Soft X-ray transients in the Hertzsprung gap

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ABSTRACT
We apply the disc instability model for soft X-ray transients to identify system parameters along evolutionary sequences of black hole X-ray binaries (BHXBs) that are consistent with transient behaviour. In particular, we focus on the hitherto neglected group of BHXBs with intermediate-mass giant donor stars. These spend a significant fraction of their X-ray active phase crossing the Hertzsprung gap.

Three case B binary sequences with a black hole accretor and $2.5 - 5M_{\odot}$ initial donor mass are presented in detail. We formulate rules which summarize the behaviour of these sequences and provide an approximate description for case B mass-transfer in intermediate-mass BHXBs. Chiefly, the time-scale of the overall radius expansion is given by the initial donor mass, while the surface appearance is determined by the current donor mass.

With these rules we obtain a general overview of transient and persistent behaviour of all intermediate-mass BHXBs by just considering single star sequences of different mass. We find that although systems in the process of crossing the Hertzsprung gap are in general persistently bright, with Eddington or super-Eddington transfer rates, there is a narrow instability strip where transient behaviour is possible. This strip extends over a secondary mass range $2.0 \leq M_2 \leq 3.5 M_{\odot}$. GRO J1655--40 might be such a system. We predict that there are no BHXB transients with (sub)giant donors more massive than $3.5 M_{\odot}$, and no neutron star transients in the Hertzsprung gap.

Key words: accretion, accretion discs – black hole physics – binaries: close – stars: evolution – stars: individual: GRO J1655--40.

1 INTRODUCTION
Low-mass X-ray binaries (LMXBs) are semidetached compact binaries with a black hole (BH) or neutron star (NS) primary that accretes mass from an accretion disc. A Roche lobe filling low-mass main-sequence or (sub)giant star feeds the disc. LMXBs appear in two main varieties: they are either persistently bright or have outbursts and long phases of quiescence (e.g. White & van Paradijs 1996). The latter, known as soft X-ray transients (SXTs: cf. Tanaka & Shibazaki 1996 and Chen, Shrader & Livio 1997 for reviews), are believed to have an unstable accretion disc which alternates between a hot and cool state (Cannizzo, Wheeler & Ghosh 1982; Lin & Taam 1984; Cannizzo, Chen & Livio 1995; King & Ritter 1998), similar to dwarf novae among cataclysmic variables.

A simple criterion for stable disc accretion is that the temperature $T_d$ at the outer disc edge is larger than the hydrogen ionization temperature $T_H = 6500$ K (e.g. King, Kolb & Burderi 1996a, hereafter KKB). As $T_d$ increases with the mass-transfer rate $\dot{M}$, a system is persistently bright if $\dot{M}$ is larger than the critical rate $\dot{M}_{\text{crit}}$ where $T_d = T_H$. The system is transient if $\dot{M} < \dot{M}_{\text{crit}}$.

An important difference between dwarf novae and SXTs is the dominant role that irradiation of the accretion disc by the central accreting source plays in LMXBs. Van Paradijs (1996) showed that the observed critical transfer rate separating transient from persistent neutron star LMXBs is much lower than the critical rate separating dwarf novae from persistently bright (nova-like) cataclysmic variables. This can be understood quantitatively by assuming that irradiation dominates over viscous heating at the outer disc rim, hence stabilizing the disc than in the absence of irradiation at a smaller mass-transfer rate. Similarly, irradiation naturally explains the observed large ratio of optical to X-ray flux from the disc and the observed long, quasi-exponential decline after an SXT outburst (King & Ritter 1998).

Models for the evolution of LMXBs predict the mass-transfer rate and thus, by comparison with $\dot{M}_{\text{crit}}$, the appearance of the system as a transient or persistently bright X-ray source (assuming that the instantaneous transfer rate is close to the evolutionary mean transfer rate). This is a powerful diagnostic tool for testing evolutionary models, or, in turn, the disc instability model for SXTs. King, Kolb & Szuszkiewicz (1997b, hereafter KKS), King et al.

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can be as long as 10^9 yr if both are small, but is typically 10^7 - 10^8 yr. Conversely, j-driven systems are short-period binaries with main-sequence donors and evolve to shorter orbital period. If the j-driven evolution is the same as for cataclysmic variables (see e.g. Kolb 1996, King 1988 for reviews) the bright phase where magnetic braking operates (P \geq 3 h) typically lasts 10^8 yr. For P \leq 3 h mass-transfer continues indefinitely at a lower rate, but the period increases again when the secondary becomes degenerate, i.e. when its mass is \leq 0.06 M_\odot.

