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On Dense Bottom Currents in the Baltic Deep Water

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A rational entrainment function for a subcritical dense bottom current is outlined. As an example the formula has been used to some orders of magnitude calculations of the deep water currents from the Darss Sill to the Stolpe Channel. It is shown that the salt and oxygen supply to the deep water of the Baltic Proper during a »normal« year stems from this bottom current and its entrained water. The renewal of the deep water in the Baltic Proper can be traced in the Bornholm Basin, and hence it is strongly recommended, that continous measurements of salinity, temperature, oxygen, phosphate etc. are performed in the Bornholm Basin, especially in the highly entraining area just north of Bornholm.

Introduction

The Baltic and its connection with the North Sea (the Cattegat, the Belts and the Sound) contains all types of estuaries, i.e. fjords (the Baltic, the Sound), partially mixed estuaries (the Cattegat, the Belts under calm conditions) and well mixed estuary (the Belts under stormy conditions). In the present article, we shall only deal with the Baltic, which is the greatest fjord in the world. The dynamics of fjords are mainly affected by the following parameters (see the author 1978):

1. The geometry.
2. The hydrology of the adjacent watershed.
3. The oceanographic conditions outside the fjord.
4. The wind field.

The above mentioned four parameters can for the Baltic very briefly be characterized as:

1. The geometry: A length $L \approx 10^6$m, a width $W \approx 10^5$m, a depth $D \approx 10^2$m and several sills with depths $10 \text{ m} < h < 10^2$m.

Hence compared with ordinary fjords, the length and the width are an order of magnitude greater. A general description of the bottom topography of the Baltic, i.e. the deep basins and the sills, has been given by Fonselius (1962) and Falkenmark and Mikulski (1975).

2. The hydrography of the adjacent watershed: The yearly fresh water inflow to the Baltic amounts to about 440 km^3 (or $14 \cdot 10^3$m^3/s), see Mikulski (1970), with a maximum in May (14%) and a minimum in Nov.-Dec. (amounting to 50% of max). The precipitation and the evaporation ($\approx$500 mm/year) equals each other on a yearly basis, see Brogmus (1952).

3. The oceanographic conditions outside the fjord: It is common, that the mouth of a fjord has a side constriction and a sill as the Baltic has. Nevertheless, the mouth of the Baltic diverges in a decisive way from ordinary fjords in the great extended shallow connection to the North Sea (the Belts, the Sound and the Cattegat of length $L \approx 10^5$m, width $W \approx 10^5$m and depth $D \approx 30$ m). The effect of this shallow connection is, that the oceanographic conditions outside the Baltic is the conditions of a partially to well mixed estuary, with a long-term average surface salinity of about $10‰ < \tilde{S}_0 < 11‰$ and bottom salinity (17 m depth) of about $13‰ \tilde{S}_{17} < 18‰$ see f.i., the Beltproject (1976). Hence contrary to ordinary fjords, the bottom water has a very low salinity. As the first sill depth (at Darss, see Fig. 1) is about 17 m, the long-term average bottom salinity of the first basin (the Arkona, see Fig. 1) is less than $\tilde{S}_{17}$due to entrainment in the dense bottom current from Darss to Arkona. Accordingly, the long-term average salinity of the deep water of the succeeding basins are decreasing, mainly due to the trapping effect of the sills, i.e. in the Bornholm Deep $\tilde{S}_{BD} \approx 16‰$ (see Fig. 3), in the Baltic Proper Deep $\tilde{S}_{BPD} \simeq 11‰$ (see Fonselius (1969), Fig. 37), decreasing to about 4‰ in the innermost Bothnian Bay deep water, Dahlin (1976). The hydrographic conditions outside the mouth of the Baltic i.e. the very important boundary conditions, have been measured nearly continously during the last 100 years (The Danish Nautical-Meteorological-Yearbooks) and very extensively during the last five years (the Belt Project (1976)).

