The runaway instability of self-gravitating tori with non-constant specific angular momentum around black holes

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ABSTRACT

We discuss the runaway instability of axisymmetric tori with non-constant specific angular momentum around black holes, taking into account self-gravity of the tori. The distribution of specific angular momentum of the tori is assumed to be a positive power law with respect to the distance from the rotational axis. By employing the pseudo-Newtonian potential for the gravity of the spherical black hole, we have found that self-gravity of the tori causes a runaway instability if the amount of the mass which is transferred from the torus to the black hole exceeds a critical value, i.e. 3 per cent of the mass of the torus. This has been shown by two different approaches: (1) by using equilibrium models and (2) by dynamical simulations. In particular, dynamical simulations using an SPH code have been carried out for both self-gravitating and non-self-gravitating tori. For non-self-gravitating models, all tori are runaway stable. Therefore we come to the conclusion that self-gravity of the tori has a stronger destabilizing effect than the stabilizing effect of the positive power-law distribution of the angular momentum.

Key words: accretion, accretion discs – black hole physics – instabilities.

1 INTRODUCTION

From the recent BATSE experiment, the sources of $\gamma$-ray bursts have been proven to distribute isotropically on the sky but non-uniformly in the radial direction (see e.g. Meegan et al. 1992; Fishman & Meegan 1995). Since that discovery, toroidal configurations around black holes have become of much importance, because it has been suggested that they can be candidates for the sources of $\gamma$-ray bursts at cosmological distances (see e.g. Murakami et al. 1988; Paczyński 1991; Narayan, Paczyński & Piran 1992).

Even before the BATSE experiment, such systems had been analysed from the standpoint of stability of mass transfer from tori to black holes by many groups. In particular, Abramowicz, Calvani & Nobili (1983) first proposed the concept of runaway instability. They argued that once a small amount of mass begins to fall into a black hole from the surrounding Roche-lobe-filling torus, the remaining mass of the torus overflows from the Roche lobe and continues falling if the mass of the torus is above 1 per cent of the black hole. Their conclusion was obtained by using the so-called Paczyński–Wiita pseudo-Newtonian potential (Paczynski & Wiita 1982) and by neglecting the angular momentum transfer. Wilson (1984) investigated relativistic tori without self-gravity around Kerr black holes and concluded that the angular momentum transfer makes the configurations stable.

To resolve the controversy between these two conclusions, Nishida et al. (1996) and Nishida & Eriguchi (1996) took a fully general relativistic approach to this problem including self-gravity of the tori. They found that massive tori are subject to runaway instability and concluded that self-gravity plays a very significant role in destabilization. It should be noted that all these investigations mentioned above have been done for tori with constant angular momentum distribution.

Recently Daigne & Mochkovitch (1997) showed that the tori the angular momentum distribution of which increases enough with distance are runaway stable. Although their analysis was carried out in the Newtonian framework and without taking self-gravity of the torus into account, it is likely that the non-zero gradient of the angular momentum distribution has a strong stabilizing effect because the removal of a very small mass at the inner part with small angular momentum scarcely affects the global structure of the tori. Abramowicz, Karas & Lanza (1998) also showed that the outwardly increasing angular momentum distribution of the torus and the spin of the black hole have a stabilizing effect by investigating non-self-gravitating tori in the Kerr geometry.

In contrast, recent dynamical simulation of the mass overflow from the self-gravitating torus to the black hole showed that Roche-lobe-filling tori around black holes are unstable against mass overflow for a wide variety of angular momentum distributions (Masuda & Eriguchi 1997). In other words, runaway instability does occur for any rotation law. In that simulation the pseudo-Newtonian potential was used and computations were started by applying rather large initial perturbations, i.e. the order of 10 per cent.

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This is a new contradiction for runaway instability. However, the reason for this discrepancy seems evident, i.e. the existence of self-gravity. Therefore we need to investigate a simple problem – is the torus with both self-gravity and an outwardly increasing angular momentum distribution stable? In this paper we will give the answer to this question by employing two different approaches. First, as in all other works done by different groups, equilibrium configurations are used and the stability is analysed. Secondly, we perform SPH dynamical computations of self-gravitating and non-self-gravitating tori to confirm the results of the equilibrium analysis.

