where the $k_{\text{max}}$ refer to the maximum deflection constants of the cases used. From the program results, or by hand computation,

$$k_{\text{max}}(\text{variable load}) = 0.63442; k_{\text{max}}(\text{inner load}) = 0.15914;$$

$$k_{\text{max}}(\text{redundant load}) = 0.11126$$

(22)

If the proper values are inserted in equation (21), and the equation is solved for plate thickness, the result is

$$h = \left\{ \frac{(61.2^2)}{0.08(70.3 \times 10^4)} \left[ 0.63442(147) + 0.15914(1430) \right] \right. - 0.11126(2736) \}^{1/2}$$

(23)

$$= [1.0037]^{1/2} = 1.030 \text{ cm}$$

The radial and tangential moments are ascertained by superposition; viz,

$$\Sigma M_r = M_r(\text{variable load}) + M_r(\text{inner load}) - M_r(\text{redundant load})$$

$$= k_r(\text{variable load}) + k_r(\text{inner load}) W - k_r(\text{redundant load}) R;$$

(24)

$$\Sigma M_t = M_t(\text{variable load}) + M_t(\text{inner load}) - M_t(\text{redundant load})$$

$$= k_t(\text{variable load}) + k_t(\text{inner load}) W - k_t(\text{redundant load}) R$$

Fig. 14 depicts the superpositioning of the moments.

From Fig. 14, the maximum unit stress at the inner radius becomes

$$\sigma_{\text{max}} = \pm \frac{6\Sigma M_{\text{re}}}{h^2} = \pm \frac{6(99.7)}{(1.030)^2} = \pm 564 \text{ kg/cm}^2$$

(25)

**Discussion of Generalized Cases**

Cases VIII and X of the four generalized cases are partially depicted in reference [5] as Cases 59 and 60. The original derivations of the four generalized cases were attained by solving the moment equations and/or the integrated equations of the equilibrium equation with the imposed boundary and continuity conditions for the six unknown constants.

The author of [5] has indicated that the deflection formulas of Cases 59 and 60 should be modified. Modifications will be incorporated in later editions of [5].

**Acknowledgment**

Grateful acknowledgment is extended to J. Gvildys for his assistance in substantiating the author's derivations of the four generalized cases and for developing the moment-deflection program.

**References**


**DISCUSSION**

K. P. Coover

Mr. Heap has presented, in papers 67-WA/DE-1 and 67-WA/DE-2, a helpful coverage of different conditions of loading and support for circular plates. These papers essentially extend the number of cases given in Roark [5] for the same type of problem, and provide considerably more assistance for the design engineer.

The only point that I would like to clarify with respect to Mr. Heap's paper is his use of the term "statically indeterminate." This term is mentioned several times, and I believe it is inconsistent, viz:

All plates are statically indeterminate internally since there are more unknowns than equations of equilibrium. If, instead, the author is referring to external indeterminacy, then cases VIII and XV should be switched, Part II.

L. J. Schlink

The author must be congratulated for his thorough way of collecting, selecting, and extending the available data on the subject of circular plates, and for providing in both papers a convenient, usable form for use by design engineers.

Many times it is possible to attain a more nearly optimum design at lower cost through a precise mathematical analysis. Obviously this incurs an increase in engineering cost and underlines the advantages of having design tools which permit rapid and reliable computations.

The collection of formulas presented here also provides a simple means for answering other related questions. For instance, if the redundant reaction of the author's numerical example 1 in Part II were elastically supported, the problem could be analyzed by replacing equation (11) with the following relationship:

$$\Sigma w = R_t = w(\text{variable load}) - w(\text{redundant load})$$

where $R$ is the spring constant of the elastic support.

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Author's Closure

The primary intents of these papers are (a) to avoid continual reiteration of laborious derivations for generalized symmetrical loaded circular plates, (b) to provide a means for ready extension to other formulas and conditions, and (c) to bring the structural design and stress analysis into the realm of designers, engineers, and scientists unfamiliar with advanced mechanics of materials with a minimum of effort. From the reviewers' comments, these papers fulfill these intents.

Statically determinate and indeterminate circular plates have been dealt with in these papers without making a positive distinction between them. A statically determinate circular plate may be considered to be a rigid body, and the reactions and total or resultant stress on any section may be found from the equations of equilibrium alone; viz., Cases II, VI, VIII, XII, and XVI are statically determinate. A circular plate is said to be statically indeterminate when the number of unknown reactions or supports exceeds the number of conditions of equilibrium, as in Cases I, III, IV, V, VII, IX, X, XI, XIII, XIV, XV. The problems of the second paper deal with statically indeterminate plates having two supports; however, plates having three or more supports can be solved using an analogous method.

The author appreciates learning that additional usage can be made of these papers beyond that initially foreseen. For additional information and discussion covered by the reviewers, reference is made to the author's book, *Formulas for Circular Plates Subjected to Symmetrical Loads and Temperatures*, TID-23984, available from the Clearinghouse for Federal Scientific and Technical Information.