formation of the main crack would continue to dominate throughout the crack's life, since the left side of the inequality remains essentially constant. Furthermore, it follows by similar reasoning that this conclusion would remain true even if the simple criterion (25) were replaced with any more general expression of the form $f(\Delta K_{II}/\Delta K_{Imax}) >$ constant, provided the function f is relatively smooth. Given that cracks of the type in question are initiated through a shear mechanism, mode II growth should therefore control, and this is indeed observed in experimental studies. Note that the small change in the ratio depicted in Fig. 5 that does occur favors this conclusion as well. Evidently, to consider further the possibility of crack branching, very long cracks (i.e., cracks quite close to the free surface in relative terms) need to be analyzed, a reliable criterion defining mode preference needs to be developed, and the possible role of factors not included here (e.g., other cracks, inclusions, etc.) should be examined. To carry out such analyses both the free surface and the details of the contact itself must be modeled to a greater degree of accuracy than was done here, since the scaling is such that the surface no longer could be considered remote.

Conclusions

An approximate characterization of the branching behavior of horizontal subsurface cracks under a moving compressive surface load has indicated that for short to medium length (relative length, c/d) cracks mode II growth is most likely to occur, although branching mode I growth is also possible. This conclusion is based on the results of stress intensity factor calculations, combined with a competing failure mode criterion in which the ratio $\Delta K_{II}/\Delta K_{Imax}$ is the critical parameter in determining the prefered growth mode. This criterion bears experimental scrutiny, and the analysis itself needs to be extended to consider relatively long (i.e., large c/d) cracks. Nevertheless, the conclusions are in agreement with experimental observations, and therefore may be viewed as providing a preliminary explanation of the phenomenon.

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DISCUSSION

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The author should be congratulated for presenting an interesting analysis to a complex problem. Although the approach is approximate, it may be viewed as providing some qualitative information about the mechanism of subsurface crack propagation. However, some important points need further clarification.

In regard to the use of linear elastic fracture mechanics as a means of predicting crack propagation, one of the main prerequisites is that the plastic zone at the crack tip should be smaller than a critical dimension such as the crack length or depth of crack location. This requirement may not be satisfied when the crack is small and/or close to the surface. In that case the stress intensity factor has no longer any physical significance. Has this requirement been satisfied in the present analysis?

Furthermore, the majority of the experimental studies has revealed that propagation of subsurface cracks eventually leads to formation of sheet-like wear particles [A1]. The length of these wear sheets is typically in the range 50–100 μm, whereas the thickness is 1–5 μm. This suggests that subsurface cracks in a compressive field grow in shear and parallel to the surface [A2, A3] until they reach a critical length (which is several times larger than the depth of the crack) before they break through the surface. Incidentally, the fact that the proposed simple criterion, equation (25), states that non-branching mode II growth is most likely the predominant mode of propagation in the case of short to medium length cracks essentially coincides with the former interpretation. For the analysis of the subsurface branching problem, therefore, a relatively large main crack must be considered, i.e., d/c values well below unity. Wear particles with length-to-thickness ratios similar to the d/c values of the present study may result, however, if a small crack is formed perpendicular to the surface and above the horizontal crack. Then the vertical crack has a strong tendency to propagate downwards towards the horizontal crack resulting thus in material flake off [A4].

Further, it is well known that small cracks may be open even during compressive portions of loading cycles. Thus the condition for the main crack to remain closed may not be representative of the situation.

Finally, a comment in reference to surface traction generation during sliding wear is appropriate. Contact stresses between mating surfaces are generated at the solid-to-solid contacts. At these contacts, termed asperity junctions, the load is distributed in a Hertzian fashion when extensive plastic deformation below the surface has not taken place. In developing a model for subsurface crack propagation it would seem, therefore, appropriate to include the size of the asperity contact and a contact stress distribution, rather than a concentrated force, especially when the subsurface crack is small and presumably of smaller size than the asperity contact width.

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Author's Closure

The points raised by the discusser essentially revolve around issues of scaling: i.e., what physical scaling limitations (such as the size of the plastic zone relative to crack length or crack depth) constrain the applicability of this idealized, nondimensional analysis to real cracks in real materials. These limitations were not considered in the analysis, and they will vary depending on the specific materials and geometries involved. For the reasons listed by the discusser, it is clear that the present analysis can not be applied directly to the mechanics of surface wear in ductile materials, in which the relevant dimensions are on the order of microns, and the material state near the surface is far from being linear or elastic. However, for cracks that are remote from the surface, or for materials that are relatively brittle, or for geometries in which the relevant dimensions are large, the analysis should be consistent with the actual mechanics involved in the failure process. Some of the scaling problems identified by the discusser are relatively simple to treat using a more detailed analysis, such as replacing the point load with a contact stress distribution, or approximately including mild residual stress effects. Such a modified analysis is in fact presently underway. It is unlikely, however, that the more complicated material behaviors could be modeled consistently with the techniques employed in this analysis.