4. The wind field, i.e. the meteorologic conditions over the Baltic has a great effect on the physical oceanography of the Baltic. The meteorological activities over Scandinavia with succeeding low- and high pressure act like a piston on the Baltic, which combined with wind setup and -down creates an oscillating in- and outflow through the mouth. This means, that a great part of the surface water in this area is pendling in and out in contrast to the saline bottom water, which is trapped in a dense bottom current due to the existance of the Darss Sill. The high correlation between the wind and the salinity in the Baltic Deep water has been demonstrated by Kullenberg (1977). The other main effect of the wind is the production of turbulence in the surface and the

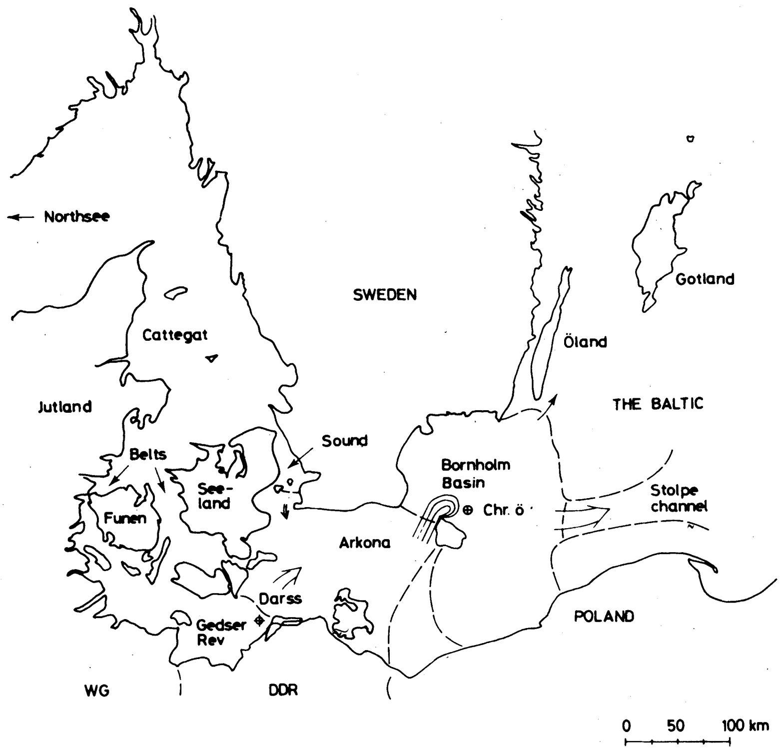

Fig. 1. Chart of the southern part of the Baltic.

bottom water (up- and downwelling and seiching) which creates entrainment (only upwards directed) and diffusion (equal amounts of water up- and downwards) of salt, oxygen, heat etc., see later, Kullenberg (1974, 76, 77) and Shaffer (1977).

The Baltic and its connection with the North Sea is one of the most intense investigated oceanographic fields in the world, but due to the briefly described, very complicated hydrodynamic, there are still som disagreements, especially in connection with the hydrodynamics of the Baltic Deep water. Until recently, the scientists were only concerned with the intermittent major deep water inflows, which are easy to trace, see f.i. Fonselius (1962, 67, 69, 70), Fonselius and Rattanasen (1970), Soskin (1963) and Wyrtki (1954). As demonstrated and discussed in deails by Fonselius (1962, 67, 69, 70) and Fonselius a.o. (1970) an extreme salt inflow has at first a positive ecologic effect (high oxygen content), but the long-term effect is negative, because the oxygen is rapidly consumed, and the high density prevents for a long time new inflowing water in supplying new oxygen to the deep water.

The observations performed by Petrén and Walin (1975) in the Bornholm Strait (see Figs. 1 and 11) and by Rydberg (1976) in the Stolpe Channel (see Fig. 1) have clearly showed the existance of a significant deep water current. These observations are from an ecologic and hydrodynamic point of view of great importance, because a nearly continous deep water inflow is the most momentous source for a regeneration of the deep water, and because a better understanding of the physics of the renewal of the deep water can be obtained.

In the present article, we shall strengthen these observations of a deep water current by connecting the observations made by Petrén a.o. (1975) with those of Rydberg (1976) and furthermore by including a simplified hydrodynamic calculation of the bottom current from the Darss Sill. As the observations in the Stolpe Channel have only been performed during one week, we shall prove the existance of this deep water current all over the year, and furthermore calculate the yearly fluxes of water, salt, oxygen and heat by this current. These calculations allow us to make an estimate of the relative importance of the entrainment and the diffusion of dissolved matter across the interface in the Baltic Proper.

