Can the turbulent galactic dynamo generate large-scale magnetic fields?

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ABSTRACT
Large-scale magnetic fields in galaxies are thought to be generated by a turbulent dynamo. However, the same turbulence also leads to a small-scale dynamo which generates magnetic noise at a more rapid rate. The efficiency of the large-scale dynamo depends on how this noise saturates. We examine this issue, taking into account ambipolar drift, which obtains in a galaxy with significant neutral gas. We argue as follows.

(i) The small-scale dynamo generated field does not fill the volume, but is concentrated into intermittent rope-like structures. The flux ropes are curved on the turbulent eddy scales. Their thickness is set by the diffusive scale determined by the effective ambipolar diffusion.

(ii) For a largely neutral galactic gas, the small-scale dynamo saturates, as a result of inefficient random stretching, when the peak field in a flux rope has grown to a few times the equipartition value.

(iii) The average energy density in the saturated small-scale field is subequipartition, since it does not fill the volume.

(iv) Such fields neither drain significant energy from the turbulence nor convert eddy motion of the turbulence on the outer scale into wave-like motion. The diffusive effects needed for the large-scale dynamo operation are then preserved until the large-scale field itself grows to near equipartition levels.

Key words: magnetic fields – turbulence – ISM: magnetic fields – galaxies: magnetic fields – cosmology: miscellaneous.

1 INTRODUCTION
The origin of ordered, large-scale galactic magnetic fields remains a challenging problem. Magnetic fields in galaxies have strengths of order $10^{-6}$ G, and are coherent on scales of several kpc (Beck et al. 1996). These fields can arise, in principle, by dynamo generation from a weak, but non-zero, seed field $\sim 10^{-19} - 10^{-20}$ G, if the galactic dynamo can operate efficiently enough to amplify the field exponentially by $\sim 30 - 40$ e-foldings (cf. Moffat 1978; Parker 1979; Zeldovich, Uznaiskin & Sokoloff 1983). The capacity of presently known turbulent dynamo mechanisms to produce the observed galactic fields has been debated, however (Cattaneo & Vainshtein 1991; Vainshtein & Rosner 1991; Ulsrud & Anderson 1992; Brandenberg 1994).

It has been argued that magnetic noise in the form of small-scale fields builds up much faster than the mean field in a turbulent flow. Magnetic noise can result from the tangling of the large-scale field by turbulence or the operation of a small-scale dynamo. The dominant source, when one starts from weak large-scale seed fields, is the operation of a small-scale dynamo. Turbulence with a large enough magnetic Reynolds number (M R N), even when mirror-symmetric on average, generically leads to an exponential growth of fields correlated on the turbulent eddy scales, independently of the large-scale field. This growth occurs on the turbulent eddy turnover time-scale, which is much smaller than the time-scale for the growth of the mean field. The kinematic dynamo paradigm will therefore become invalid long before the large-scale field has grown.
Galaxies have a significant neutral gaseous component. As magnetic fields grow, the Lorentz force on the charged component will cause a slippage between it and the neutrals. Its magnitude is determined by the friction between the components caused by ion-neutral collisions. This drift, called ambipolar drift (Mestel & Spitzer 1956; Spitzer 1978; Draine 1980, 1986; Zweibel 1986), causes important non-linear feedback on both the small- and the large-scale dynamos by the generated magnetic field. In another paper (Subramanian 1997, hereafter Paper II), we give a derivation of the equations for both the mean field and the magnetic correlations, incorporating the effects of ambipolar drift. We also give there the solution of these equations in several contexts. Some pertinent results of this work will be quoted here, where needed. We will see below that the presence of neutrals may be very important in leading to a saturated state for the small-scale dynamo that preserves large-scale dynamo action.

We begin in Section 2 by introducing the turbulent galactic dynamo. In Section 3, we summarize the properties of the kinematic small-scale dynamo action in Kolmogorov-type turbulence and point out the problem that they raise. The influence of ambipolar drift on the small-scale dynamo is considered in Section 4. Section 5 considers back reaction effects and saturation of the small-scale dynamo due to the Lorentz forces acting on the fluid as a whole. The last section contains a discussion and summary of our results. We argue that, in galaxies, magnetic noise generated by the small-scale dynamo may indeed saturate in a fashion that preserves large-scale dynamo action.

2 THE TURBULENT GALACTIC DYNAMO

Spiral galaxies are differentially rotating systems. The magnetic flux is to a large extent frozen into the fluid and so any radial component of the magnetic field will be efficiently wound up and amplified to produce a toroidal component of the field. This only results in a linear amplification of the field, however; to obtain the observed galactic fields starting from small seed fields we must find a way to generate the radial components of the field in the galaxy from the toroidal one. If this can be done, the field can grow exponentially and one has a dynamo.

The standard picture involves the effects of cycloidal turbulence in the galactic gas. The galactic interstellar medium is assumed to be turbulent, as a result of, for example, the effects of supernovae randomly exploding in different regions. In a rotating, stratified (in density and pressure) medium, like a disc galaxy, such turbulence becomes cycloidal and acquires a net helicity. Isotropic and homogeneous turbulence with helicity, in the presence of a large-scale magnetic field, \( B_0 \), leads to an extra electromotive force of the form \( E = n_i B_x - n_j V \times B_y \), where \( \pi \) depends on the helical part of the turbulence and \( n_i \) is the turbulent diffusion, which depends on the non-helical part of the turbulent velocity correlation function (Krause & Radler 1980; Moffat 1978; Parker 1979). It should be noted that both the alpha effect and turbulent diffusion depend crucially on the diffusive (random walk) property of fluid motion (cf. Field 1996), so that if for some reason (see below) the fluid motion becomes wave-like, then the alpha effect and turbulent diffusion will be suppressed (Chandran 1996).

The induction equation, with the extra turbulent component of the electric field, and a prescribed large-scale velocity field, can have exponentially growing solutions for the large-scale field. These have been studied extensively in the literature (cf. Ruzmaikin, Shukurov & Sokoloff 1988, Mestel & Subramanian 1991, Beck et al. 1996 for recent reviews). It had been assumed in most earlier works that the turbulent velocities are not affected by Lorentz forces until the mean large-scale field builds up sufficiently. However, this may not turn out to be valid because of the more rapid build-up of magnetic noise compared with the value of the mean field, a problem to which we now turn.

3 THE KINEMATIC SMALL-SCALE DYNAMO AND THE PROBLEM OF MAGNETIC NOISE

We split up the magnetic field, \( \mathbf{B} = \mathbf{B}_0 + \mathbf{aB} \), into a mean field \( \mathbf{B}_0 \) and a fluctuating component \( \mathbf{aB} \). Here the mean field, \( \mathbf{B}_0 = \langle \mathbf{B} \rangle \), is defined either as a spatial average over scales larger than the turbulent eddy scales or, more correctly, as an ensemble average. Kazantsev (1968) was the first to show that even purely mirror-symmetric turbulence leads to dynamo amplification of the small-scale fluctuating fields, for a sufficiently large magnetic Reynolds number. Some of the subsequent work is summarized by Zeldovich et al. (1983). The statistical properties of the small-scale field are most clearly expressed in terms of the magnetic correlation function \( M(r, t) = \langle aB(x,t)aB(y,t) \rangle \), where \( r = |x-y| \). Small-scale dynamo action is described by the evolution of the longitudinal component \( M_x(r, t) = r^2 M_x(r, t) \), where \( r^2 = x^2 - y^2 \).