A critical bifurcation period P_b = 1.2 d (Plyser & Savonije 1988, 1989) separates j-driven and n-driven LMXBs. Systems born with P < P_b evolve towards shorter periods, and those born with P > P_b towards longer P. Similarly, the initial donor mass M_2 plays a role in determining the group membership. If M_2 \leq 0.8 M_\odot the system is j-driven, as the main-sequence lifetime of the donor is longer than a Hubble time. If M_2 \geq 1.5 M_\odot the system cannot evolve to shorter orbital periods as magnetic braking does not operate in such massive stars, and gravitational radiation alone is too weak to dominate the evolution. In the intermediate range 0.8 \leq M_2 \leq 1.5 both groups exist. Mass-transfer stability defines an upper limit for the mass ratio q = M_2/M_1 (M_1 is the donor mass, M_1 the primary mass), hence for M_2 and M_1. This limit depends on the response of the donor to mass loss (e.g. Hjellming 1989), i.e. it is a non-trivial function of the evolutionary state. As a rule, the limit is roughly M_2 \leq M_1 in the case of conservative mass-transfer, but can be somewhat larger with mass loss from the system (cf. Kalogera & Webbink 1996, where the NS case is discussed in detail). If the donor is more massive the system suffers a short, violent phase of mass-transfer/loss and is unlikely to appear as an X-ray source. Obviously, neutron star X-ray binaries with Roche lobe filling donors more massive than 2 - 3 M_\odot do not exist, while there is no such upper limit for BHXBs; hence no clear separation between low-mass and high-mass BHXBs. The importance of BHXBs with intermediate-mass donors was first pointed out by Romani (1994).

Recently, two limiting cases of LMXB evolution have been considered to determine the incidence of transient behaviour among LMXBs. KKB and KKS described the j-driven evolution of completely unevolved zero-age main sequence (ZAMS) donors. They found that BH systems are always transient, consistent with observations, and that donor stars in NS systems need to be very close to the end of core H burning in order to be transient, consistent with the observed small fraction of transient NS LMXBs. King et al. (1997a) and KKS studied n-driven LMXBs with low-mass donor stars well-established on the first giant branch, and found that essentially all of them are transient. They considered giant donors with a thin H burning shell source above a degenerate He core where the stellar radius R and luminosity L are unique functions, R \propto M_1^{2/5}, L \propto M_1^{8/5}, of the slowly growing core mass M_1, essentially independent of the total stellar mass (e.g. Webbink, Rappaport & Savonije 1983). Stars with mass \lesssim 2 M_\odot indeed establish such a structure soon after the end of central H burning and spend a considerable time ascending the first giant branch, before the ignition of helium burning in the centre terminates their radius expansion. Most n-driven NS LMXBs are well described in this way, but not BHXBs with more massive donor stars (M \gtrsim 2.0 - 2.5 M_\odot). These ignite core helium burning before the core becomes highly degenerate, i.e. before the first giant branch evolution governed by the above core mass relations takes hold. Simplified descriptions for the evolution of these stars fail, and full stellar models are needed.

We consider these n-driven intermediate-mass BHXBs in the next sections.

### 2 EVOLUTION OF LMXBs

The mechanism driving mass-transfer naturally divides LMXBs into two distinct classes. In the first group, nuclear expansion of the secondary maintains the semidetached state, while in the second group, orbital angular momentum losses such as gravitational radiation and magnetic braking drive mass-transfer. For convenience we denote neutron star or black hole LMXBs in the two groups by n-driven or j-driven systems.

Generally, n-driven LMXBs are long-period systems with (sub)-giant donor stars on the first giant branch. Hydrogen burns in a shell source above a nuclearly inactive helium core. The orbital period increases until mass-transfer terminates when core helium burning ignites, or when the hydrogen-rich envelope is fully transferred to the primary, whichever happens earlier. The final state is a wide, detached binary with a white dwarf secondary and a BH or NS primary (possibly a millisecond pulsar). The LMXB lifetime depends on the initial secondary mass and orbital distance and can be as long as 10^9 yr if both are small, but is typically 10^7 - 10^8 yr.