The final and definite answer will be obtained only when a fully general relativistic effect is included. However, it is out of the scope of the present paper.

2 ASSUMPTIONS, METHODS AND RESULTS

2.1 Assumptions

We assume that the system which consists of a non-rotating black hole and a self-gravitating torus is axisymmetric. Since both the black hole and the torus can be the source of gravity, the gravitational potential of the system \( \Phi(\sigma, z) \) is written as

\[
\Phi(\sigma, z) = \Phi_{BH}(\sigma, z) + \Phi_T(\sigma, z),
\]

where \( \Phi_{BH} \) and \( \Phi_T \) correspond to the gravitational potentials of the black hole and the torus, respectively, and the cylindrical coordinates \((\sigma, z, \varphi)\) are used. For \( \Phi_{BH} \), we use the pseudo-Newtonian potential follow as (Paczyński & Wiita 1982):

\[
\Phi_{BH}(\sigma, z) = -\frac{G M_{BH}}{r} - \frac{G M_{BH}}{r - r_g} + \frac{G M_{BH}}{r_g},
\]

where \( G, M_{BH}, c \) and \( r \) are the gravitational constant, the mass of the black hole, the speed of light and the spherical radius \( r = \sqrt{\sigma^2 + z^2} \), respectively. This pseudo-Newton potential was also adopted by Abramowicz et al. (1983) and Daigne & Mochkovitch (1997). The Newtonian potential of the torus \( \Phi_T \) is calculated as

\[
\Phi_T(\sigma, z) = -\int \frac{G \rho(\sigma', z')}{r} dV',
\]

where \( \rho \) and \( V \) are the density and the volume of the torus, respectively.

In order to compare our results with those of Daigne & Mochkovitch (1997), we choose the same equation of state in the torus as theirs, i.e.

\[
p = K(T_p)^{4/3},
\]

where \( p \) is the pressure, the constant is \( K = 1.2 \times 10^{15} \) and the electron concentration \( Y_e = 0.5 \).

2.2 Initial state

Under these assumptions, a critical equilibrium state with the distribution of the angular momentum

\[
j(\sigma) = j(\sigma_m)(\frac{\sigma}{\sigma_m})^{0.2},
\]

is constructed by specifying the masses of the black hole and of the torus \( M_T \). Here the critical state means the Roche-lobe-filling state. It implies that the size of the torus cannot be elongated any more.

The radius \( \sigma_m \) is the distance of the inner edge of the torus from the rotational axis and the quantity \( j(\sigma_m) \) is a constant. Once we obtain a critical state, we can calculate the total angular momentum of the torus \( J_T \), the mass of the torus inside the coordinate distance \( \sigma, m(\sigma) \), and therefore the specific angular momentum distribution as a function of \( q = m M_T, j(q) = j(\sigma_m(q)) \), where \( \sigma(q) \) is the inverse of \( q(\sigma) = m(\sigma)/M_T \). In this way the angular momentum–mass relation is obtained for the initial critical state. We will check the stability of the obtained models by the following two approaches described below.

In actual computations we have solved the initial state by applying the HSCF scheme (Hachisu 1986). This initial state is the same as that of Daigne & Mochkovitch (1997). The model quantities are as follows:

\[
M_T = 0.36 M_\odot, \quad M_{BH} = 2.44 M_\odot,
\]

\[
J_T = 4.02 \times 10^{49} \text{g cm}^2 \text{s}^{-1}.
\]

2.3 Equilibrium approach

First, stability of initial models is investigated by using the equilibrium states. This is basically the same approach as Nishida & Eriguchi (1996), Nishida et al. (1996) and Daigne & Mochkovitch (1997).