As the renewal of the Baltic Deep water can be traced in the Bornholm Basin, we shall start the analysis there.

The Deep Water Outflow from the Bornholm Basin

Next to the Cattegat, the Belts etc., the Bornholm Basin has the most intense series of measurements. This allows us to give a general description of the conditions during this century in this important area of the Baltic. Fonselius (1962) collected all available data for the salinity and the temperature at 80 m depth, which is a representative depth for the deep water. These data are reproduced in Fig. 2. A cronologic description of the processes responsible for the major changes in salinity has been given by Fonselius (1962, 67, 69, 70) and Fonselius a.o. (1970) and shall not be repeated. Instead, we shall focus on the normal, nearly continous inflow.

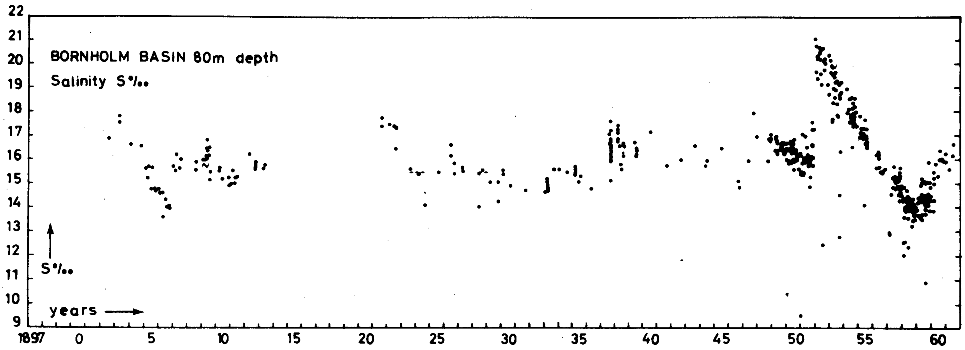

Fig. 2. Salinity values from 1900-1961 in the Bornholm Basin at 80 m (from Fonselius 1962).

From Fig. 2 it is realized, that there has been no significant trend in the long-term averaged salinity of the deep water, which is about 16‰ at 80 m depth. Significant deviations from this mean value can be caused by a major salt inflow but also by a wind set-up or -down, an internal wave or an error, but in a long continous time serie of the salinity this can easily be separated from a major salt inflow.

The existence of a significant nearly continous inflow to the Baltic Deep water is not taken for granted by all scientists, but its existence is easily proved. To this end we make a statistical calculation on the measured salinity, oxygen and temperature in the Bornholm Basin for the periods 1949 to 54 and 1956 to 61. During these periods we have the mentioned measurement from a Danish hydrographic station located 1,5 nm ESE of Christiansö (Nautical-Meteorological Yearbooks 1950-61). The measurements have been performed occasionally (approximately once a month), so in order to present the data in a more condensed form, the average values and the standard deviations have been calculated for each month as well as for the total period (by Christian Bo Pedersen), see Figs. 3, 4 and 5. If first we focuse on Fig. 3 (the salinity distribution) it is evident that a continous outflow through the Stolpe Channel takes place all over the year from the layer located between the sill depth 60 m and say 45 m's depth. This flow situation is illustrated in Fig. 6, where the 12 years average values have been noted as well. The average salinity S_i of the discharge Q_i can be depicted from Fig. 3 as the average of the salinity between the interfacial and the sill level, because the high stability in the Bornholm Deep water creates a blocking effect.