In the case when the turbulent velocity, say \( \mathbf{v}_t \), has a delta-function correlation in time (Markovian), it is relatively straightforward to derive the evolution equation for \( M_x \) (cf. Kazantsev 1968; Vainshtein & Khachaturov 1986; Paper II). Suppose we also assume \( \mathbf{v}_t \) to be an isotropic, homogeneous, Gaussian random velocity field with zero...
mean. We can then specify its two-point correlation function by\( \langle c_\xi(x, t)c_\eta(y, s) \rangle = T(r)|\delta(t-s)|, \) with
\[
T^k(r) = T_{nn} \left[ \frac{\partial^3}{(rT)} \right] + T_{LL} \left( \frac{rT}{rT} \right) + C_{vr}T.
\]
(1)

Here,\( T_{nn}(r) \) and\( T_{nn}(r) \) are the longitudinal and transverse correlation functions for the velocity field and\( C(r) \) represents the helical part of the velocity correlations (cf. Landau & Lifshitz 1987). If\( v_t \) is assumed to be divergence-free, then
\[
T_{nn}(r) = \frac{1}{2r} \frac{\partial}{\partial r} [r^2T_{LL}(r)].
\]

The evolution of\( M_r \) is given by
\[
\frac{dM_r}{dt} = \frac{2}{r} \frac{\partial}{\partial r} \left( r^2 \frac{\partial M_r}{\partial r} \right) + GM_r
\]
where we have defined
\[
M = \kappa + T_{LL}(0) - T_{LL}(r),
\]
\[
G = -4 \frac{d}{dr} \left( \frac{T_{nn}}{r} \right) + \frac{1}{2} \frac{d}{dr} (rT_{LL}).
\]
(4)

The term involving\( \kappa \) in equation (3) represents the effects of diffusion. The diffusion coefficient includes the effects of microscopic diffusion (\( \eta \)) and a scale-dependent turbulent diffusion\( [T_{LL}(0) - T_{LL}(r)] \). The term proportional to\( G(r) \) allows for the rapid generation of magnetic fluctuations through shearing action and the existence of a small-scale dynamo independent of the large-scale field.\( H^\infty L \) the evolution equation for\( M_r \), given above, we have neglected its coupling to the helical part of the magnetic correlation caused by a scale-dependent alpha effect. In the galactic context this has a negligible effect. We have also neglected the subdominant coupling to the large-scale field (see Paper I).)

Suppose\( V \) and\( L \) are the velocity and correlation lengths of the dominant energy-carrying eddies of the turbulence, which is assumed to have a Kolmogorov energy spectrum. For numerical estimates we generally take\( V = 10 \) km s\(^{-1} \) and\( L = 100 \) pc. For Kolmogorov turbulence, the eddy velocity at any scale\( l \) is\( v_t \propto 1^{1/3} \) in the inertial range. The turbulence is cut off at a scale\( l_c = L/2^{3/4} \), where\( R_s = VL/v \) is the fluid Reynolds number and\( v \) is the kinematic viscosity. We will use a neutrally stable galactic gas and for this,\( v \) is dominated by the neutral contribution. We take the neutral–neutral collisions to be dominated by\( H-H \) collisions with a cross-section\( \sigma_{HH} \sim 10^{-30} \) cm\(^2 \), leading to a kinematic viscosity\( v \sim \nu_0/(\nu_0a_{\nu}), \) for\( v_t \sim L/10 \) km s\(^{-1} \), leading to a neutral hydrogen number density\( n_h \sim 10^{12} \) cm\(^{-3} \), so\( v \sim 10^{23} \) cm s\(^{-1} \), so
\[
R_s = \frac{VL}{v} \approx 3 \times 10^7 V_{10} L_{100}.
\]
(5)

where\( V_{10} = (V/10 \) km s\(^{-1} \) and\( L_{100} = (L/100 \) pc). The magnetic Reynolds number at the outer scale of the turbulence is defined to be\( R_m = (VL/\eta) \). For the Spitzer value of the resistivity,\( \eta = 10^{-7}(T/10^4 K)^{-3/2} \) cm\(^2 \) s\(^{-1} \), with turbulence parameters as above,\( R_m = 3 \times 10^{10} \). Since\( v_t \propto 1^{1/3} \), the magnetic Reynolds number associated with eddies of size\( l_s \) scales as\( R_{ms}(l) = \nu_0 l_s R_m (L/ll_s)^{3/2} \).

The evolution of\( M_r(r, t) \) has been studied in detail by several authors (Zeldovich et al. 1983 and references therein) for the case when\( T_{nn}(r) \) has a single scale. We also study in Paper II, using WKB technique, the evolution of\( M_r(r, t) \) for a model Kolmogorov-type turbulence. We look at the properties of exponentially growing modes,\( M_r(r, t) \sim e^{\lambda r}, \) which are regular at the origin, and tend to zero as\( r \to \infty \). Some results of this work and the earlier works of Kazantsev (1968) and Zeldovich et al. (1983) pertinent to the present context are summarized here.

(i) There is a critical value for the magnetic Reynolds number\( (MRN) \):\( R_m = R_s \approx 60 \) for the excitation of the small-scale dynamo. Above this critical\( MRN \) the small-scale dynamo can lead to an exponential growth of the fluctuating field correlated on a scale 1. Furthermore, the equations determining\( R_s \) are the same if we replace\( (L, R_m) \) by\( (l, R_m(l)) \). The critical\( MRN \) for excitation of a mode concentrated around\( r = 1 \) is therefore also\( R_m(l) \sim R_s, \) as expected from the scale invariance in the inertial range.

In the galactic context\( R_m \gg R_s \), in fact, one also has\( R_m(l) = R_m/\eta = R_m/R_s \gg 1 \). (Here\( v_t \) is the eddy velocity at the cut-off scale.) Small-scale dynamo action hence excites modes correlated on all scales, from the cut-off scale\( l_c \) to the external scale of the turbulence.

(ii) A result of small-scale dynamo action the fluctuating field, tangled on a scale 1, grows exponentially on the corresponding eddy turnover time-scale, with a growth rate\( \Gamma_1 \sim v_t/\eta \). Since\( v_t \approx 1^{1/3}, \) the growth rate is\( \Gamma_1 \approx 1^{1/3}, \) and so increases with decreasing\( L \). In the galactic context, with\( R_m(l) = R_m/R_s \gg R_s, \) the small-scale fields tangled at the cut-off scale grow more rapidly than any of the large-scale modes.

(iii) The WKB analysis gives a growth rate\( \Gamma_1 = (v_t/l)_0 \left[ \log \left( R_m/R_s \right) \right]^{-2/3} \) with\( c_0 = 0.26/12 \) for the fastest mode. Also the growth rate for the small-scale dynamo is only weakly (logarithmically) dependent on\( R_m \), provided\( R_m \) is large enough.

