterations)	0.098	0.131	0.229
Decentralized MPC	0.007	0.328	0.335

Table 5 | Performance indices for the two reservoirs network with a non-perfect inflow prediction

Control algorithm	Upstream cost	Downstream cost	Total cost
Centralized MPC	0.102	0.130	0.232
Comm. based MPC	Unstable	Unstable	Unstable
Coop. based MPC (5 iterations)	0.108	0.141	0.249
Coop. based MPC (50 iterations)	0.105	0.136	0.241
Decentralized MPC	0.025	0.335	0.360

Table 6 | Comparison of the computational load for solving the two reservoirs network with different MPC configurations

Control algorithm	Average computational time per controller (s)
Centralized MPC	0.65
Comm. based MPC	n/a
Coop. based MPC (5 iterations)	0.25
Coop. based MPC (50 iterations)	0.31
Decentralized MPC	0.19

controller in the upstream reservoir. This is because the centralized one optimizes the total cost of all the reservoirs in the system, with the risk of not guaranteeing the best performance in each subsystem, while the decentralized strategy seeks only the local optimum. The centralized controller exploits the resources not employed by the upstream controller, improving the overall performance at the cost of the upstream reservoir. On the other hand, the decentralized controller ignores the effect of the upstream reservoir on the downstream reservoir by optimizing the upstream performance locally; a high cost is thus incurred at the downstream reservoir (Figure 6).

The average computational requirements of the various algorithms are shown in Table 6. Similar to the quadruple tanks system, the centralized control scheme requires a very high computational effort, whereas for the decentralized control scheme the computational demand is significantly reduced. The additional computational resources required for enabling communication and cooperation among the individual controllers is marginally higher than the decentralized control scheme and still much less compared with the centralized control scheme.

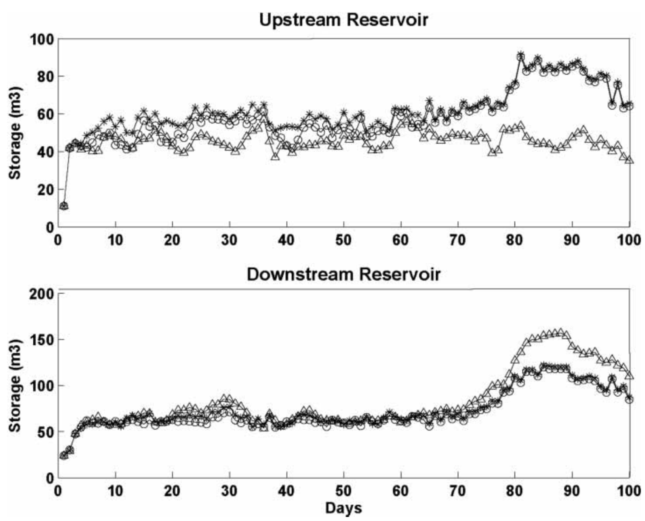

Figure 6 | Storage (m^3) comparison at the two reservoirs. Centralized MPC (-o-), decentralized MPC (-Δ-) and 50 iterations of cooperation-based MPC (-*-).

CONCLUSIONS

With the purpose of exploring the potential of coordination techniques for the management of large-scale water systems, this work provides a numerical evaluation of the communication-based and cooperation-based algorithms first proposed in Venkat *et al.* (2006) on two different case studies. The coordination of multiple MPCs is shown to significantly improve the performance of decentralized control strategies, driving them towards the control performance of a centralized controller. The exchange of information between the controllers (communication) is seen to be insufficient to guarantee closed-loop stability. To overcome this drawback, the objective functions of the local controllers have to be modified to enable the subsystems to cooperate. Cooperation-based coordination can asymptotically converge to the centralized controller performance (Anand *et al.* 2011) and provide a closed-loop stable solution at each iteration. The main advantage is that it can thus be stopped at any arbitrary iteration depending on the available computational resources and desired level of performance enhancement. Though the coordination of multiple controllers improves the closed-loop performances significantly, this comes at the cost of increased communications between the controllers and a higher computational effort than a completely decentralized control strategy. Through the numerical simulation studies it is observed that the increase in computational demand required for communication and cooperation is significantly lower than a centralized control strategy, making the application of such a strategy a very attractive prospect.

The system dynamics and the level of interaction between the subsystems are found to have a significant effect on the performance of the coordination algorithms. For example, in the quadruple tank system with non-minimum phase system behavior, the cooperation-based coordination strategy is found to be closed-loop. In other words, a good understanding of the system dynamics is necessary before choosing an appropriate coordination algorithm, especially for systems with multivariable process zeros, like the quadruple tank system. This will help control practitioners to select the best coordination algorithm based on *a priori* knowledge of the system behavior and the extent parametric uncertainties. As for the two reservoirs network, the coordinated control algorithm is able to significantly improve the controller performance in the upstream reservoir by explicitly accounting for the linking variables between the two reservoirs. It is also shown that the downstream reservoir contributes more to the overall costs, and the coordinated control strategy is able to improve the overall controller performances by compromising between the upstream and downstream costs to achieve the overall optimum.

In the current formulation of the coordination strategies, the key underlying assumption is that the interaction model between the different subsystems is linear, or that it can be linearized without an excessive loss of modeling accuracy. However, in the presence of strongly nonlinear interactions, this assumption does not hold and the algorithm cannot guarantee convergence. Since in most of the water reservoirs operation problems the interactions are usually nonlinear, the design and application of nonlinear coordinators is indispensible. Future research activities will thus concentrate on the development and testing of nonlinear coordinators, for which a mathematical framework was recently proposed by Stewart *et al.* (2011). Other relevant theoretical aspects requiring further investigation are an analysis of the minimum amount of communication load between the subsystems, and the robustness of coordinators to errors and losses in communication. Future efforts will also be devoted to the application of these coordination techniques to larger reservoir systems, which could also allow for a comparison against the decomposition-aggregation techniques generally adopted for DP and SDP problems.

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