The knowledge of the mean value of the salinity flowing over the sill S_i = 11 ‰ enables us to estimate the associated discharge by use of Knudsen's hydrographical theorem (steady state conditions):

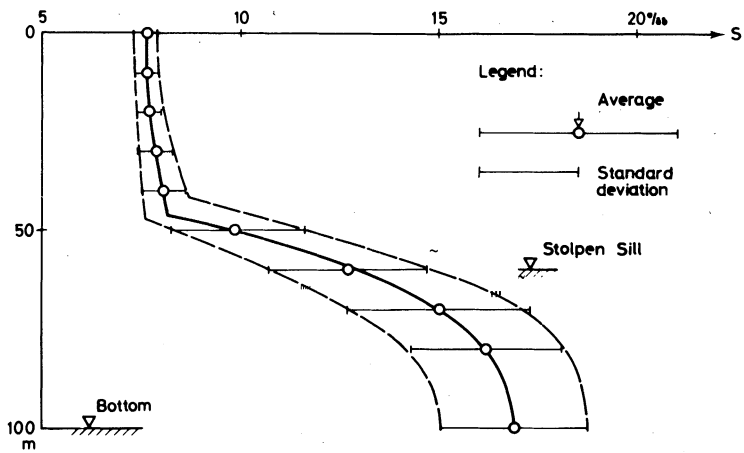

Fig. 3. Salinity distribution in the Bornholm Basin, 1.5 nm ESE of Christiansø 1949 to 55 and 1956 to 61.

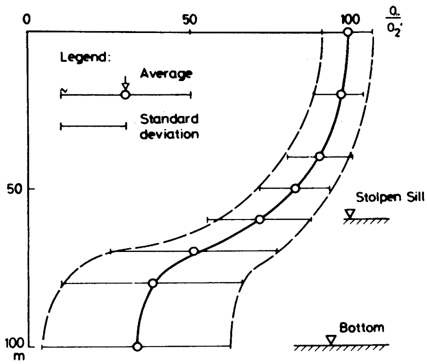

Fig. 5. Temperature distribution in the Bornholm Basin, 1.5 nm ESE of Christiansø 1949 to 55 and 1956 to 61.

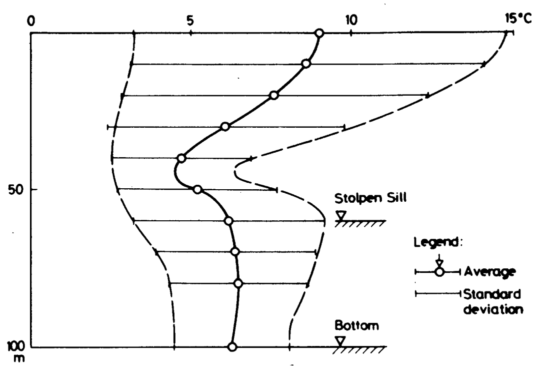

Fig. 4. Oxygen distribution in the Bornholm Basin, 1.5 nm ESE of Christiansø 1949 to 55 and 1956 to 62.

$$Q_i = Q_0 - Q_F \tag{1}$$

$$S_i \, Q_i = S_0 \, Q_0 \tag{2}$$

where

Q_F = the fresh water discharge
Q_0 = the surface water discharge (at Stolpe)
S_0 = the surface water salinity (7.8 0/00 (see Fig. 3)).

In Eqs. (1) and (2) we have assumed, that the volume and the salinity of the bottom water have not changed significantly during the 12 years period. The fresh water supply for the same period has been calculated by Mikulski (1970):

$$Q_F = 440 \text{ km}^3/Y = 14 \cdot 10^3 \text{m}^3/\text{s} \tag{3}$$

Hence as an estimate for the nearly continous inflow to the Baltic Proper deep basin, we get:

$$Q_i = Q_F \frac{S_0}{S_i - S_0} = 34 \cdot 10^3 \text{m}^3/\text{s} \ (1100 \text{ km}^3/Y) \tag{4}$$

In order to estimate the standard deviation of this inflow, we need a hydrodynamic model, which relates the discharge to the salinity and the interfacial depth. As the flow in the Stolpe Channel can be described by the hydrodynamic model outlined later in the article, we can conclude, that the discharges Q_i in two flow situations (index 1 and 2) are related by

$$\frac{Q_{i,1}}{Q_{i,2}} = \frac{y_1}{y_2} \sqrt{\frac{(\Delta\rho)_1 y_1}{(\Delta\rho)_2 y_2}} = \frac{y_1}{y_2} \sqrt{\frac{(S_{i,1} - S_{0,1}) y_1}{(S_{i,2} - S_{0,2}) y_2}} \quad 1 \tag{5}$$