In this approach, we will check whether an equilibrium state can be recovered or not after transferring a small amount of mass from the torus to the black hole. This can be done as follows. Let the transferred mass and the transferred angular momentum be \( M_{BH} \) and \( J_{BH} \), respectively. Thus the mass and the angular momentum of the torus \( M_T \) and the angular momentum of the torus \( J_{T_0} \) can be written as

\[
M_T = M_{BH} + \Delta M,
\]

\[
J_{T_0} = J_T - \Delta J
\]

\[
= J_T - \int_0^1 dm \left[ \int dV' \frac{m}{M_T} (\frac{\sigma}{\sigma_m})^{0.2} \right] (\frac{m}{M_T})
\]

The new specific angular momentum–mass relation should be

\[
j_{BH}^\text{new} \equiv m(M_T - \Delta M) \Rightarrow j(m + \Delta M)/M_T\] ,

where \( 0 \leq m \leq M_T - \Delta M \).

Since there is no guarantee of existence of an equilibrium state which satisfies the above conditions, we try to find trial equilibrium states which satisfy weaker conditions by the following procedure. When the mass of the black hole and the rotation law of the torus are given, the equilibrium state of a torus can be specified by two quantities: one corresponds to the strength of gravity and the other to the amount of rotation.

Therefore once these two parameters are given we can get one equilibrium state, if any. Let the mass and the angular momentum of the torus and the transferred mass and the transferred angular momentum be \( M_T, J_T, \Delta M \) and \( \Delta J \), respectively. Thus the mass of the black hole \( M_{BH} \) is expressed as

\[
M_{BH} = M_T + \Delta M
\]

We search for a trial equilibrium state which satisfies the following conditions:

\[
\frac{\Delta M}{M_T} = \frac{\Delta M}{M_{BH}}
\]

\[
\frac{\Delta M}{M_{BH}} = \frac{\Delta M}{M_T}
\]
The transferred angular momentum $\Delta \mathcal{J}/10^{59}$ (in cgs units) is plotted against $\log \rho_{\text{max}}$ (g cm$^{-3}$) where $\rho_{\text{max}}$ is the maximum density of the torus. Different equilibrium sequences with different $\Delta M$ are shown by different curves. Solid curve: a sequence with $\Delta M/M_T = 0.0014$, long-dashed curve: a sequence with $\Delta M/M_T = 0.012$, dash–dotted curve: a sequence with $\Delta M/M_T = 0.038$, and dotted curve: a sequence with $\Delta M/M_T = 0.084$. Horizontal lines denote the values of $\Delta J$ which are accompanied by the corresponding transferred mass $\Delta M$. The same type of line denotes the corresponding sequence and value. When $\Delta M/M_T$ is 0.0014 or 0.012, the curves reach the corresponding $\Delta J$ lines and so they are stable. On the other hand, when $\Delta M/M_T$ is 0.038 or 0.084, the curves cannot extend to the corresponding $\Delta J$ lines. Therefore they are unstable. The crosses denote the mass-shedding states.

As for the other model parameter, in actual computations we have chosen the maximum density of the torus. Thus by varying these two parameters we have tried to find equilibrium states which satisfy $\alpha = 1$ and $M'_T = M'^{\text{new}}_T$. These two conditions ensure that the obtained equilibrium state satisfies the required conditions, i.e. equations (6), (7), (8) and (9). Consequently, if there exist equilibrium states which satisfy all the conditions mentioned above, the initial state can be said to be stable against mass overflow because the matter can settle down to a new equilibrium configuration.

In contrast, if the equilibrium models which satisfy the conditions cannot be obtained by physical mechanisms such as mass shedding of the matter from the inner edge, we can conclude that the initial state is unstable against the mass overflow from the Roche lobe.

We have examined four different mass transfer processes which correspond to differences of the transferred mass. Our results are shown in Fig. 1 and summarized in Tables 1–4.

Concerning the angular momentum distribution, we will choose the following formula $\tilde{f}(q)$:

$$\tilde{f}(m'(\sigma)) = \alpha [(m' + \Delta M')(M'_T + \Delta M')],$$

where $m'(\sigma)$ is the mass of the torus inside a distance $\sigma$ for the trial equilibrium state. The constant $\alpha$ is one of two model parameters mentioned before and can be varied to obtain the required configurations.