(iv) For the parameters adopted above, we have\( \Gamma_1 \approx 10^{-4} \) yr\(^{-1} \). For the modes tangled at the cut-off scale, this is\( \Gamma_1 \approx 10^{10} \) yr\(^{-1} \). These times are much smaller than the time-scale for the growth of the large-scale field.

The spatial structure of the dynamo-generated small-scale is important in determining how the small-scale dynamo saturates. To examine the spatial structure for various eigenmodes of the small-scale dynamo, it is more instructive to consider the function\( w(r, t) = \langle \delta B(r, t) \cdot \delta B(y, t) \rangle \), which measures the correlated dot product of the fluctuating field\( \langle w(0) \rangle = \langle \delta B \rangle \). First, there is a general constraint that can be placed on\( w(r) \). Since the fluctuating field is divergence free, we have (Kleeorin, Ruzmaikin & Sokoloff 1986; Paper II)
\[
w(r, t) = \frac{1}{r^2} \frac{d}{dr} \left[ r^2 M_1 \right],
\]
(6)
so
\[
\int_0^\infty w(r)^2 dr = \int_0^\infty \frac{d}{dr} \left[ r^2 M_1 \right] = 0,
\]
(7)
since $M_1$ is regular at the origin and vanishes faster than $r^{-3}$ as $r \to \infty$. The curve $r^2 w(r)$ should therefore have zero area under it. Since $w(0) = \langle (dB)^2 \rangle$, $w$ is positive near the origin and the fluctuating field points in the same direction for small separation. As one goes to larger values of $r$, there must then be values of $r$, say $r \sim d$, where $w(r)$ becomes negative. For such values of $r$, the field at the origin at a separation $d$ are pointing in opposite directions on average. This can be interpreted as indicating that the field lines on average are curved on the scale $d$.

(v) In the case $\lambda / \ell_c \gg 1$, $w(r)$ is strongly peaked within a region $r = r_0 \approx \ell_c (\lambda / \ell_c)^{-1/2}$ about the origin, for all the modes. Note that $r_0$ is the diffusive scale satisfying the condition $\eta \ell_c^2 \sim v_c / \lambda$.

(vi) For the most rapidly growing mode, $w(r)$ changes sign across $r \sim l_c$ and rapidly decays with increasing $r/l_c$.

(vii) For slower growing modes, with $\Gamma_1 \sim v_c / \lambda$, $w(r)$ extends up to $r \sim l_c$ after which it decays exponentially.

We should point out that a detailed analysis of the eigenfunctions, for the simple case when the longitudinal velocity correlation has only a single scale, can be found in Keeleorin et al. (1986). Their analysis is also applicable to the mode near the cut-off scale in Kolmogorov-type turbulence. Furthermore, these authors elaborate on a pictorial interpretation of the correlation function, in terms of the Zeldovich rope-dynamo (cf. Zeldovich et al. 1983). We have shown, schematically, $w(r)$ for the fastest growing mode in Fig. 1(a), and its pictorial interpretation in terms of a flux rope in Fig. 1(b). For the fastest growing mode there is only one node where $w(r) = 0$. For higher order modes with smaller growth rates, several nodes for $w(r)$ will occur and can be interpreted in terms of several scales on which the field is curved (cf. Keeleorin et al. 1986, see also Ruzmaikin, Shukurov & Sokolov 1989). The extent of $w(r)$, after which it decays exponentially, gives the largest scale on which the flux rope is curved.

(viii) If one adopts this interpretation of the $w(r)$ for the various modes, the small-scale field can be thought of as being concentrated in rope-like structures with thickness of order of the diffusive scale $r_0 \ll l$ and curved on a scale up to $\sim l_c$ for modes extending up to $r \sim l_c$.

To end this section we give a qualitative picture of the mechanism for the dynamo growth of small-scale fields. When the field starts from an arbitrary initial configuration, in the kinematic stage, the initial field growth is just a result of random stretching by the turbulence, together with flux freezing. Eddies of scale $l$ stretch and tangle the field on corresponding scales. The field grows at this stage because the stretching of a ‘flux rope’ leads to a decrease in its cross-section (as a result of near-incompressibility of the flow), and hence, from flux freezing, an increase in the field strength. As the field grows the magnetic field gets concentrated into smaller and smaller scales until diffusion comes into play.

Consider to begin with the effect of eddies at the cut-off scale, $l_c$, of the turbulence. The initial amplification caused purely by stretching of the field by these eddies stops when the field has been concentrated into a small enough scale $r_0$, such that the rate of diffusion across $r_0$ becomes comparable to the stretching rate by these eddies. This occurs when

$$\eta \ell_c^2 \sim v_c / l_c.$$  

This gives $r_0 \sim l_c / R_{\ell_c}^{1/2}$. We will refer to $r_0$ as the thickness of the flux rope curved on a scale $l_c$. Further growth of the field can only be achieved by the operation of the small-scale dynamo, which increases the field exponentially for an $M_R$ above $R_{\ell_c}$. This is explicitly demonstrated by the solutions for the kinematic small-scale dynamo discussed above and can be thought of as the operation of the Zeldovich-Stretch-Twist-Fold–rope dynamo at random locations.

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**Figure 1.** (a) Schematic representation of the magnetic correlation function $w(r)$ for the fastest growing mode in the kinematic regime. The fact that $w(r)$ is positive at small $r$ and concentrated into a region of scale $r_0$ near the origin implies that the field points in the same direction on average for separations smaller than the diffusive scale $r_0$. The anticorrelation tail, with $w(r) < 0$, is related to the vanishing of field divergence; for points separated by the eddy scale $l_c$, the field points in opposite directions on average. (b) The pictorial interpretation of $w(r)$ as a flux rope, with $r_0$ as the rope thickness and $l_c$ as curvature scale of the flux rope.

For galactic gas with $R_{\text{in}}/R_{\text{s}} \gg 1$, even the eddies at the cut-off scale have an M R N greater than the critical value needed for dynamo action. These eddies increase the field exponentially at a growth rate $v_{i}/l$. Larger eddies of scale $l \gg l_{s}$ also lead to stretching, twisting and folding of the field at a slower rate, $v_{s}/l$. This leads to dynamo growth of fields tangled at scale $l$, with a slower growth rate $v_{s}/l$. The field curved on a scale $l$ is also, at the kinematic stage, chopped up further by smaller scale eddies (a scale dependent turbulent diffusion) until its energy can be dissipated by microscopic diffusion at scale $r_{\text{g}}$. On any flux rope of length $l$, therefore, one has smaller scale ‘wiggles’ until the diffusive scale $r_{\text{g}}$ is reached.