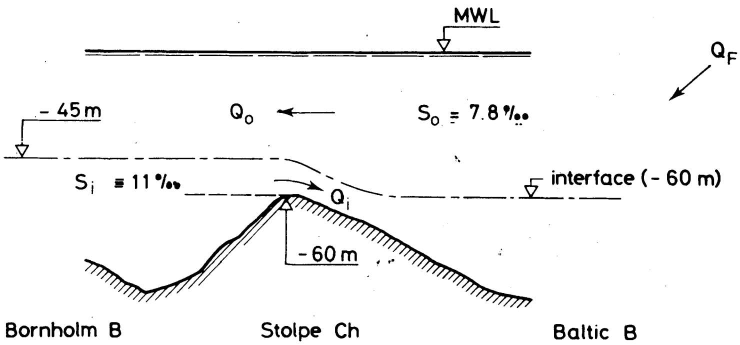

Fig. 6. The average flow conditions through the Stolpe Channel.

where $\Delta\rho$(= 0.75ΔS) = the mass difference between the upper and lower layers,
y = the depth of the deep water flow.

We take flow situation 1 as the 12-years average

$$(Q_{i,1} = 34 \cdot 10^3 m^3/s, \quad y_1 = 14\ m, \quad (S_{i,1} - S_{0,1})y_1 = 45 \cdot 10^{-3} m)$$

From Fig. 3 we estimate the ranges for flow situation 2 as

$$13\ m \leq y_2 \leq 18\ m \tag{6}$$

and

$$23 \cdot 10^{-3} m \leq (S_{i,2} - S_{0,2})y_2 \leq 68 \cdot 10^{-3}\ m \tag{7}$$

which inserted in Eq. (5) gives the following range of Q_i (within the standard deviation):

$$23 \cdot 10^3 \leq Q_i \leq 54 \cdot 10^3 m^3/s \tag{8}$$

The discharge measured by Rydberg (1976) during one week in august - $Q_i \simeq 50 \cdot 10^3 m^3/s$ - is in agreement with the above calculated values, and so is the range of salinities observed. The average salt flux in the 12-years period SF_i can be evaluated to

$$SF_i = S_i \cdot Q_i \simeq 375\ t/s \tag{9}$$

The amount of oxygen supply to the Baltic Deep water from the nearly continous inflow can be estimated from Fig. 4 (the average oxygen distribution). The average oxygen content in the selective withdrawal layer (45 m to 60 m depth) is 76% corresponding to approximately $6.6 \cdot 10^3 t/km^3$. Hence, the normal total yearly supply of oxygen to the Baltic Deep water from the Bornholm Basin amounts to

$$O_2 F_i \simeq 7.3 \cdot 10^6 t/Y \tag{10}$$

If we look at Fig. 5 (the temperature distribution), it is seen that the average temperature in the selective withdrawal layer is about 5.7° C, which is a little higher than the average temperature of the bottom water in the Baltic Proper Deep, see Fonselius (1962, Fig. 5), in accordance with the fact, that entrainment of cold water (2° C) takes place in the Stolpe Channel before the bottom current reaches the Baltic Proper deep water.

The reason for the steadiness of the deep water flow during the year is, that the »active« volume of the Bornholm Basin (45 m to 60 m depth) acts like a buffer, which decreases the ratio between max. and min. flow compared with the values for the inflow. A measure for the time scale T_{BB} of this reservoir can be obtained by dividing the active volume by the time average max. outflow, (as for a linear reservoir):

$$T_{BB} \simeq 10 \text{ weeks} \tag{11}$$

Hence, rapid variations in the outflow from the Bornholm Basin can only occur due to up- or downwelling and seiching at the Stolpe Sill, not to variations in the inflow.

Now, let us compare the nearly continous inflow to the Baltic Proper with the greatest inflow ever observed, see Wyrtki (1954). The hydrographic conditions in the Bornholm Basin in the very end of this inflow is shown in Fig. 7. By applying Eq. (5) on this extreme situation ($y_2 \simeq 26$ m, $(S_{i,2} - S_{0,2})y_2 \simeq 180 \cdot 10^{-3}$m) we get

$$Q_{i,\max} \simeq 126 \cdot 10^3 \mathrm{m}^3/\mathrm{s} \tag{12}$$

or 327 km³ for the total period (30 days).