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### 3 ILLUSTRATIVE CASE B MASS-TRANSFER SEQUENCES WITH BLACK HOLE ACCRETORS

The binary evolutionary phase we consider here is case B mass-transfer: the donor star fills its Roche lobe after termination of core H burning but before the ignition of core He burning. Case B mass-transfer is well-studied (e.g. Kippenhahn & Weigert 1967; Plyser & Savonije 1988; Kolb & Ritter 1990; De Greve 1993) in the context of Algol evolution. All these studies assume that the donor is more massive than the accretor, simply because the faster evolving, more massive component always fills its Roche lobe first. In BHXBs, with a more complicated evolutionary history, case B mass-transfer begins with an already inverted mass ratio, i.e. the donor is less massive than the BH. Hence there is no initial rapid phase with extremely high transfer rate.

In the following we present calculations of three case B mass-transfer sequences on to a BH primary from a secondary with
initial mass close to the transition region between degenerate and non-degenerate helium ignition. We report briefly on the numerical technique and associated problems (Section 3.1), describe in detail the evolution of the secondary star (Section 3.2), and then focus on the resulting appearance in X-rays along the sequences in Section 3.3.

3.1 Input parameters and computational technique

We used Mazzitelli’s stellar evolution code in a version as described by Mazzitelli (1989; see also references therein) with pre-OPAL opacities. The computations started from chemically homogenous ZAMS models with a Population I mixture (X = 0.70, Y = 0.28). Convection is treated by the standard mixing-length theory; a calibration to a solar model determines the mixing length parameter to be 1.4 (no overshooting was allowed).

To allow the application to binary evolution the mass-transfer rate \( M \) was calculated for each time-step according to

\[
M \propto \exp\left( \frac{\Delta R}{H} \right) \tag{1}
\]

(cf. Kolb & Ritter 1990), where \( \Delta R = R - R_L \) is the difference between the radius \( R \) of the secondary and the Roche lobe radius \( R_L \), \( H \) the atmospheric pressure scaleheight, and \( k \) is a numerical constant, usually taken as \( k = 1 \). Although (1) is strictly valid only for \( \Delta R < 0 \) and has to be replaced by an expression with an approximate power-law dependence on \( \Delta R/H \) for \( \Delta R > 0 \), we use (1) for any \( \Delta R \). Furthermore, to avoid numerical instabilities at high transfer rates we set \( k = 10 \) for \( M \geq 10^{-7} \, M_\odot \, \text{yr}^{-1} \). This procedure is justified as long as \( \Delta R/R \ll 1 \) (Kolb & Ritter 1990), i.e. as long as the stellar radius is close to the critical Roche radius. For the sequences presented here this is always the case. A high-resolution reference sequence with \( k = 1 \) was calculated to check the validity of sequences obtained with \( k = 10 \) explicitly in the case of sequence S3 (see below). Attempts to retain the damping factor \( k = 10 \) also in later phases (where \( \Delta R < 0 \) failed as \( H/R \) becomes non-negligible for extended giants. A damped evolution would proceed qualitatively differently from the true evolution with \( k = 1 \), leading to higher mass-transfer rates and a correspondingly earlier termination of mass transfer.

3.2 Detailed description of the sequences

All three binary sequences, S1–S3, begin mass-transfer with a 2.5-\( M_\odot \) donor star and a 6.8-\( M_\odot \) black hole. The initial orbital separation is \( a_1 = 13.2 \) (S1), 15.2 (S2) and 18.5 R_\odot \) (S3); see the summary in Table 1. Mass transfer is conservative, i.e. the total binary mass and orbital angular momentum are constant. Sequence S3 follows the donor star until ignition of central helium burning. The subsequent evolution is detached, and we evolved the secondary further through central helium burning and the brief subsequent asymptotic giant-branch phase with weak thermal pulses. In contrast, sequences S1 and S2 have been terminated once the secondary was established on the first giant branch.

In Figs 1–5 we show characteristic parameters along the sequences S1–S3, sometimes together with the evolution of a 2.5-\( M_\odot \) single star (for convenience referred to as sequence S0).

