Wisps and knots in the central Crab nebula

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Accepted 1997 September 19. Received 1997 July 28; in original form 1997 May 9

ABSTRACT

The fast-spinning Crab pulsar (~30 turn s⁻¹), which powers the massive expansion and synchrotron emission of the entire Crab nebula, is surrounded by quasi-stationary features such as fibrous arc-like wisps and bright polar knots in the radial range of 2 × 10¹⁶ ≤ r ≤ 2 × 10¹⁷ cm, as revealed by high-resolution (~0.1 arcsec) images from the Wide Field and Planetary Camera 2 (WFPC2) on board the Hubble Space Telescope (HST). The spin-down energy flux (~5 × 10³⁸ erg s⁻¹) from the pulsar to the luminous outer nebula, which occupies the radial range 0.1 ≤ r ≤ 2 pc, is generally believed to be transported by a magnetized relativistic outflow of an electron–positron e⁻ pair plasma. It is then puzzling that mysterious structures like wisps and knots, although intrinsically dynamic in synchrotron emission, remain quasi-stationary on time-scales of a few days to a week in the relativistic pulsar wind. Here we demonstrate that, as a result of slightly inhomogeneous wind streams emanating from the rotating pulsar, fast magnetohydrodynamic (MHD) shock waves are expected to appear in the pulsar wind at relevant radial distances in the forms of wisps and knots. While forward fast MHD shocks move outward with a speed close to the speed of light c, reverse fast MHD shocks may appear quasi-stationary in space under appropriate conditions. In addition, Alfvénic fluctuations in the shocked magnetized pulsar wind can effectively scatter synchrotron beams from gyrating relativistic electrons and positrons.

Key words: MHD – relativity – stars: neutron – pulsars: general – ISM: individual: Crab – supernova remnants.

1 INTRODUCTION

Variations of light ripples, referred to as wisps in modern parlance (Scargle 1969), were first noticed in the underluminous inner region of the Crab nebula by Lampland (1921) more than 80 years ago. These wisp activities were rediscovered by Baade (Oort & Walraven 1956) in the early 1940s. It was boldly speculated (Oort & Walraven 1956) well ahead of time that a central active ‘star’, which is now known as the Crab pulsar (Pacini 1967; Gold 1968; Staelin & Reifenstein 1968; Cocke, Disney & Taylor 1969), must be held responsible for these wisp activities. Modern observations of wisps started with those of Scargle (1969) who reported, among other findings, violent wisp activities (Scargle & Harlan 1970; Scargle & Pacini 1971) several months after the first reported Crab pulsar glitch (Boynton et al. 1972). Over the years, wisps in the inner Crab nebula have been scrutinized in radio (e.g. Bietenholz & Kronberg 1992; Kronberg et al. 1993), infrared and optical (e.g. Van den Bergh & Pritchett 1989; Hester et al. 1995; Tanvir, Thomson & Tsikarishvili 1997) bands, and a similar radio wisp structure was also detected around the Vela pulsar (Bietenholz, Frail & Hankins 1991). Besides unprecedented high-resolution observations of wisps in the inner Crab nebula, the most remarkable result from WFPC2/HST reported by Hester et al. (1995) was the discovery of two brightened quasi-stationary knots south-east of the Crab pulsar, apparently aligned along the spin axis of the pulsar (parallel to a jet-like structure in X-ray images: cf. Hester et al. 1995, Aschenbach & Brinkmann 1975, Harnden & Seward 1984 and Pelling et al. 1987) with the closest knot being ~2.3 × 10¹⁷ cm away from the Crab pulsar.