In order to calculate the inflow from the Belts and the Sound we have to subtract the entrainment to the dense bottom current from Darss to Bornholm. Again with reference to the hydrodynamic model outlined later, we get for the ratio between the total entrainment Q_E in the two situations (index 1 and 2):

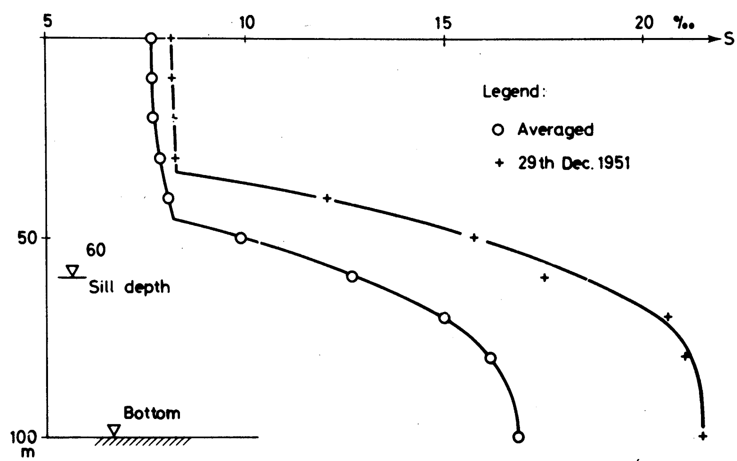

Fig. 7. The salinity distribution in the Bornholm Basin after a unique inflow 1951 compared with the 12 years average salinity.

$$\frac{Q_{E1}}{Q_{E2}} = \sqrt[3]{\frac{(\Delta\rho)_1 Q_1}{(\Delta\rho)_2 Q_2}} = \sqrt[3]{\frac{(S_{i,1} - S_{0,1})Q_1}{(S_{i,2} - S_{0,2})Q_2}} \qquad (13)$$

As the entrainment under average conditions amounts to $Q_{E1} = 24 \cdot 10^3 m^3/s$ (see later), the entrainment under the extreme conditions can be calculated by Eq. (13), which yields

$$Q_{E2} = 48 \cdot 10^3 m^3/s \qquad (14)$$

The inflow from the Belts and the Sound was therefore in Dec. 1951:

$$Q_{max} = (126-48)10^3 = 78 \cdot 10^3 m^3/s \qquad (15)$$

or 202 km^3 in 30 days in good agreement with the volume estimated by Wyrtki (1954).

Hence, this unique inflow corresponds to only three to four months of the nearly, continous inflow.

Finally an estimate of the relative importance of the entrainment and the diffusion across the interface in the Baltic Proper can be given by applying the method described by Walin (1977). If the interface is defined in the same way as Hela (1966) did, is the average salinity in the primary halocline $S_{PH} = 7.8‰$ (see Fig. 3). Hence the continuity of salt below the 7.8 0/00 isohaline gives an upwards salt transport SF_E through the interface by entrainment

$$SF_E = Q_i S_{PH} = 265\ t/s \quad \text{or} \quad 2.65 \cdot 10^{-6}\ kg/s/m^2 \qquad (16)$$

and therefore an upwards salt transport by diffusion SF_D which is the difference between the inflow (375t/s, see above) and SF_E,

$$SF_D = 110\ t/s \quad \text{or} \quad 1.1 \cdot 10^{-6}\ kg/s/m^2 \qquad (17)$$

where an area of $A \simeq 10^5$ km^2 has been used for the interface of the Baltic Proper, see Ehlin and Mattisson (1976, areas 4-13 in 60 m depth). The diffusion measured by Kullenberg (1974, 76, 77) for the open sea and referred to by Kullenberg (1977) for the coastal zone is in good agreement with the above values, while the entrainment and diffusion measured by Shaffer (1977) for the coastal zone seams to be too great.

If we apply the salt diffusion coefficient at the oxygen diffusion, we find, that the net transport of dissolved oxygen is nil across the primary halocline, which means that the deep water oxygen is supplied only by the dense bottom current.