Fig. 1 (upper panel) reveals that the overall radius evolution with time is very similar in all sequences S0–S3, except for the ‘plateau phase’ soon after mass-transfer turn-on, which corresponds to a brief radius-contraction phase at the red end of the Hertzsprung gap of S0. This phase occurs at a characteristic radius, the ‘plateau radius’ \( R_p \), and will be examined in more detail below. The similarity shows that significant mass loss from the outer envelope

<table>
<thead>
<tr>
<th>sequence</th>
<th>( M_1/M_\odot )</th>
<th>( M_2/M_\odot )</th>
<th>( a/R_\odot )</th>
<th>int</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1 (initial)</td>
<td>6.8</td>
<td>2.5</td>
<td>13.2</td>
<td>b860</td>
</tr>
<tr>
<td>S2 (initial)</td>
<td>6.8</td>
<td>2.5</td>
<td>15.2</td>
<td>b849</td>
</tr>
<tr>
<td>S3 (initial)</td>
<td>6.8</td>
<td>2.5</td>
<td>18.5</td>
<td>b830/32</td>
</tr>
<tr>
<td>S3 (final)</td>
<td>8.726</td>
<td>0.574</td>
<td>213.1</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Radius \( R \) (upper panel) and effective temperature \( T_{\text{eff}} \) (lower panel) of a 2.5-\( M_\odot \) single star (dashed line) and the secondary star in the binary sequence S1 (dash–dotted line), S2 (thick solid line) and S3 (thin solid line), cf. Table 1, as a function of time elapsed since the (donor) star left the main sequence.

Figure 2. Surface luminosity \( L \) (top), nuclear luminosity \( L_{\text{nuc}} \) (middle panel) and absolute value of the gravothermal luminosity \( L_g = L - L_{\text{nuc}} \) (bottom) of a 2.5-\( M_\odot \) single star (dashed line) and the secondary star in binary sequences S1–S3, as a function of time elapsed since the (donor) star left the main sequence. The line style is as in Fig. 1.
Figure 3. Surface luminosity $L$ as a function of core mass $M_c$ (defined as the mass enclosed by the shell where the H burning energy-generation rate is maximal). Full triangles: selected models along sequence S3. The triangles with smallest and largest $M_c$ correspond to models at the luminosity minimum and close to central He ignition, respectively. The open triangle marks a 2.5-$M_\odot$ single star at its luminosity minimum. The positions of models with larger $M_c$ along the single star sequence coincide with the positions of sequence S3. The solid curve is the core-luminosity relation for low-mass ($<1.5$ $M_\odot$) giant stars, cf. King et al. (1997a).

Figure 4. Mass-transfer rate $\dot{M}$ as a function of time (elapsed since the donor left the main sequence) for sequences S1 (bottom panel), S2 (middle panel) and S3 (upper panel). The line style is broken when the mass-transfer rate is smaller than the critical rate $\dot{M}_\text{crit}$ for disc instability (from KKS) for sequence S1. The mass-transfer rate $\dot{M}$ as a function of core mass $M_c$ (defined as the mass enclosed by the shell where the H burning energy-generation rate is maximal). Full triangles: selected models along sequence S3. The triangles with smallest and largest $M_c$ correspond to models at the luminosity minimum and close to central He ignition, respectively. The open triangle marks a 2.5-$M_\odot$ single star at its luminosity minimum. The positions of models with larger $M_c$ along the single star sequence coincide with the positions of sequence S3. The solid curve is the core-luminosity relation for low-mass ($<1.5$ $M_\odot$) giant stars, cf. King et al. (1997a).

Figure 5. Mass-transfer rate $\dot{M}$ as a function of effective temperature $T_{\text{eff}}$ for sequences S1 (dash–dotted line), S2 (thick solid line) and S3 (thin solid line). Also shown is the critical rate for disc instabilities (from KKS) for sequence S1. We distinguish transient from persistent BHXBs by comparing the mass-transfer rate $\dot{M}$ with the critical rate $\dot{M}_\text{crit}$ for which hydrogen shell. The net effect is that thin hydrogen-shell burning is established earlier, at a slightly smaller core mass. Once thin H burning is fully established the luminosity grows with further growing core mass. The corresponding effective core mass–luminosity relation is much steeper than the standard relation $L \propto M_c^8$ for fully degenerate cores, and meets the latter one only immediately before core helium ignition (Fig. 3). As a result, the secondary in S1–S3 begins its ascent along the first giant branch at a significantly smaller surface luminosity than the 2.5-M$_\odot$ single star. The drop of $L$, a familiar feature of case B mass transfer, occurs along with a correspondingly fast decrease of $T_{\text{eff}}$ to log $T_{\text{eff}}$/K = 3.70, i.e. the secondary in the mass-loss sequence crosses the Hertzsprung gap even faster than the 2.5-M$_\odot$ single star (Fig. 1, lower panel). Roughly, at any time along S1–S3 in the Hertzsprung gap, $L$ and $T_{\text{eff}}$ are the same as for a single star with the same mass and radius as the secondary star at that time.

