$$\mathrm{Nu}_d' = \frac{2}{\ln\left(1 + \frac{2}{\mathrm{Nu}_d}\right)} \quad (19)$$

where Nu_d' is the corrected Nusselt number and Nu_d is the prediction for full-sized cylinders (e.g., Fig. 3). For normal applications, $2/\mathrm{Nu}_d \ll 1$, so

$$\ln\left(1 + \frac{2}{\mathrm{Nu}_d}\right) \approx \frac{2}{\mathrm{Nu}_d}, \quad \mathrm{Nu}_d' \approx \mathrm{Nu}_d$$

Comparison With Vertical Plate Heat Transfer. For film condensation on a vertical plane of height L, the dimensionless heat-transfer prediction of boundary-layer theory takes the form

$$\mathrm{Nu}_L \bigg/ \left[\frac{g\rho h_{fg} L^3}{k\nu(T_{sat} - T_w)}\right]^{1/4} = 0.943[-\theta'(0)] \left[\frac{c_p\Delta T/h_{fg}}{\mathrm{Pr}}\right]^{1/4} \quad (20)$$

where $\theta'(0)$ is the same function of Pr and $c_p\Delta T/h_{fg}$ as appears in equation (16) and

$$\mathrm{Nu}_L = \frac{\bar{h}L}{k}, \quad \bar{h} = \frac{Q}{L(T_{sat} - T_w)}$$

So, for a given Pr and $c_p\Delta T/h_{fg}$, it follows that

$$\mathrm{Nu}_L \bigg/ 0.943 \left[\frac{g\rho h_{fg} L^3}{k\nu(T_{sat} - T_w)}\right]^{1/4} = \mathrm{Nu}_d \bigg/ 0.733 \left[\frac{g\rho h_{fg} d^3}{k\nu(T_{sat} - T_w)}\right]^{1/4} \quad (21)$$

As a consequence of this correspondence, Fig. 3 can be used equally well for the flat plate as for the horizontal cylinder.

It is interesting to inquire about the relationship between plate height and cylinder diameter required to produce equal *heat-transfer coefficients* for plate and cylinder. Equating heat-transfer coefficients, temperature differences, and fluid properties in equation (21), it is found that[4]

$$L = 2.78\, d \quad (22)$$

Using this equivalence, correlations of the heat-transfer coefficient for cylinders may also be used for vertical plates and vice versa.

Circumferential Variation of Film Thickness. For a particular physical situation (i.e., given fluid, ΔT and r), the circumferential variation of the film thickness δ is found from equation (18) to be

$$\delta \sim \frac{1}{\gamma(X)} \quad (23)$$

The function $\gamma(X)$ is graphed on Fig. 2, and its reciprocal provides us with the variation of δ. Utilizing Fig. 2, it is seen that the condensate layer thickness, starting from a *finite value* at $X = 0$, grows rather slowly over the upper portion of the cylinder and then increases quite rapidly over the lower part of the cylinder. At the lower stagnation point ($X = \pi$), the theoretical prediction of an infinite film thickness can be interpreted as signifying the dropping away of the condensate from the cylinder surface.

Comparison With Experimental Heat Transfer. Experiments on film condensation over a horizontal cylinder have been carried out primarily for fluids with Prandtl numbers lying above unity. Data for these high Prandtl number experiments are summarized by McAdams (ref. [5], p. 340). In general, the conditions of the tests corresponded to small values of $c_p\Delta T/h_{fg}$. Since the analytical predictions of Nusselt closely correspond to those of the boundary-layer formulation for low $c_p\Delta T/h_{fg}$, the comparison of McAdams (table 13-4) between experimental data and Nusselt's theory also applies here.

For the low Prandtl number range, experiments on sodium and mercury have been carried out by Misra and Bonilla for values of $c_p\Delta T/h_{fg}$ up to 0.03. Their heat-transfer results were only 5 to 15 per cent of equation (17). While boundary-layer theory predicts a lowering of the Nusselt number for these low Prandtl number fluids (see Fig. 3), the predicted reduction is much smaller than that found by experiment. So, it would appear that the effect of the inertia forces is by no means sufficient to explain the experimental findings. There still remains the need for further experiments to clearly define the departures between the test conditions and the analytical model.