We emphasize that the time-scale for mean-field growth is $\sim 10^{2}$ yr, of the order of a few rotation time-scales of the disc, and is much larger than the time-scale for the growth of the fluctuating field ($T_{\text{c}}^{-1} \sim 10^{5}$ yr). The operation of the small-scale dynamo will hence imply that the magnetic field is rapidly dominated by the fluctuating component. Chandran (1996) has shown that the presence of small-scale magnetic fields could change hydrodynamic turbulence into a small-scale dynamo will hence imply that the magnetic field is rapidly dominated by the fluctuating component. Chandran (1996) has shown that the presence of small-scale magnetic fields could change hydrodynamic turbulence into magneto-elastic waves with a phase velocity $v_{\text{p}}$, where $v_{\text{p}}$ is two-thirds of the magnetic energy per unit mass. If the energy density in the small-scale magnetic noise builds up to equipartition levels, therefore, the fluid motions could become predominantly wave-like, with a wave period of order of the eddy turnover time. This could then lead to a reduced alpha effect and turbulent diffusion. Unless the build up of magnetic noise is curbed in a way which leaves the turbulence still having a diffusive property, large-scale dynamo action will be severely affected. We now turn to the effect of ambipolar drift and the possible ways in which the small-scale dynamo can saturate.

4 THE EFFECT OF AMBIPOLAR DRIFT

In a partially ionized medium the magnetic field evolution is governed by the induction equation,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (v \times \mathbf{B}) - \eta \nabla \times \mathbf{B},$$

(9)

where $v$ is the velocity of the ion component of the fluid. The ions experience the Lorentz force resulting from the magnetic field. This will cause them to drift with respect to the neutral component of the fluid. If the ion-neutral collisions are rapid enough, one can assume that the Lorentz force on the ions is balanced by their friction with the neutrals. Under this approximation, the Euler equation for the ions reduces to

$$\rho_{i} v_{i}(v_{i} - v_{n}) = \frac{(V \times \mathbf{B}) \times \mathbf{B}}{4\pi},$$

(10)

where $\rho_{i}$ is the mass density of ions, $v_{n}$ is the ion-neutral collision frequency and $v_{i}$ is the velocity of the neutral particles.

The ion-neutral elastic scattering frequency is given by

$$\nu_{n} = \frac{\rho_{i}\langle \sigma v \rangle}{(m_{i} + m_{n})},$$

where $\rho_{i}$ is the neutral fluid density and $m_{i}, m_{n}$ are the ion and neutral particle masses (cf. Mestel and Spitzer 1956; Draine 1980, 1986). We will assume that the galaxy had very nearly primordial composition in its early stage of evolution; in this case the ions are mostly just protons and the neutrals are mostly hydrogen atoms. Elastic scattering occurs with the ion polarizing the neutral atom, and interacting with the induced dipole. For H–H– interactions, in addition to elastic scattering, there can also be charge exchange reactions, which increase the ion–neutral cross-section. Draine (1980) adopts the maximum of these two rates, and gives a momentum transfer rate coefficient of $\langle \sigma v \rangle \approx 3.2 \times 10^{-4} \text{ cm}^{2} \text{s}^{-1}$ for $v < 2 \text{ km s}^{-1}$, and $\langle \sigma v \rangle \approx 2.0 \times 10^{-4} \text{ (v/km s}^{-1})^{0.5} \text{ cm}^{2} \text{s}^{-1}$ for 2 km s$^{-1} < v < 1000 \text{ km s}^{-1}$. In the galactic disc, we expect the gas to have a temperature $T < 10^{4}$ K with $v \sim 10 \text{ km s}^{-1}$ and so $\langle \sigma v \rangle (H–H) \approx 10.74 \times 10^{-4} \text{ cm}^{2} \text{s}^{-1}$. Furthermore, the interaction with helium atoms will not give a significant addition to the collision rate, because the polarizability of helium is lower because of its symmetry and helium is four times heavier than hydrogen. However, helium will contribute about 25 per cent of the total density of the fluid. Let $\rho_{i}, \rho_{n}$ be the proton and hydrogen densities and $n_{i} = \rho_{i}/m_{i}$. We then have

$$\rho_{i} v_{i} = \frac{\rho_{i} \rho_{n} \langle \sigma v \rangle + n_{i}}{2m_{i}},$$

(11)

with $\langle \sigma v \rangle \approx 4 \times 10^{-9} \text{ cm}^{2} \text{s}^{-1}$.

For the evolution of the small-scale field, we show in Paper II that ambipolar drift adds to the diffusion coefficient, $\kappa$, a term proportional to the energy density in the fluctuating fields. This changes $\eta$ to an effective value

$$\eta_{\text{ambi}} = \eta + \frac{w(0, t)}{6\pi\rho_{i} v_{i}^{2}} = \eta + \frac{\langle \nabla \times \mathbf{B} \times \mathbf{B} \rangle}{6\pi\rho_{i} v_{i}^{2}},$$

(12)

and replaces $\kappa$ in equation (3) for $M_{i}(r, t)$ by a new $\kappa_{\text{ambi}} = \eta_{\text{ambi}}$, by the effective magnetic Reynolds number, incorporating the effect of ambipolar drift, for fluid motion on any scale of the turbulence by

$$R_{\text{ambi}}(l) = \frac{v_{i}}{\eta_{\text{ambi}}} = \frac{v_{i}^{2}}{\eta_{\text{ambi}}} \frac{16\pi\rho_{i} v_{i}^{2} \rho_{i}}{\langle \nabla \times \mathbf{B} \times \mathbf{B} \rangle}.$$  

(13)

Here $v_{i} = (l l_{6})^{1/3} v$ as before, and we have assumed $\eta_{\text{ambi}} \gg \eta_{i}$, a condition which soon obtains as the field builds up.

As the energy density in the fluctuating fields increases, $R_{\text{ambi}}(l)$ decreases. First, this makes it easier for the field energy to reach diffusive scales from a general initial configuration. At this stage, the initial amplification resulting purely from stretching stops, and further growth of the field can only be achieved by the operation of the small-scale dynamo. If, as the field grows, $R_{\text{ambi}}(L)$ decreases sufficiently, a stationary state with $\partial M_{i}/\partial t = 0$ could, in principle, be achieved. In such a state, $M_{i}$ is independent of time, so the condition on the critical MRN for the stationary state to be reached will be identical to that obtained in the kinematic stage; that is, if $R_{\text{ambi}}(L)$ decreases to a value $R_{\text{ambi}} \sim 100$, dynamo action will stop completely.

However, for galactic turbulence,

$$R_{\text{ambi}}(l) = \frac{1}{f(l)} \frac{3\beta v_{i}^{2} \rho_{i}}{2\rho_{i} v_{i}} = \frac{Q(l)}{f(l)},$$

(14)

where $f(l) = B_{i}^{2} / (4\pi\rho_{i} v_{i}^{2})$ is the ratio of the local magnetic energy density of a flux rope curved on scale $l$ to the turbu-
lent energy density \( \rho_v l^2/2 \) associated with eddies of scale \( l \).

Using the value of \( \nu_m \) determined in equation (11) and putting in numerical values we get

\[
Q(l) = \frac{3\rho_v \nu_m l}{2\rho_v \nu_0} \sim 1.8 \times 10^9 n_{-2} \left( \frac{l}{L} \right)^{2/3} L_{10}^2 V_{10}^{-2},
\]

where \( n_{-2} = (n/10^{-2} \text{ cm}^{-3}) \) and we have assumed a Kolmogorov scaling for the turbulent eddy fluctuations.