As for the other model parameter, in actual computations we have chosen the maximum density of the torus. Thus by varying these two parameters we have tried to find equilibrium states which satisfy $\alpha = 1$ and $M'_T = M'^{\text{new}}_T$. These two conditions ensure that the obtained equilibrium state satisfies the required conditions, i.e. equations (6), (7), (8) and (9). Consequently, if there exist equilibrium states which satisfy all the conditions mentioned above, the initial state can be said to be stable against mass overflow because the matter can settle down to a new equilibrium configuration.

In contrast, if the equilibrium models which satisfy the conditions cannot be obtained by physical mechanisms such as mass shedding of the matter from the inner edge, we can conclude that the initial state is unstable against the mass overflow from the Roche lobe.

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In Fig. 1 the transferred angular momenta for models which satisfy equations (11), (12), (13) and $M'_T = M'^{\text{new}}_T$ are plotted against the maximum density of the torus. Horizontal lines correspond to the prescribed values of the transferred angular momentum $\Delta J$ associated with the transferred mass $\Delta M$. Therefore if the trial equilibrium curve reaches the corresponding horizontal line, it means that

$$\Delta J' = \Delta J,$$

or that there exists an equilibrium state which satisfies the conditions required for the final state after the mass transfer. On the contrary, if the trial equilibrium curve cannot reach the corresponding horizontal line due to mass shedding, it implies

$$\Delta J' < \Delta J,$$

or that the initial state cannot settle down to a new equilibrium state but will suffer from runaway instability.

In Tables 1–4, the maximum density $\rho_{\text{max}}$, the radii of the inner edge $r_{\text{in}}$ and the outer edge $r_{\text{out}}$ and the transferred angular momentum $\Delta J'$ for trial equilibrium sequences are shown for $\Delta M' = 4.95 \times 10^{-3} M_\odot$, $4.23 \times 10^{-3} M_\odot$, $1.39 \times 10^{-2} M_\odot$ and $3.05 \times 10^{-2} M_\odot$, respectively. The mass and the angular momentum of the initial torus are $M_T = 0.36 M_\odot$ and $J_T = 4.024 \times 10^{50}$ g cm$^2$ s$^{-1}$, respectively. The initial radii of the inner edge and the outer edge are $r_{\text{in}}/r_{\odot} = 2.27$ and $r_{\text{out}}/r_{\odot} = 40.04$, respectively.

From Fig. 1 we can see that trial equilibrium sequences of $\Delta M/M_T = 0.0014$ and $\Delta M/M_T = 0.012$ can reach the state $\Delta J' = \Delta J$. Therefore the torus can shrink to a new equilibrium state after mass transfer and subsequent mass overflow will not occur, i.e. they are stable states. On the other hand, we cannot extend sequences of $\Delta M/M_T = 0.038$ and $\Delta M/M_T = 0.084$ up to the state which satisfies the condition $\Delta J' = \Delta J$. These sequences
Table 1. Quantities of the equilibrium sequence where $\Delta M = 4.95 \times 10^{-6} M_\odot$ or 0.14 per cent of the mass of the critical torus is transferred from the torus to the black hole. The corresponding $\Delta f$ is $4.382 \times 10^{56}$ g cm$^2$ s$^{-1}$.

<table>
<thead>
<tr>
<th>$\rho_{\text{max}}$ (g cm$^{-3}$)</th>
<th>$r_{\text{in}}/r_g$</th>
<th>$r_{\text{out}}/r_g$</th>
<th>$\Delta f$ (g cm$^2$ s$^{-1}$)</th>
</tr>
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<td>$3.497 \times 10^{11}$</td>
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</tr>
<tr>
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<td>3.400</td>
<td>3.946 \times 10^4</td>
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Table 2. Same as Table 1 but for $\Delta M = 4.23 \times 10^{-3} M_\odot$, i.e. 1.2 per cent of the mass of the torus, and the corresponding $\Delta f$ is $3.893 \times 10^{57}$ g cm$^2$ s$^{-1}$.