With the ignition of helium burning the radius of the secondary decreases sharply and mass-transfer terminates. At this point the binary parameters of S3 are $M_1 = 8.726\, M_\odot$, $M_2 = 0.574\, M_\odot$ (with core mass $0.320\, M_\odot$) and $P = 118.3$ d, $a = 213.1$ R$_\odot$ (see Table 1). For the next 270 Myr the secondary burns helium in the centre and is well inside its Roche lobe ($R = 8–10$ R$_\odot$ versus $R_L = 34.6$ R$_\odot$). Subsequently, He burning moves to a shell source and the radius increases again, at a core mass of $\approx 0.420\, M_\odot$. The ensuing double-shell burning is unstable and leads to thermal pulses. We find that these pulses are rather weak, with decreasing maximum radius. The radius maximum in subsequent pulses is always smaller than the Roche radius (though not very much), and quickly decreasing. Eventually the secondary contracts towards the white dwarf stage. If there are no significant wind losses during the thermal pulses the final state of S3 is a wide, detached binary with a 9-M$_\odot$ black hole primary, a $\approx 0.55\, M_\odot$ carbon–oxygen white dwarf companion and a 5.6 month orbital period.

### 3.3 Transient versus persistent accretion

We distinguish transient from persistent BHXBs by comparing the mass-transfer rate $\dot{M}$ with the critical rate $\dot{M}_\text{crit}$ for which hydrogen...
just ionizes at the outer disc edge. A system is transient if $M < M_{\text{crit}}$. In Fig. 4 we plot $M$ as a function of time along sequences S1–S3. The linestyle is broken if $M < M_{\text{crit}}$ with $M_{\text{crit}}$ taken from KKS. Obviously, the accretion disc is stable for about $4-5$ Myr when the secondary crosses the Hertzsprung gap, and the transfer rate is close to (or slightly above) the Eddington limit

$$M_{\text{edd}} = 2 \times 10^{-8} \, M_\odot \, \text{yr}^{-1} \left( \frac{M_i}{M_\odot} \right)$$

(assuming an effective accretion efficiency $\eta = L_{\text{acq}}/M_\odot c^2 = 0.1$). The system becomes a transient source soon after it begins to ascend along the Hayashi line, and remains transient until helium ignites in the centre $\approx 12$ Myr later. Significantly, there is also a short transient phase (lasting $\approx 1$ Myr) right in the Hertzsprung gap, centred around $\log T_{\text{eff}}/K = 3.73$. This phase is caused by the transition from thick-to thin-shell source burning, the same effect that leads to a brief radius-contraction when the 2.5-M$_\odot$ single star crosses the Hertzsprung gap. The associated change of the internal structure of the star is highly non-linear and complex. Just as the ultimate cause for the expansion to the red giant state cannot be easily derived from a simple analysis of the stellar structure equations, there is no easy way to understand this temporary halt in the expansion either.

The Hertzsprung-gap transient phase in S1–S3 and the radius-contraction phase of S0 all occur at the same range of $T_{\text{eff}}$ (Fig. 5), while the corresponding plateau radius $R_p$ is largest for S0, being progressively smaller the earlier mass transfer begins in the Hertzsprung gap. The secondary mass in the plateau phase is $1.9, 2.05$ and $2.25$ M$_\odot$ for sequences S1, S2 and S3, respectively. In fact, $R_p(M_2)$ is practically the same as the plateau radius of a single star with the mass of the secondary in the plateau phase.