One can see from equations (14)–(15) that, for typical parameters associated with Galactic turbulence, the MRN incorporating ambipolar drift is likely to remain much larger than \( R_c \), for most scales of the turbulence, even when the field energy density becomes comparable to the equipartition value, so ambipolar drift by itself cannot saturate the small scale dynamo. Rather, one expects the field to continue to grow rapidly, even taking into account ambipolar diffusion. Note also that the growth rates for the small-scale dynamo generally depend only weakly on the MRN, provided the MRN is much larger than \( R_c \), (see Section 3, Kleerin et al. 1986, Ruzmaikin et al. 1989). We therefore still expect the small-scale dynamo generated field to grow almost exponentially on the eddy turnover time-scale, as long as \( R_{\text{ambi}} \gg R_c \).

The spatial structure of the fluctuating field will also remain ropey, as argued in Section 3, as long as \( R_{\text{ambi}} \gg R_c \). However as the field strength in a flux rope, curved on a scale \( > l \), grows to near equipartition with the turbulent energy associated with eddies smaller than \( l \), these smaller scale eddies would no longer be effective in causing ‘turbulent diffusion’ of the larger scale field. The thickness \( r_d(l) \) of a flux rope curved on a scale \( l \) is therefore determined eventually by demanding that the ambipolar diffusion timescale across \( r_d(l) \) becomes comparable to the stretching time-scale \( l/t_v \); that is,

\[
r_d^2(l)/h_{\text{ambi}} \sim l/t_v.
\]

This determines the thickness to be \( r_d(l)/R_{\text{ambi}}^2(l) \). As \( R_{\text{ambi}}(l) \gg 1 \), we expect flux ropes to remain relatively thin with a thickness \( \sim R_{\text{ambi}}^{1/2}(l) \ll l \), even taking account of the ambipolar drift.

In summary, we have argued here that, in galaxies, the small-scale dynamo continues to exponentiate the field fluctuations even in the presence of ambipolar drift. This fluctuating field, however, does not fill the volume but is concentrated into intermittent rope-like structures. The ropes are curved on the turbulent eddy scale and their thickness is set by the diffusive scale \( r_d(l) \) determined by the effective ambipolar diffusion.

We have to consider how other non-linear feedback processes could limit the growth of this small-scale dynamo generated field.

5 Saturation of the small-scale dynamo

5.1 Inefficient random stretching and damping

The first of these restraining processes is the reduction in the efficiency of stretching of a flux rope as the field in the rope, say \( B_r \), grows in strength. A turbulent eddy of scale \( l \) produces a correlated winding-up of the field for a time of order \( l/v_s \). Note that this is just a consequence of the induction \( \nabla \times (v \times B) \) term and velocity shear. No dynamics is involved. However, suppose that the growing tension component of the Lorentz force on the flux rope tangled on this scale can untangle (or straighten) the rope and damp away its wrinkle on a comparable time-scale. The random stretching of the flux rope by these eddies will be suppressed and the small-scale dynamo will not operate efficiently.

To estimate the straightening time-scale, say \( t_s \), we have to look at the dynamics of the flux rope. We give below a plausible picture of this dynamics, but caution that a more detailed calculation and/or numerical simulation will be needed to put this picture on a firmer footing.

The motion of the flux rope through the surrounding medium is influenced not only by the tension force but also by friction. (We are neglecting, for now, the effects of rotation and gravity.) Suppose \( v_s \) is the velocity associated with

![Figure 2](https://academic.oup.com/mnras/article-abstract/294/4/718/1026065/18002665)

**Figure 2.** (a) A schematic illustration of a curved flux rope, with a radius of curvature \( l \) and thickness \( r_d \). Magnetic tension acts to straighten the rope and aerodynamic drag damps the magnetic energy associated with the wrinkle in the rope. This leads to inefficient random stretching when \( v_s \) becomes comparable to \( v_t \). (b) The collapse of flux loops is illustrated schematically. Such collapse results in an irreversible removal of small-scale magnetic noise and limits the value of \( N \).
we have equipartition values. However, the effect of drag leads to a smaller 'terminal' velocity, which can be estimated by equating the tension force to the drag on the flux rope. Also, the work done by the rope against the frictional drag leads to a damping of the energy associated with the wrinkle of the flux rope on the time-scale $t_{\text{cur}}$, which is comparable to the rapid eddy turnover time when $v_s \sim v_r$. We illustrate schematically this process of flux rope straightening and damping in Fig. 2(a).

The force drag per unit length on the flux rope is $\sim C_s \rho_s v_s^2 t_\text{s}/2$ (cf. Parker 1979, equation 8.59). The coefficient $C_s$ depends on the fluid Reynolds number on the scale of the radius of the flux rope, that is on $R_\text{rad} = r_e r_d/\nu$. Note that the drag formalism is not well developed for the case when the external medium is itself turbulent (cf. Parker 1979, section 8.7). One expects eddies with a scale smaller than the rope radius to have a different effect compared with larger eddies. In our case $r_e \ll l$, and so one expects momentum transfer by eddies with scale $< r_e$ to be subdominant compared with the turbulent drag induced by larger eddies. Here we adopt the drag formula given above, with the assumption that the complications mentioned above do not drastically change the results.

Equating the magnetic tension component of the Lorentz force in the rope and drag then gives

$$B_\text{s}^2 \left( \pi r_\text{s}^2 \right) \sim \frac{C_s}{2} \rho_s v_s^2 t_\text{s}.$$  \hspace{1cm} (17)

Here we have taken the curvature radius associated with the field tangled on scale $l$ to be $l/2$. The dynamic action on these scales will be affected when $t_\text{e} \sim l/v_s \sim l/v_r$ or when $v_s \sim v_r$, since, in this case, the flux rope is able to straighten and damp (as a result of friction) its wrinkle (curvature) on a time-scale comparable to the stretching time-scale $l/v_s$. The random stretching of the field by eddies of scale $l$ will therefore become inefficient when the peak field has grown to a value found by substituting $v_s \sim v_r$ in equation (17). Straightforward rearrangement of the various quantities in equation (17) then implies an upper limit on the magnetic field in the flux rope of

$$f_s(l) \approx \left( \frac{10 (4\pi)^{1/2} Q_{39}^{2/7}}{R_\text{rad}^{2/7}} \right).$$ \hspace{1cm} (18)

Since we expect $r_d \ll l$, the peak field in the rope can exceed equipartition values.

When ambipolar drift is causing the flux rope to thicken, we have $l v_s = R_\text{rad}^{2/3}$, which itself depends on the magnetic field strength in the rope. Furthermore, the drag coefficient $C_s$ depends on $R_\text{rad}$, so on $r_d$, and therefore implicitly on the field in the rope.