[4] According to Nusselt's simple theory, as quoted by McAdams (ref. [5], p. 341) the equality of coefficients is achieved when $L = 2.87\, d$; a result which is very close to equation (22).

References

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2 R. Hermann, "Heat Transfer by Free Convection From Horizontal Cylinders in Diatomic Gases," NACA TM 1366, 1954.

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5 William H. McAdams, "Heat Transmission," third edition, McGraw-Hill Book Company, Inc., New York, N. Y., 1954.

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DISCUSSION

L. A. Bromley[5]

The authors are to be commended for bringing laminar-film condensation on a horizontal cylinder "within the framework of boundary-layer theory." I believe it would be of interest to compare the result reported here with an approximate one I derived[6] and corrected after discussions with Rohsenow[7] for high Prandtl number fluids.

In the authors' nomenclature, it may be written:

$$\mathrm{Nu}_d = 0.7280 \left\{ \frac{\left(1 + \frac{3}{8}\frac{c_p\Delta T}{h_{fg}}\right)}{\left(1 + \frac{11}{40}\frac{c_p\Delta T}{h_{fg}}\right)^{3/4}} \right\} \left[\frac{g\rho h_{fg} d^3}{k\nu(T_{sat} - T_w)}\right]^{1/4}$$

the term in braces has the following values as a function of $c_p\Delta T/h_{fg}$

$\frac{c_p\Delta T}{h_{fg}}$	0.01	0.1	0.5	1.0	2.0
{ }	1.0017	1.017	1.078	1.146	1.260

From Fig. 3 in the text, it appears that the agreement is good for high Prandtl numbers.

The results for Prandtl number near unity should apply to film boiling (within the limits imposed on the derivation) with the

[5] Associate Professor of Chemical Engineering, University of California, Berkeley, Calif.

[6] L. A. Bromley, *Industrial and Engineering Chemistry*, vol. 44, 1952, pp. 2966–2969.

[7] W. M. Rohsenow, "Heat Transfer and Temperature Distribution in Laminar-Film Condensation," Trans. ASME, vol. 78, 1956, pp. 1645–1648.

physical properties of liquid and vapor interchanged. Of course, the assumption of no drag at the liquid-vapor interface would be poor.

Although I am sure the author is aware of other limitations of the derivation as applied to "laminar-film" condenser calculations, I think it is well to point out some of them to the reader.

1 Surface tension produces droplets covering appreciable areas on the underside of a horizontal tube.

2 Resistance at the vapor-liquid interface may be present either due to "accommodation or evaporation coefficient" effects or to presence of impurities at the interface.

3 Ripples may be present.

4 Vapor drag is seldom negligible.

Authors' Closure

The authors extend their thanks to Professor Bromley for his interesting discussion. From these comments, it will be possible to gain further physical insights into the problem.

First of all, it is found that, when the values tabulated by Professor Bromley are plotted on Fig. 3, good agreement is obtained with the Pr = 100 curve. Now, the Bromley-Rohsenow formulation was carried out under the assumption that the inertia forces (fluid accelerations) are negligible. This situation does occur either for very viscous liquids or for very thin condensate films. The viscous liquids are characterized by high Prandtl numbers; and so it is reasonable that the Bromley predictions should agree with the high Prandtl number results of the present analysis, where the effects of inertia forces are included. Thin films are characterized by small values of $c_p\Delta T/h_{fg}$; and according to Fig. 3, the results for all Prandtl numbers approach the Nusselt asymptote, which neglects the inertia terms.

The restricting assumptions mentioned at the end of Professor Bromley's discussion are, unfortunately, common to almost all condensation analyses. In terms at our present analytical skills these problems would be intractable were these simplifications not made.