Details of the exact location (e.g. the radius $R_p$), duration and depth of the radius-contraction phase of a single star must necessarily depend on details of the stellar input physics, in particular on opacities and treatment of convection. As an example, the 2.5-M$_\odot$ single star track obtained by Salaris et al. (1997) with OPAL opacities shows a much less pronounced radius-contraction phase at a slightly higher effective temperature ($\log T_{\text{eff}}/K = 3.75$) than our sequence S0. Hence we do not expect our models to reproduce the precise location of the real transient phase in the Hertzsprung gap and the actual mass-transfer rate in this phase. Nevertheless, our sequences certainly show the differential change of that phase with varying initial orbital distance: the earlier mass-transfer starts in the Hertzsprung gap, the less pronounced the decrease of $M$ in the transient phase. If the secondary is already very close to the plateau phase when it fills its Roche lobe for the first time, the system might almost detach in the transient phase and not appear as an X-ray source at all; but if the system begins mass transfer early in the Hertzsprung gap it certainly will appear as an X-ray source.

4 GENERALIZATION: INTERMEDIATE-MASS BHXB EVOLUTION

We formulate three rules which summarize the behaviour of the above evolutionary sequences and provide an approximate description for the case B evolution of intermediate-mass BHXBs.

(R1) The overall time evolution $R(t)$ of the donor star radius in a binary sequence with initial donor mass $M_2$ is the same as the time evolution of the radius of a single star with mass $M_2 = M_3$,

(R2) the plateau phase (the brief contraction phase or slow-down of the expansion phase) in the Hertzsprung gap occurs at a donor radius $R_p$ equal to the plateau radius of a single star with the mass of the donor

(R3) a donor star with mass $M_2$ and radius $R$ has the same effective temperature $T_{\text{eff}}$ and radius $R$.

In other words, the time-scale of the overall radius expansion is given by the initial donor mass, while the surface appearance and the plateau phase are given by the current donor mass.

In the following we use these rules to investigate the incidence of transient and persistent behaviour in intermediate-mass BHXBs with arbitrary initial mass and initial separation, simply by considering the evolution of single stars in this mass range.

In order to do so we note that in the case of stationary mass transfer $(R/R = \bar{R}_i/\bar{R}_s)$ the transfer rate is given by

$$\dot{M} = M_2 \frac{K}{\bar{R}_i} - \bar{R}_s$$

(cf. e.g. Ritter 1996). Here $K = (d \ln R/dt)_{\text{const.}}$ is the radius expansion in the absence of mass loss (e.g. due to nuclear evolution), $\bar{R}_s$ is the thermal equilibrium mass–radius exponent and $\bar{R}_i$ is the Roche-lobe index ($\bar{R}_i = 2M_2/M_1 - 5/3$ for conservative evolution). We set $\bar{R}_s = 0$ and, using (R1), approximate $K$ by $d \ln R/dt$ along the single star sequence. It is clear that these secondaries are not in thermal equilibrium (Fig. 2), and their radius expansion is not purely nuclear. However, the mass loss perturbs the overall radius evolution only slightly.

Then we obtain the approximate binary evolution for given initial donor mass $M_2$ by integrating $\dot{M}(t)$, i.e. $K(t)$ of a single star with mass $M_0 = M_2$, over time. To decide the stability of the accretion disc we need to know the mass-transfer rate $\dot{M}$ as a function of orbital period $P$. The main characteristics of the real evolutionary tracks $M(P)$ are already evident from Fig. 6 where we plot the mass-transfer rate $\dot{M}$ soon after turn-on of mass transfer, estimated from

![Figure 6](https://academic.oup.com/mnras/article-abstract/297/2/423/988364/19988354/download)

**Figure 6.** Transfer rate $\dot{M}$ versus orbital period $P$ at turn-on of mass transfer, for three different donor masses (full line: $3.5$ M$_\odot$; dashed line: $2.5$ M$_\odot$; dotted line: $1.3$ M$_\odot$), estimated from equation (3) and assuming a 8-M$_\odot$ primary. The horizontal line indicates the corresponding Eddington transfer rate, see (2). The thick solid line is the critical mass accretion rate $M_{\text{crit}}$ separating transient ($M < M_{\text{crit}}$) from persistent ($M > M_{\text{crit}}$) systems, taken from KKS.
Figure 7. Orbital period–secondary mass (P–M₂) plane for BHXBs, showing the exclusion zone for transients (unhatched). A BH mass of 8 M☉ was assumed. Systems in the unhatched region are always persistent; systems in the narrow hatched instability strip are always transient. Systems that are born in the large hatched area are always transient, but systems that have evolved from the unhatched into the large hatched area will remain persistent for some time before they, too, become transient. Evolutionary tracks for sequences S1–S3 are also shown (solid where persistent, dashed where transient). The heavy dot along each sequence marks the location of the transient phase in the Hertzsprung gap. The dash–dotted line is the plateau phase. This strip terminates at 5 log \( \dot{M}/M_{\odot} \) = 2, while the flat branch moves to longer \( \dot{M}/M_{\odot} \). As Fig. 7, but with the critical boundary between transient and persistent systems for different BH masses short-dashed line: 5 M☉; solid line: 10 M☉; dotted line: 15 M☉. The cross indicates the observed system parameters of GRO J1655–40 (Orosz & Bailyn 1997). The solid curves represent evolutionary tracks assuming conservative evolution (full line) and evolution with constant black hole mass (the mass leaving the system carries the specific orbital angular momentum of the black hole; long-dashed line).