The value of $C_s$ decreases from about 30 at $R_\text{rad}^{2/3} \sim 1$ to 4 at $R_\text{rad}^{2/3} \sim 10$ to 1 at $R_\text{rad}^{2/3} \sim 10^4$ and ranges between 1 and 0.1 for larger values (cf. Parker 1979, section 8.7). For our problem, an adequate estimate of the drag coefficient is obtained by taking $C_s \sim 10(R_\text{rad}^{2/3})$. Taking $v_s \sim v_r$, we have $R_\text{rad} = R_\text{rad}(l) = r_d l/2$. Using $\nu = \nu_{\text{mag}}(l)$, and rearranging, we obtain

$$f_s(l) \approx \frac{10}{4\pi} \left( \frac{4\pi^{1/2} Q_{39}^{2/7}}{R_\text{rad}^{2/7}} \right) \sim 3.1 n_{37}^{2/7} \left( \frac{l}{L} \right)^{5/7} L^{32/7} V_{30}^{5/7}. \hspace{1cm} (19)$$

One can now go back and estimate the Reynolds number $R_\text{rad}^{2/3}$. We get

$$R_\text{rad}^{2/3} \sim 392(l/L)^{32/7} n_{37}^{2/7} L^{32/7} V_{30}^{5/7}. \hspace{1cm} (20)$$

At the cut-off scale $l_1$, $R_\text{rad}^{2/3} \sim 0.4$, while at the outer scale $L$, $R_\text{rad}^{2/3} \sim 392$, so the approximation used for the drag coefficient should be reasonably accurate.

We see from equation (19) that when the field in the flux rope of scale $l$ grows to a few times the equipartition field associated with eddies on that scale, the random stretching of the rope will become inefficient, limiting its further growth by dynamo action. Since larger eddies carry larger energy, this also implies that the saturated value of the field in a flux rope tangled on a scale $l$ will be larger for larger $l$.

The average energy density in the field tangled on the scale $l$, however, need not exceed the energy density of the turbulent eddies because of the ropiness of the field. Since the ropes have a thickness of order $r_d$, and are curved on a scale $l$, the average energy density contributed by a flux rope to a sphere of radius $l$ is

$$E_s(l) \sim \frac{B_\text{s}^2}{8 \pi} \left( \frac{\pi r_\text{s}^2 l}{l} \right) \frac{B_\text{s}^2}{8 \pi} \left( \frac{3 \pi r_\text{s}^2 l_1}{l_1} \right).$$ \hspace{1cm} (21)

Note that, although $B_\text{s}^2 \propto l/l$ and can exceed equipartition value, the average energy density of a flux rope in a correlation volume $E_s(l) \propto B_\text{s}^2 l^4/l^4 \propto E_s(l)$. This can be much smaller than equipartition because $r_d/l \ll 1$.

In principle, the length of the flux rope tangled on a scale $l$ may be larger than the value $l$ assumed above, and equal to $N l$. In this case, the ratio of the average magnetic energy density of the field tangled on scale $l$ to the turbulent energy on the same scale is

$$F(l) = \frac{E_s(l)}{\left( \rho_\text{s} v_\text{s}^2 /2 \right)} \sim N \left( \frac{3 C_s}{16} \right) \left( \frac{l}{l_1} \right).$$ \hspace{1cm} (22)

Putting in numerical values, we have

$$F(l) = 0.9 \times 10^{-3} N = \frac{1}{2} \left( \frac{l}{l_1} \right)^{-1/6} L^{32/7} V_{30}^{-1/4}. \hspace{1cm} (23)$$

We can also ask if the energy dissipated in ambipolar drift and frictional drag is comparable to the turbulent power in the case when the small scale field saturates as a result of the stretching constraint. Since the ambipolar drift rate across the rope is comparable to the stretching rate, $e/\ell$, the power dissipated by ambipolar drift is $(n/\nu)(l/l_1)^{-1/2} E_s(l) \sim E_s(l)$. The power lost as a result of frictional drag is also $E_s(l)$, so the ratio of the total power dissipated to the turbulent power is

$$P_s = 2 \frac{E_s(l)}{\left( \rho_\text{s} v_\text{s}^2 /2 \right)} \sim 2 F(l).$$ \hspace{1cm} (24)

Both the energy density in the flux ropes and the energy dissipated as a result of ambipolar drift and friction depend on how large $N$ can become, or how many flux ropes of scale $l$ are packed into a correlation volume of radius $l$ in the final saturated state.

5.2 The limiting effect caused by flux loop collapse

An important process, which limits $N$ from becoming too large, enters as the peak field in the flux rope increases to the value given by equation (18). As one tries to pack more and more flux ropes curved on a scale $l$ into a volume of the same scale, the probability that the rope intersects itself, or another cohabiting rope, increases. This will result in loops of flux of radius $l$. A also, note that the very process by which the small-scale dynamo may operate, namely the stretching-twisting-folding actions associated with the Zeldovich rope dynamo, will generically result in flux loops of scale $l$.

These loops of flux, when not being stretched, can collapse to a small radius (cf. De Luca, Fisher & Patten 1993; Vishniac 1995). This is schematically illustrated in Fig. 2(b). The time-scale for the collapse is of order $l/v$, where $v$ is the eddy turnover time, since $v \sim v_l$ when the peak field in the rope is given by equation (18). The collapse of the loop will be halted when it has shrunk to a radius comparable to its thickness. At this stage, the effects of diffusion could convert most of the remaining energy in the loop into kinetic energy and heat. This process results in the irreversible removal of energy from the small-scale magnetic field, exchanging with the external pressure of the gas has to be predominantly neutral with an ion density less than $10^{-5}$ per cent of a neutral hydrogen density as large as $10^{-5}$.

Whether the field in the ropes will be limited by inefficient random stretching as given by equation (19) or by external pressure (equation 25) will depend on the parameters of the problem. The rope field will be given by the lower of the limiting fields implied by equations (19) and (25).

The total pressure of the interstellar medium in a galaxy could contain a number of components: a thermal component, pressure resulting from turbulence itself and possibly pressure resulting from non-thermal 'cosmic-rays'. The ratio of the gas pressure to the turbulent energy density is

$$\frac{P_g}{E_T} \sim 1.7 \left( \frac{T}{10^4 \text{ K}} \right) V_{10^2}^{-2}.$$

If $P_{\text{ext}}$ in a galaxy is a factor $F$ times the gas pressure, then the peak field given by equation (19) begins to exceed that given by equation (25), when the ion density exceeds a critical value $n_i^c$. This critical value is given by

$$n_i^c \sim 6.4 \times 10^{-2} \left( \frac{n_l}{1 \text{ cm}^{-3}} \right)^{2/3} \text{ cm}^{-3} \times \frac{V_{10^2}}{n_l^{1/3}} \left( \frac{L_{120}}{1} \right)^{1/4} \left( \frac{l}{1} \right)^{1/4} \left( \frac{T}{10^4 \text{ K}} \right)^{7/3}.$$

For a larger ion density than $n_i^c$, the peak field for the flux ropes tangled on the largest scale $l$ saturates to a value $B_{ls} \sim (8\pi P_{\text{ext}})^{1/2}$, lower than given by equation (19), so, for the small-scale dynamo generated field to saturate to subequilibrium level by the processes described above, the galactic gas has to be predominantly neutral with an ion density less than $n_i$.

5.3 The limit on the field in flux ropes resulting from the external pressure of the gas

There is one caveat to the above discussion. We have assumed that the field in the rope can grow sufficiently that the tension in the rope begins to play an important role in the rope dynamics. However, there is an upper limit to the growth of the magnetic field in the ropes, from the effect of its magnetic pressure on the dynamo process. As a result of the increasing importance of this pressure, stretching of field lines can lead to a partial decrease in fluid density in the ropes rather than a decrease in the rope cross-section and the associated increase in the rope magnetic field (cf. Vishniac 1995). 