The calculated values should have been corrected for the entrainment into the dense bottom current from the Bornholm Basin to the Baltic Proper basin, but as it only amounts to about 10% of Q_i; this correction can be neglected in an order of magnitude calculation.

As demonstrated above, the knowledge of the conditions in the Bornholm Basin is essential for the understanding of the hydrodynamics of the deep water renewal of the inner Baltic Basins. As the dense bottom current through the Stolpe Channel is the carrier of oxygen, temperature, phosphate etc., it is important to measure these quantities continously in the Bornholm Basin, especially the position of the interface and the conditions in the selective withdrawal layer.

The amount of water flowing through the Stolpe Channel exceeds the amount of dense water coming from the Belts and the Sound. This is due to entrainment in the dense bottom current from Darss sill to the Bornholm Basin. As this entrainment contributes considerably to the discharge, it is of great importance for the conditions in the Bornholm Basin and hence for all the Baltic Basins. In the next chapter, an entrainment function and a hydrodynamic model for a dense entraining bottom current is outlined.

The Hydraulics of a Dense Bottom Current

Immiscible fluids (Fig. 8)

A dense bottom current of a fluid, which does not mix with the ambient water, behaves like a free surface flow subject to a reduced gravity $g' = \Delta g$, where Δ is the relative mass difference between the two fluids, see e.g. Engelund and Christensen (1969). Hence, the differential equation for the depth y reads (for $y << D$):

$$\frac{dy}{dx_1} = I_0 \frac{1-(y_n/y)^3}{1-(y_c/y)^3} \tag{18}$$

where I_0 = bottom slope
y_n = natural depth
y_c = critical depth

Natural depth is the depth of a uniform bottom current ($dy/dx_1 = 0$), and is determined by the balance between the longitudinal pressure gradient $\Delta\rho g y I_0$ and the shear stresses at the bottom τ_b and at the interface τ_i, i.e.

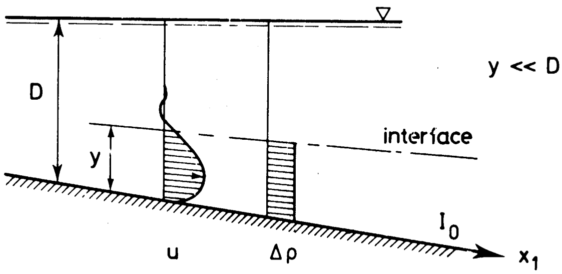

Fig. 8. A dense, non-entraining bottom current.

$$(\tau_b+\tau_i) = \Delta\rho g y_n I_0 \tag{19}$$

The shear stresses are related to the mean velocity V in the common way by introducing a friction factor $f/2$ for the bottom and the interface (subscript b and i respectively):

$$(\tau_b+\tau_i) = \rho(\frac{f_b}{2} + \frac{f_i}{2})\, V^2 \tag{20}$$

where it has been assumed, that the density difference is small ($\Delta << 1$). By use of the continuity equation ($Q = ByV$), Eqs. (19) and (20) yield:

$$y_n = \sqrt[3]{\frac{(f/2)\, Q^2}{B^2 g \Delta\, I_0}} \tag{21}$$

where $f/2 = f_b/2 + f_i/2$
Q = the discharge
B = the width.

The critical depth is

$$y_c \equiv \sqrt[3]{\frac{Q^2}{B^2 g \Delta}} \tag{22}$$

Miscible fluids (Fig. 9)

If the density difference is created by salt, temperature or suspended matter, the bottom current will entrain ambient water, the effects of which are increased discharge and decreased density difference.

In that case the continuity equations for volume and mass read

$$\frac{dQ}{dx_1} = V_E B \tag{23}$$

and

$$\frac{d(\bar{\Delta}Q)}{dx_1} = 0 \tag{24}$$

respectively, where V_E is the entrainment velocity.