Since the late 1960s, a theoretical paradigm has gradually emerged in which a magnetized relativistic pulsar wind consisting mainly of an electron–positron e⁻ pair plasma provides the vital energetic and dynamic linkage between the Crab pulsar and its brilliant supernova remnant (Michel 1969; Goldreich & Julian 1970; Rees & Gunn 1974; Arons
Gold 1968; Gunn & Ostriker 1971). While theoretical star with a misaligned dipole magnetic field (Pacini 1967; Gold 1968; Gunn & Ostriker 1971) must overwhelm the particle luminosity in the immediate environs \((r \lesssim 1.5 \times 10^8 \text{ cm})\) of the rapidly spinning neutron star by \(B_\parallel = A_\parallel / r^2, \Omega_r\) is the angular spin rate of the pulsar, the constant effective mass flux of the wind is \(A_{\|} = \text{const. N } \nu U_r r/c^2\) with \(T_\parallel\), \(N\) and \(\nu\) being the Lorentz factor of the bulk flow, the proper particle number density and the proper specific enthalpy, respectively, and \(r_\parallel\) is the radial location where \(U_r\) and the generalized radial Alfvén speed \(C_{A,\|} = B_{\parallel} / (4\pi A_{\|}/(rU_r))^{1/2}\) are equal. Note that both \(U_r\) and \(B_{\parallel}\) scale as \(\sim r^{-1}\) at large \(r\). Symptotically, \(U_r \sim c\) dominates \(U_r\), while \(B_{\parallel}\) overwhelms \(B_{\parallel}\) in the distant pulsar wind. It is straightforward to verify that by setting \(\theta = \pi/2\) and taking the limit of large \(r\), equations (2.1) and (2.2) are consistent with the previous results for an equatorial pulsar wind (Michel 1969; Goldreich & Julian 1970).

Given the above three-dimensional magnetized pulsar wind, one can study the propagation and advection of MHD waves in the relativistic pulsar wind. Conceptually, one readily perceives the relativistic generalization of Alfvénic fluctuations (Lou 1993a,b) and fast and slow MHD disturbances (Lou 1996) in non-relativistic magnetized stellar winds. Technically, an important aspect of this generalization is to include properly the effects of the displacement current and the bulk Lorentz factor \(\Gamma_r\) of the pulsar wind. Without resorting to lengthy calculations, the basic results may be succinctly summarized as follows. Alfvénic fluctuations are predominantly transverse and incompressible; they can propagate in either direction along magnetic field lines relative to the pulsar wind. Slow MHD fluctuations scarcely propagate relative to the wind because of the dominance of \(B_{\parallel}\) at large \(r\); they are basically advected outward by the pulsar wind. Fast MHD fluctuations are compressible and can lead to shock formation as a result of steepening; they can propagate in both forward and reverse directions relative to the pulsar wind (that is, perpandicular to the toroidail magnetic field \(B_{\parallel}\)).

The emphasis here is on large-scale fast MHD disturbances in the pulsar wind. In particular, reverse fast MHD disturbances, under appropriate conditions, may propagate with a speed close to \(c\) relative to the wind such that the resulting spatial structures appear quasi-stationary in space (Hester et al. 1995). Meanwhile, fast forward MHD disturbances move outward in space (Tanvir et al. 1997) with a speed very close to \(c^2\). The fast MHD wave speed \(C_r\) relative to the pulsar wind is given by \(C_r = \left(C_A^2 + C_s^2\right)^{1/2}/(1 + C_s^2/c^2)\) where \(C_s\) is the sound speed, the \(C_s^2/c^2\) term results from the displacement current perturbation and \(C_A = B_{\parallel} / [4\pi A_{\|}/(rU_r)]^{1/2}\) is the generalized azimuthal Alfvén speed. At large radii, \(C_A\) can approach a constant value much greater than \(c\) and thus \(C_s\), and hence \(C_r\) can be very close to \(c\). In an inertial reference frame, the quasi-stationarity criterion for reverse fast MHD disturbances is simply \(U_r \lesssim \Gamma_r C_r\). In terms of parameters of a pulsar and its wind, this criterion can be readily translated to

\[
U_r \Gamma_r \lesssim \left(\frac{A_\parallel \Omega_r^2 \sin^2 \theta}{4\pi m_e A_{\|}}\right)^{1/3},
\]

approximately, where \(U_r\) represents the radial pulsar wind speed profile, \(A_\parallel\) is the constant radial magnetic flux related to the radial magnetic field \(B_{\parallel}\) by \(B_{\parallel} = A_{\|} / r^2, \Omega_r\) is the angular spin rate of the pulsar, the constant effective mass flux of the wind is \(A_{\|} = \text{const. N } \nu U_r r/c^2\) with \(T_\parallel\), \(N\) and \(\nu\) being the Lorentz factor of the bulk flow, the proper particle number density and the proper specific enthalpy, respectively, and \(r_\parallel\) is the radial location where \(U_r\) and the generalized radial Alfvén speed \(C_{A,\|} = B_{\parallel} / (4\pi A_{\|}/(rU_r))^{1/2}\) are equal. Note that both \(U_r\) and \(B_{\parallel}\) scale as \(\sim r^{-1}\) at large \(r\). Symptotically, \(U_r \sim c\) dominates \(U_r\), while \(B_{\parallel}\) overwhelms \(B_{\parallel}\) in the distant pulsar wind. It is straightforward to verify that by setting \(\theta = \pi/2\) and taking the limit of large \(r\), equations (2.1) and (2.2) are consistent with the previous results for an equatorial pulsar wind (Michel 1969; Goldreich & Julian 1970).