(3), as a function of initial orbital period \( P_i \) for secondaries in the mass range of interest (\( M_2 = 1.3–3.5 \) M☉). Roche geometry determines the initial period to \( \log(P_i/\text{d}) = 1.5 \log(R/R_{\odot}) - 0.5 \log(M_2/M_{\odot}) - 0.433 \). Also shown is the critical period \( M_{\text{crit}} \) for disc instability, taken from KKS, and the Eddington accretion rate (2) for a typical 8-M☉ BH accretor.

For \( M_2 \approx 1.9 \) M☉ we find that \( M_i < M_{\text{crit}} \) at any \( P_i \), while for \( M_2 \approx 1.9 \) M☉ the transfer rate is smaller than \( M_{\text{crit}} \) either if the donor is in the plateau phase, \( P_i = P_p(M_2) \), or if \( P_i \) is sufficiently long, i.e. longer than a critical period \( P_{\text{crit}} \) which is the total binary \( M_{\text{crit}} \) (cf. the example in Fig. 8).

A simple corollary of (R2) is that a system will reach the transient plateau phase when its evolutionary track crosses the instability strip in Fig. 7. This is because \( P_{\text{crit}} \) is a unique function of the donor mass, independent of the initial donor mass. In contrast, as the initial donor mass dictates the mass-transfer rate outside the plateau phase, we can expect that the critical period where the system changes from a persistently bright to a transient X-ray source is close to \( P_{\text{crit}}(M_2) \). Then, as a consequence of the opposite slopes of evolutionary tracks and the curve \( P_{\text{crit}}(M_2) \) in Fig. 7, a system remains persistently bright when its track leaves the unhatched region, and becomes transient only later when its period roughly equals \( P_{\text{crit}}(M_2) \). The sequences S1–S3 (solid where persistent, broken where transient; heavy dots mark the plateau phase) and actual integrations of (3) confirm this. Hence no system in the unhatched region is transient; it is a ‘transient exclusion zone’, bisected by the Hertzsprung gap instability strip. The hatched region, on the other hand, may contain both persistently bright and transient systems.

Fig. 8 shows how the transient exclusion zone depends on the BH mass \( M_i \). As \( M_i \) increases, the steep branch of \( P_{\text{crit}}(M_2) \) moves to larger \( M_2 \), while the flat branch moves to longer \( P \). The BH mass does not affect the location of the instability strip.
5 SUMMARY AND DISCUSSION

In extension of the work by KKB, KKS and King et al. 1997a we studied the evolution of BHXBs with intermediate-mass giant donor stars in the context of the disc instability model for soft X-ray transients. These represent a hitherto somewhat neglected group of systems with very high mass-transfer rates and donors spending a significant fraction of the X-ray active time in crossing the Hertzsprung gap. We find that although systems in the process of crossing the Hertzsprung gap are in general persistently bright with Eddington or super-Eddington transfer rates, there is a narrow instability strip where transient behaviour is possible. This strip extends over a secondary mass range 2.0 \( \leq M_2/M_2 \leq 3.5 \) and is roughly given by \( P/d = 4.3 M_2/M_2 - 6.2 \), but the precise location is subject to uncertainties in the stellar input physics. We predict that BHXBs in the Hertzsprung gap are not transient if the donor is more massive than \( = 3.5 M_2 \), and that neutron star LMXB transients in the Hertzsprung gap do not exist. The latter is a consequence of a number of factors: in neutron star LMXBs the upper (stability) limit for the donor mass is smaller, the mass-transfer rate is larger as the systems are closer to mass-transfer instability, and the critical rate \( M_{\text{crit}} \) is possibly lower by almost an order of magnitude (see KKS).