<table>
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<tr>
<th>$\rho_{\text{max}}$ (g cm$^{-3}$)</th>
<th>$r_{\text{in}}/r_g$</th>
<th>$r_{\text{out}}/r_g$</th>
<th>$\Delta f$ (g cm$^2$ s$^{-1}$)</th>
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Table 3. Same as Table 1 but for $\Delta M = 1.39 \times 10^{-2} M_\odot$, i.e. 3.8 per cent of the mass of the torus, and the corresponding $\Delta f$ is $1.318 \times 10^{58}$ g cm$^2$ s$^{-1}$.

<table>
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<th>$r_{\text{out}}/r_g$</th>
<th>$\Delta f$ (g cm$^2$ s$^{-1}$)</th>
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Table 4. Same as Table 1 but for $\Delta M = 3.05 \times 10^{-2} M_\odot$, i.e. 8.4 per cent of the mass of the torus, and the corresponding $\Delta f$ is $2.959 \times 10^{58}$ g cm$^2$ s$^{-1}$.

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3 DISCUSSION AND CONCLUSIONS

Our results show that critical tori are stable against mass overflow as long as the amount of transferred mass is small, i.e. less than 3 per cent of the mass of the torus. This is consistent with the results of Daigle & Mochkovitch (1997). In other words, if the distribution of the specific angular momentum has a rather large dependence on the distance, tori around black holes can exist stably. This is in clear contrast to the case for constant specific angular momentum tori which are unstable.

However, if the transferred mass exceeds a critical value, roughly 3 per cent of the mass of the torus, tori suffer from instability. Since the difference between our models and those of Daigle & Mochkovitch (1997) lies only in treatment of gravity, this
Runaway instability of self-gravitating tori

Figure 2. (a) Snapshots of the particle positions projected on to the equatorial plane. Positions of the gas particles of about 3 per cent of the mass of the torus are perturbed from their equilibrium positions. The upper panel shows particle positions projected on to the equatorial plane at the initial stage and the lower panel those at $t = 1.45 t_{\text{rot}}$. Here $t_{\text{rot}}$ is the rotational period of the torus at the place of the initial maximum density. It can be seen that the torus shrinks into the black hole in about 1–2 rotational periods. Solid circles denote the outer edge and the inner edge of the torus for the initial state and only the outer edge for the evolved state because the mass overflows on to the black hole. (b) The same as (a) but for a torus without self-gravity. Even after mass transfer of about 3 per cent of the mass, the torus settles down to a new equilibrium state.

Table 5. Comparison of our results with those of other groups.

<table>
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<th>Author(s)</th>
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<th>angular momentum</th>
<th>self-gravity</th>
<th>brief result</th>
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<td>Yes</td>
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<tr>
<td>Daigne &amp; Mochkovitch (1997)</td>
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<td>No</td>
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</tr>
<tr>
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<td>$j \propto \sigma^{\alpha}$</td>
<td>No</td>
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<td>Yes</td>
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</tbody>
</table>
implies that self-gravity of the tori has a stronger destabilizing effect than the stabilizing effect of the distribution of the angular momentum with a positive power law. This is because the existence of self-gravity changes the Roche lobe as well as the shape of the tori. In other words, even if the gas shrinks due to mass transfer, the shrinking of the Roche lobe due to self-gravity overcomes the configuration change of the tori.

The importance of self-gravity is concluded not only from the present investigation but also from previous studies. As shown in Table 5 which summarizes the results of investigations about the runaway instability thus far made, we can see that this instability arises only for analyses including self-gravity.

Therefore from our present investigation, together with other studies, we can conclude that if self-gravity of tori around black holes cannot be neglected, the critical tori are unstable against the finite amount of mass overflow. Of course, although the final answer to the runaway instability problem needs to be analysed in the framework of general relativity, it is very likely that self-gravitating tori around black holes will suffer from runaway instability.

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