An upper limit to the magnetic pressure in the ropes is given by the external pressure $P_{\text{ext}}$. This implies that the field in the rope is limited to

$$B_s < (8\pi P_{\text{ext}})^{1/2}.$$

The time-scale for the collapse is of order $l/v$, where $v$ is the eddy turnover time, since $v \sim v_l$ when the peak field in the rope is given by equation (18). The collapse of the loop will be halted when it has shrunk to a radius comparable to its thickness. At this stage, the effects of diffusion could convert most of the remaining energy in the loop into kinetic energy and heat. This process results in the irreversible removal of energy from the small-scale magnetic field, exchanging with the external pressure of the gas has to be predominantly neutral with an ion density less than $10^{-5}$ per cent of a neutral hydrogen density as large as $10^{-5}$.

Whether the field in the ropes will be limited by inefficient random stretching as given by equation (19) or by external pressure (equation 25) will depend on the parameters of the problem. The rope field will be given by the lower of the limiting fields implied by equations (19) and (25).

The total pressure of the interstellar medium in a galaxy could contain a number of components: a thermal component, pressure resulting from turbulence itself and possibly pressure resulting from non-thermal 'cosmic-rays'. The ratio of the gas pressure to the turbulent energy density is

$$\frac{P_g}{E_T} \sim 1.7 \left( \frac{T}{10^4 \text{ K}} \right) V_{10^2}^{-2}.$$

If $P_{\text{ext}}$ in a galaxy is a factor $F$ times the gas pressure, then the peak field given by equation (19) begins to exceed that given by equation (25), when the ion density exceeds a critical value $n_i^c$. This critical value is given by

$$n_i^c \sim 6.4 \times 10^{-2} \left( \frac{n_l}{1 \text{ cm}^{-3}} \right)^{2/3} \text{ cm}^{-3} \times \frac{V_{10^2}}{n_l^{1/3}} \left( \frac{L_{120}}{1} \right)^{1/4} \left( \frac{l}{1} \right)^{1/4} \left( \frac{T}{10^4 \text{ K}} \right)^{7/3}.$$

For a larger ion density than $n_i$, the peak field for the flux ropes tangled on the largest scale $l$ saturates to a value $B_{ls} \sim (8\pi P_{\text{ext}})^{1/2}$, lower than given by equation (19), so, for the small-scale dynamo generated field to saturate to subequilibrium level by the processes described above, the galactic gas has to be predominantly neutral with an ion density less than $n_i$.

Note that the value of $n_i^c$ is critically dependent on the turbulence parameters that obtain in the ISM of the galaxy, especially the turbulent velocity scale. For example, for $V = 5$ km s$^{-1}$ and all other parameters as above, the critical density becomes $n_i^c \sim 0.5$ cm$^{-3}$, so even for an ionized hydrogen density as large as $10^{-5}$ per cent of a neutral density (taken here to be $1$ cm$^{-3}$) saturation could occur as a result of inefficient stretching.

Let us now ask what happens if the ion densities exceed the critical value $n_i$. In this case $B_s/8\pi \sim P_{\text{ext}}$ and the stretching constraint can be satisfied only if the flux rope thickens further than the value implied by ambipolar drift, to a radius $r_o = R \sim \left( C_{\Omega}/4\pi \right) \left[ n_0 l^2 (2P_{\text{ext}})^{1/2} \right]$. If the flux ropes can thicken to this radius, the average energy magnetic energy density will be

$$\frac{E_B}{E_T} \sim \frac{3C_{\Omega}^2}{128\pi F} \left( \frac{M_2}{M_1} \right)^2, a \sim 7.3 \times 10^{-6} \text{ cm}^2 \text{ s}^{-1}.$$
where $M_\ast$ is the Mach number of the turbulence and we have adopted the parameters $F \sim 4$ and $P_{\ast}/E_\ast \sim 1.7$, given above for the numerical estimate. The value of $N$ should again be limited by the collapse of loops as discussed above.

It is not clear, however, if flux ropes can thicken further than the radius implied by ambipolar drift when dynamo action begins from weak seed fields. One possibility is that the magnetic pressure in the ropes acting on the fluid as a whole can thicken the rope, but this can only happen if the pressure in the rope becomes larger than the pressure outside, at least temporarily. Even if this were possible, as the flux rope thickens flux freezing leads to a decrease of the field strength in the rope, and a consequent decrease in the thickening rate. The fluid pressure in the rope will also decrease as the rope expands. Note that this problem does not arise when ambipolar drift is causing the thickening. In a predominantly neutral medium, the ion pressure is much smaller than that caused by neutrals. The Lorentz force term in the Euler equation for the ions can be much larger than the ion pressure gradient term, and can cause a relative drift of the ions with respect to the neutrals, carrying the field and hence thickening the flux rope.

If the ion density is larger than $n_\ast$ and the flux ropes cannot thicken sufficiently to resist stretching, then it is not clear exactly how the small-scale field saturates. The field may be packed locally into a radius $\sim R$ by the folding motions associated with the turbulence, and then resist further stretching. On the other hand, the small-scale field may only saturate if the length of flux ropes, $N$, increases sufficiently to achieve equipartition with the turbulence. In either case the small-scale field will be highly intermittent. The effects of reconnection of this highly intermittent field (cf. Vishniac 1995; Lazarian & Vishniac 1996) could be important in deciding whether the turbulence can still lead to large-scale dynamo action. In an interesting paper that came to our notice during the completion of the present work, Blackman (1996) discusses the possible effects of reconnection in greater detail, albeit in the case where the flux ropes are assumed to have a thickness $\sim R$, and the gas is assumed to be largely ionized.

6 DISCUSSION AND CONCLUSIONS

The large-scale galactic field is thought to be generated by a turbulent dynamo. However, the same turbulence will produce magnetic noise at a more rapid rate. We have examined whether the Lorentz forces associated with the growing small-scale fields can lead to their saturation in a manner that preserves large-scale dynamo action. In doing this, we have also taken account of the ambipolar drift induced by the presence of a neutral component of the galactic gas.

The saturated state of the small-scale dynamo generated field, which we have motivated above, does indeed preserve large-scale dynamo action. The crucial property of the small-scale dynamo generated field that allows this to happen is its spatial intermittency. The field can build up locally to a level that will lead to small-scale dynamo saturation, while at the same time having a subequipartition average energy density.

Numerical simulations of dynamo action resulting from mirror-symmetric turbulence (Meneguzzi et al. 1981) or convection (Brandenburg et al. 1996) have indeed hinted at a saturated state of the small-scale dynamo as described above: a magnetic field concentrated into flux ropes, occupying a small fraction of the fluid volume, having peak fields comparable to or in excess of the equipartition value but an average magnetic energy density only about 10 per cent of the kinetic energy density.