In our considerations we relate the evolutionary state of a system to the stability of its accretion disc, hence its appearance in X-rays, by comparing the evolutionary mean mass-transfer rate \( M \) with the critical transfer rate \( M_{\text{crit}} \) for disc instability. Both the identification of the actual, instantaneous mass-transfer rate with \( M \) and the normalization of \( M_{\text{crit}} \) are subject to some uncertainties.

If the actual mass-transfer rate deviated from the calculated evolutionary mean the system could of course be transient where it is predicted to be persistently bright, and vice versa. Although such deviations from the secular mean rate are quite common in cataclysmic variables (CVs) (e.g. Warner 1987) there is no observational evidence for a similar large scatter of the mass-transfer rate at a given orbital period in LMXBs (cf. van Paradijs 1996; Chen et al. 1997). Also, the most promising theoretical explanation for the observed spread of \( M \) in CVs is that the systems undergo mass-transfer limit cycles caused by weak irradiation of the secondary (Ritter, Zhang & Kolb 1995; King et al. 1996b). In LMXBs this mechanism does not work as the irradiating flux on the secondary is too large (cf. King 1998). Hence it seems not unlikely that the actual transfer rate is close to the evolutionary mean.

The theoretical value of the critical rate \( M_{\text{crit}} \) depends on a number of not very well determined quantities. Most notably these are the hydrogen ionization temperature in the disc, the relative disc thickness, the accretion efficiency \( \eta \) and the albedo. A major justification for the adopted normalization comes from the fact that it matches the observed location of the critical rate separating transient from persistent sources (van Paradijs 1996). The remaining uncertainty in the normalization of \( M_{\text{crit}} \) does not affect the main results of this study, i.e. the existence and location of the transient instability strip in the Hertzsprung gap. This is a result of the steep gradient \( dM/dP \) of the transfer rate in this phase.

The transient X-ray source GRO J1655–40 is the first confirmed member of the class of intermediate-mass BHXBs considered in this paper. GRO J1655–40 is also close to the SXT instability strip and, given the uncertainties in both the theoretical and observed values, might actually be a system right in the instability strip, consistent with its transient nature (e.g. Harman et al. 1995; Tavani et al. 1996; Levine et al. 1996). Implications of this interpretation are discussed elsewhere (Kolb et al. 1997). Particular problems arise from the apparent lack of persistently bright systems with similar parameters in the Hertzsprung gap. It was suggested that these are not seen in X-rays as their effective photosphere might radiate at much longer wavelengths. Alternatively, GRO J1655–40 could be one of the predicted systems in the Hertzsprung gap with a stable accretion disc. It is by no means clear if such systems do appear as persistently bright sources. Instabilities in the accretion flow might cause dramatic changes in the effective photosphere, causing variability in a given waveband. Kolb et al. (1997) also suggested such a mechanism for GRS 1915+105, the other X-ray source with apparent superluminal motion.

A second possible intermediate-mass BHXB is 4U 1543–47. Recently, Orosz et al. (1998) determined its orbital period as 1.123 d and found that the components are likely to be a \( = 7 M_\odot \) BH and a 2.1–2.5 \( M_\odot \) main-sequence donor. This places the system below the turn-off main-sequence period (\( P_{\text{TMS}} \)) line in Fig. 7 (dashed-dotted line; note that \( P_{\text{TMS}} \) as shown in the equivalent fig. 2 of Kolb et al. 1997 is slightly too small due to a calibration error). Therefore 4U 1543 – 47 is, unlike GRO J1655–40, in a phase of case A mass transfer where the transfer rate is determined by the nuclear timescale of the donor. With \( M_2 = 2.3 M_\odot \) and replacing \( K \) in (3) by the nuclear expansion \( \Delta n/dt \) on the main sequence we estimate \( M_2 = 4 \times 10^{-8} M_\odot \) yr\(^{-1} \), well below the critical rate KKS find for transient behaviour in BHXBs (but slightly above the critical rate claimed for neutron star systems, cf. KKB). A more detailed account of case A mass transfer in BHXBs is in preparation.

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