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in observations, the projection on to the plane of the sky can lead to apparently lower speeds than \(c\) for outward-moving features in the pulsar wind.
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where $m_e$ is the electron mass and $A_n = \frac{\Gamma_n N U_n r^2}{c}$ is the constant $e^-$ particle injection rate into the pulsar wind. For the Crab pulsar and its wind, we take $A_n \sim 10^{18} \text{G} \text{cm}^2$; $\Omega_p \sim 190 \text{ rad s}^{-1}$, $m_e \sim 9 \times 10^{-30} \text{g}$, $U_n \sim 3 \times 10^{10} \text{ cm s}^{-1}$ and $A_n \sim 10^{18} \text{ cm}^{-2}$. Around the spin equatorial wind zone with $\theta = \pi/2$, $T_{\text{eq}} \sim 2 \times 10^4$ by equation (2.3). With these estimates, it is quite plausible that reverse fast MHD shocks may appear quasi-stationary in the pulsar wind zone which occupies the central Crab nebula.

In order to relate spatial structures of reverse fast MHD disturbances to observed wisps and knots, it is also important to identify the origin of disturbances in the pulsar wind and explain why wisps and knots appear where they are observed (Scargle 1969; Van den Bergh & Pritchet 1989; Bietenholz & Kronberg 1992; Hester et al. 1995; Tanvir et al. 1997). The most likely source of fluctuations can be attributed to slightly inhomogeneous wind streams emanating from the rapidly spinning pulsar. This is not entirely implausible; for example, successive radio pulses from a pulsar vary in their shapes, even though the average radio pulse shape is quite stable and unique for a given pulsar (Manchester & Taylor 1977). In our scenario, a slower wind stream will be eventually caught up by a faster wind stream to trigger forward and reverse fast MHD shock waves in the distant pulsar wind. The initial radial speed difference $\Delta U$ of wind streams determines the rough radial distance $R_1$ from the pulsar where forward and reverse fast MHD shocks are initiated; the larger $\Delta U$ is, the closer the fast MHD shocks appear relative to the central pulsar. To be specific, $R_1 \sim U_1 U_2 \Delta t (U_2 - U_1)$ where $U_2$ and $U_1$ are the radial speeds of faster and slower streams, respectively, and $\Delta t$ is the time interval between two successive streams. For several wisps surrounding the Crab pulsar (Scargle 1969; Van den Bergh & Pritchet 1989; Bietenholz & Kronberg 1992; Hester et al. 1995; Tanvir et al. 1997), $R_1 \sim 10^{17} \text{ cm}$, $U_2 \sim U_1 \sim 3.0 \times 10^{10} \text{ cm s}^{-1}$, $\Delta t - 16 \text{ ms}$ is taken to be half of the spin period: it follows that $\Delta U \approx U_2 - U_1 \sim 1.4 \times 10^5 \text{ cm s}^{-1}$ near the pulsar. For the closest polar knot (Hester et al. 1995), $R_1 \sim 2.3 \times 10^{16} \text{ cm}$ and thus $\Delta U \approx U_2 - U_1 \sim 6.3 \times 10^{10} \text{ cm s}^{-1}$ around the polar region of the pulsar. Since these speed differences in streams are extremely small compared with the pulsar wind speed $U_2 \approx c$, the relativistic Crab pulsar wind must be regarded as remarkably homogeneous (e.g. Kennel & Coroniti 1984a), yet a slight $\Delta U$ can lead to the formation of various MHD shock structures in the distant pulsar wind.