We have described in Sections 4 and 5 the approach to this saturated state. As we noted in Section 4, for conditions appropriate to galactic gas the effective magnetic Reynolds number, even including ambipolar diffusion, is much larger than a critical value needed for small-scale dynamo action. In such a case, however, as the small-scale field grows in strength it continues to be concentrated into thin rope structures, as in the kinematic regime. These flux ropes are curved on the turbulent eddy scales, while their thickness is set by the diffusive scale determined by the effective ambipolar diffusion. The growing magnetic tension associated with the curved flux ropes acts to straighten them out. Frictional drag damps the magnetic energy associated with the wrinkle in the rope. A iso, small-scale flux loops can collapse and disappear. These non-local effects operate on the eddy turnover time scale, when the peak field in a flux rope has grown to a few times the equipartition value. Their net effect is to make the random stretching needed for the small-scale dynamo inefficient and hence saturate the small-scale dynamo. However, the average energy density in the saturated small-scale field is subequipartition, as it does not preserve. This picture of small-scale dynamo saturation obtains when the ion density is less than a critical value of $n_\ast \sim 0.06–0.5 \text{ cm}^{-3}$ $(n_i/\text{cm}^{-3})^{2/3}$.

For very large ion densities $n_i > n_\ast$, the small-scale field is expected to saturate only when its energy density grows comparable to that of the turbulence. This is because in this case the peak field cannot grow sufficiently (without its pressure exceeding the interstellar pressure) for the stretching constraint to apply. However, the spatial structure of the small-scale field is still likely to be highly intermittent. The large-scale dynamo action will depend on how such a field responds to turbulent motions, especially whether the field can reconnect efficiently (cf. Vishniac 1995, Blackman 1996).

Note that while the arguments presented in this paper appear plausible, they constitute a series of simplified models of a complex non-linear phenomenon. One should therefore keep in mind the possibility that some important effect may have been overlooked. It would be very useful to try to find independent evidence for the different processes outlined in Section 5, perhaps through numerical simulations along the lines of Brandenburg et al. (1996). Of course, such simulations will necessarily be limited by the magnetic and fluid Reynolds numbers that they can achieve. Even if one could achieve fluid and magnetic Reynolds numbers of $\sim 10^9$ (as would be appropriate for a medium with viscosity and an effective ambipolar diffusivity determined by the presence of neutrals), however, some of the ideas presented here could be tested.
We now discuss briefly the implications of the above results for the origin of galactic fields. The viability of the saturation mechanism discussed here for limiting magnetic noise depends on the ionization of the gas and the turbulence parameters. In the context of the G Galaxy, a study of warm clouds by Spitzer & Fitzpatrick (1993) gives a range of electron densities for the clouds, with an average of ~0.07 cm\(^{-3}\). They also deduce an average neutral density ~0.2 cm\(^{-3}\). For such parameters, the magnetic noise will indeed saturate if the turbulent velocity is ~5 km s\(^{-1}\) and other parameters are as in equation (27).

In the case of a young galaxy with mass ~10\(^{11}\) M\(_{\odot}\) that has just collapsed into a region of size R\(_g\) ~10 kpc, the average density is larger and is ~1 cm\(^{-3}\). If the radius were larger, say R\(_g\) ~20 kpc, the density would be smaller by a factor ~8, but collapse to a disc of height h\(_g\) ~1 kpc would increase the density by a factor R\(_g\)/h\(_g\). We therefore take a density n\(_h\) ~1 cm\(^{-3}\) as a typical density of neutral gas in a young galaxy. The average column density for H I is ~10\(^{-2}\)–10\(^{-5}\) cm\(^{-2}\). The damped Ly \(\alpha\) systems seen in the spectra of high redshift quasars, and thought to be young galaxies, do indeed have such H I column densities (cf. Wolfe 1995). The ionization fraction is more uncertain. For such a high H I column density as that inferred above, the gas is expected to become self-shielded to external ionizing flux. However, ionization will result from UV emission from young stars embedded in the gas, the importance of which depends on the uncertain star formation rates and stellar mass functions. From an observational point of view, a recent study of metal lines in these systems (Lu et al. 1996) deduces an upper limit to their electron density, consistent with the average electron density in warm clouds in the Milky Way, n\(_e\) ~0.07 cm\(^{-3}\), mentioned above. Further velocity widths deduced from 21 cm absorption resulting from neutral hydrogen in several damped Ly \(\alpha\) systems also limit the turbulent component of the line width to be about 10 km s\(^{-1}\) (cf. De Bruyn, O’Dea & Baum 1996). Here, also, therefore, the densities and turbulence parameters are expected to be in the range wherein the small-scale dynamo generated fields can saturate as a result of tension forces, in a way which preserves large-scale dynamo action.

In our analysis, so far, we have ignored the small-scale field generated by the tangling of the large-scale field by the turbulence. Even when the dynamo-generated small-scale field has saturated, this will provide an additional source of small-scale magnetic noise. As the large-scale field grows, so does this component of the small-scale field with an energy density ultimately decided by the nature of the M HD turbulence (cf. Zeldovich et al. 198). We hope to return to this issue in a later work. One then expects two components to the small-scale magnetic field in the interstellar medium of a galaxy: first a ropey, intermittent component, with flux ropes curved on scale L ~100 pc, say, and thickness r\(_g\) ~10\(^{-2}\)L ~1 pc, with peak field a few times equipartition, and secondly a more diffuse small-scale field related in strength to the large-scale field. It would be interesting to search for both of these components in the interstellar medium of galaxies.

The effect of ambipolar drift, together with the small-scale dynamo, can also influence galactic magnetic field generation, indirectly, in another fashion. Note that any dynamo needs a seed field to act upon. Since the small-scale dynamo acts to generate fields more rapidly than the large-scale dynamo, the magnetic noise so generated may itself provide a significant seed for the large-scale dynamo (cf. Beck et al. 1994). However, the small-scale dynamo also leads to highly ropey fields with a rope thickness r\(_g\). In a fully ionized gas, r\(_g\) ~L/R\(_g\)\(^{1/2}\) will be very small, since R\(_g\) >> 1. The overlap of such a field with a large-scale dynamo eigenfunction will be small. However, if small-scale dynamo action proceeds in the presence of neutrals, r\(_g\) ~r\(_g\) ~L/R\(_g\)\(^{1/2}\) in general, since R\(_g\)/r\(_g\) ~1, so when neutrals are present the small-scale dynamo generated magnetic noise will provide a more coherent seed field for large-scale dynamo action. This will act to shorten the time-scale for the generation of a large-scale galactic magnetic field to the microgauss level at higher redshift.

Note that, after recombination, the residual ionization fraction of the intergalactic medium drops to about 10\(^{-4}\)–10\(^{-5}\). One may be tempted to apply some of the results obtained here to the first generation of objects, which collapse at high redshifts. The limitations of our semi-quantitative arguments, and our assumption of a homogeneous galactic interstellar medium, in reaching the above conclusions, need little emphasizing. It would also be fruitful to find ways of incorporating more fully the dynamics of the velocity correlations, as we have done for the magnetic correlations. This full M HD turbulence problem appears formidable at present. Nevertheless, the results obtained here encourage the belief that the turbulent galactic dynamo could indeed be made to produce large-scale fields, in the presence of a significant neutral component.

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