In reality, the Crab pulsar wind may well consist of several streams with slightly different speeds during one spin period at various rotational latitudes. After multiple stream interactions, several relativistic MHD shocks can appear at different radial distances in the wind and at different rotational latitudes. For example, the two conspicuous polar knots (Hester et al. 1995) may actually result from three successive fast MHD shocks surrounding the polar region with slightly different speeds $U_2$, $U_3$, and $U_4$ such that $U_3 > U_2 > U_1$. The first knot appears at a closer distance where $U_2$ catches up $U_1$, and the second knot appears further away where $U_3$ catches up the remnant stream after the first knot. One can apply similar reasoning to account for the existence of several wisps in the pulsar wind, except that the relevant latitude is considerably away from the spin axis of the pulsar. Also, the shock structures can appear more extended along the toroidal magnetic field lines.

Synchrotron emissions result from relativistic charged particles gyrating in a magnetic field and thus require a continuous supply of extremely relativistic electrons and positrons, as lighter particles lose their energies more rapidly. In the present context, the original source of $e^-$ pair plasma resides in the close vicinity of the strongly magnetized neutron star. The intense electric field associated with a fast-spinning magnetized pulsar (Goldreich & Julian 1969) provides a persistent boost to accelerate electrons and positrons, as well as any species of heavier ions present near the pulsar, into the relativistic energy range. Collectively, these charged particles give rise to a bulk relativistic outflow; individually, all particles gyrate wildly in a strongly magnetized environment. Out into the pulsar wind zone, charged particles can be further energized by electromagnetic plasma turbulence/waves, forward and reverse fast relativistic M HD shocks and magnetic reconnections. As a result of these acceleration processes in the distant pulsar wind, the most energetic electrons and positrons can be continuously injected into the outer part of the Crab nebula (0.1 $\leq r \leq$ 2 pc) to power synchrotron emissions there. Likewise, heavier ions can be accelerated to the relativistic energy range by the same processes in the pulsar wind and thus become part of cosmic rays that populate our Galaxy.

3 DISCUSSION

Most arguments presented above, although devised for an aligned pulsar, are applicable to the more realistic misaligned pulsar wind model. Several important consequences of a misaligned model are noted below. First, the system is then intrinsically unsteady; electromagnetic waves (with $\sim$ 30-Hz frequency) and the pulsar wind are mixed, and a pulsar wind with a slightly inhomogeneous speed distribution is even more plausible. Secondly, one expects a wavy current sheet with a radial wavelength $\sim 10^5 \text{ cm}$ floating around the spin equator in the pulsar wind with alternating magnetic stripes (Kennel & Coroniti 1984a,b; Coroniti 1990; Michel 1994) the widths of which are $\sim 5 \times 10^5 \text{ cm}$. The swing amplitude in latitude of this current sheet is roughly proportional to the angle between the spin and magnetic axes of the pulsar (Hester et al. 1995). Thirdly, in the presence of fast MHD shocks within the latitude range of the wavy current sheet, magnetic reconnection proceeds to convert more efficiently the magnetic energy carried by the wind into $e^-$ plasma heating and particle acceleration (Coroniti 1990; Michel 1994). Meanwhile, enhanced synchrotron emission is expected as a result of pitch-angle scattering of MHD shock-energized relativistic particles.

For the microphysics of energy distributions of relativistic particles associated with magnetized collisionless shocks, we note the following. Enhanced and sustained non-thermal emissions from dynamic wisps and knots in the inner Crab nebula derive their energy from the low-entropy magnetized pulsar wind through relativistic shocks, concurrent particle acceleration, as well as thermalization therein, and a certain degree of isotropization of particle distribution functions [through, e.g., Weibel-type instabilities (Weibel 1959)]. Furthermore, a somewhat isotropized, power-law-
like energy distribution of the $e^-$ thus produced holds the promise of explaining several macroscopic properties of the brilliant outer Crab nebula (cf. Kenzel & Coroniti 1984a,b; Arons & Tavani 1994). Several investigations (Langdon, Arons & Max 1988; Hoshino & Arons 1991; Hoshino et al. 1992; Gallant et al. 1992; Gallant & Arons 1994) have shown that while the $e^-$ are quickly 'thermalized' with relativistic M axwellian distributions downstream of a perpendicular magnetosonic shock in the case of a pure $e^-$ pair plasma, the energy distribution of positrons can be sustained as power-law-like in the high-energy range in an appropriate mixture of $e^-$ with heavy ions (protons). It remains a challenge to derive two-dimensional numerical results in the high $m_i/m_e$ mass ratio regime in order to assess quantitatively the efficacy of this synchrotron masing mechanism.

The facts that several wisps appear to circle around the spin axis (Scargle 1969; Van den Bergh & Pritchet 1989; Bietenholz & Kronberg 1992; Hester et al. 1995; Tanvir et al. 1997), that magnetic fields as inferred from polarization studies (Bietenholz & Kronberg 1992) tend to orient along wisps, and that fresh relativistic particle accelerations as suggested by spectral radio observations (Bietenholz & Kronberg 1992; Kronberg et al. 1993) appear in wisp regions are all in accordance with the above scenario. The visual impression that fibrous wisps become gradually incomplete in the direction perpendicular to the projected spin axis within the plane of the sky can be explained by the azimuthal magnetic field pointing in and out of the plane of the sky such that detectable synchrotron emissions become weakest along the magnetic field direction.

In contrast to wisps at lower rotational latitudes (Hester et al. 1995; Tanvir et al. 1997), reverse fast MHD shocks associated with the polar knots do not involve magnetic reconnections. For a more dipole-like distribution of magnetic flux over the surface of the pulsar, a helical magnetic flux rope entrained in the polar wind is expected to surround the spin axis. Fast MHD waves, before steepening into shocks, can be viewed as dynamic processes of adjusting uneven stresses among magnetic helices. Synchrotron polarizations from the knots are aligned with the spin axis, which is consistent with a helical magnetic field configuration (Hester et al. 1995). Synchrotron intensities of counter knots or jets are weaker, probably as a result of relativistic boosting and beaming effects.

To understand physically synchrotron emissions from quasi-stationary reverse fast MHD shocks in the pulsar wind, it is crucial to distinguish the Lorentz factors of the bulk flow $\Gamma_B$ and of individual $e^-$ particles $\Gamma_p$, especially in the presence of a strong toroidal magnetic field $B_T$. For those gyrating particles that are non-relativistic in the comoving reference frame of the relativistic pulsar wind, their trajectories appear more or less straight in the sidereal reference frame. For those gyrating particles that are relativistic in the comoving reference frame of the pulsar wind with $\Gamma_p > \Gamma_B$, their trajectories appear as a series of stretched loops in the sidereal reference frame. In the former case, synchrotron emissions are more or less beamed in the forward direction of the bulk flow with an angular spread of the order of $\sim 1/\Gamma_B$. In the latter case, the synchrotron beam ahead of a relativistic particle tangentially follows the stretched-loop path with the forward beam width much smaller than the backward beam width. For synchrotron structures to be seen in a receding pulsar wind with a toroidal magnetic field, there must exist a considerable population of $e^-$ with $\Gamma_p > \Gamma_B$.

In relation to synchrotron appearances of wisps and knots, several factors may actually come into play. First, Alfvénic fluctuations (Lou 1993a,b; Kronberg et al. 1993) are likely to pervade a misaligned global pulsar wind so that toroidal magnetic field lines in the wind are wiggling about their means all the time. This effect inevitably leads to scattering of synchrotron beams from relativistic particles with $\Gamma_p > \Gamma_B$ from different portions of randomly fluctuating toroidal magnetic field lines. Secondly, fast relativistic MHD shocks in the pulsar wind tend to randomize pitch angles of $e^-$ pair particles (e.g. Weibel 1959), also causing synchrotron beam scattering. Thirdly, a low azimuthal bulk speed $u_B$ at large radii as given by equation (2.1) refers to the mean and should not imply that individual $e^-$ pair particles cannot travel at relativistic speeds along wavy magnetic field lines. These effects together give rise to synchrotron appearances of wisps and knots, that is, a series of brightened arcs and blobs accumulated at the radial locations of quasi-stationary reverse fast relativistic MHD shocks in the pulsar wind. Meanwhile, forward fast relativistic MHD shocks together with slow MHD disturbances propagate outward (Tanvir et al. 1997) to produce ever-changing yet diffuse variations in synchrotron brightness. Naturally, the synchrotron brightnesses of these MHD shock structures fluctuate as the successive quasi-periodic disturbances arrive on spots.

**ACKNOWLEDGMENTS**

This research was supported by grants from the US NSF and NASA to the University of Chicago.

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