Ellison and Turner (1959) and Wilkinson (1970) have performed an experimental and theoretical study of entraining dense bottom currents with high densimetric Froude's number

$$F_\Delta = \frac{V}{\sqrt{\bar{\Delta} g y}} \tag{25}$$

They found, that the entrainment velocity V_E was negligible for low F_Δ. Lofquist (1960) on the contrary found an entrainment velocity for low F_Δ, but as his measure-

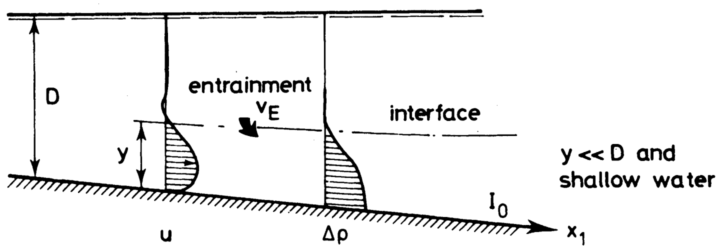

Fig. 9. An entraining dense bottom current.

ments were highly influenced by side-wall effects and low Reynolds' numbers and furthermore show a great scatter, they cannot be used directly. Hence, instead we will make use of the bulk flux Richardson number (R_f^T) concept, as suggested by the author (1976). In the actual case (subcritical flow) the bulk flux Richardson number degenerates to the ordinary flux Richardson number, and hence the determination of V_E is simplified to:

$$V_E = \frac{R_f^T \cdot \mathrm{PROD}}{1/2\xi\Delta\rho g y} \tag{26}$$

where PROD = the rate of production of turbulent energy,

$\frac{1}{2}\xi y$ = the distance from the interface to the center of gravity of the mass-deficit distribution.

PROD: The PROD-term can be evaluated by the following integration by parts

$$\mathrm{PROD} = \int_\Omega (\tau \frac{\partial u}{\partial x_3})\, d\Omega = \int_\Omega (u \frac{\partial \tau}{\partial x_3})\, d\Omega - [u\tau]_{x_3,\mathrm{lower}}^{x_3,\mathrm{upper}}$$

$$= V(\tau_i + \tau_b) - u_i\tau_i = V\tau(1-\beta) \tag{27}$$

in which the approximate linear shear stress distribution has been utilized. The term

$$\beta = \frac{u_i}{V}\left(\frac{\tau_i}{\tau_i + \tau_b}\right) \tag{28}$$

is for the type of flow considered here equal to a constant, the order of magnitude of which is

$$0.15 < \beta < 0.25 \tag{29}$$

R_f^T: The bulk flux Richardson number R_f^T depends on the distance between the main production of turbulent energy and the entrainment zone. For internal generated turbulence (short distance) is $R_f^T \simeq 0.13$ (see the author (1976)), while it decreases to approximately 0.05 for external generated turbulence (long distance), see Edwards and Darbyshire (1973), Kullenberg (1976) and Turner (1973). Hence for a dense bottom current, where the primary production is external (for F_Δ low)

$$R_f^T = \phi \cdot 10^{-1} \qquad 0.5 < \phi < 1.3 \tag{30}$$

$\frac{1}{2}\xi$: For a uniform distribution of the density deficit, $\frac{1}{2}\xi$ takes the value 1. In the actual case, the distribution shown in Fig. 9 is more realistic, see Petrén a.o. (1975) and hence

$$\frac{2}{3} < \frac{1}{2}\xi < 1 \tag{31}$$

where the lower limit corresponds to a triangular Δ-distribution.

In summary, the entrainment velocity is (Eq. 26)

$$V_E = \frac{\phi(1-\beta)}{1/2\ \xi}\ 10^{-1}\left(\frac{\tau}{\rho}\right)\frac{V}{\bar{\Delta} gy} \tag{32}$$

$$\frac{V_E}{V} = \eta\left(\frac{f}{2}\right)F_\Delta^2\ ; \qquad 0.05 < \eta < 0.17 \tag{33}$$

This equation for the entrainment explains to a great extent the scatter in Lofquist's (1960) data, as he plotted the relative entrainment velocity as a function of $F_\Delta{}^2$. If we reanalyse Lofquist's data in the light of Eq. (33), the scatter is remarkably reduced, see Fig. 10. The constant evaluated for Lofquist's data corresponds to the upper limit in η ($\simeq$0.2), see Eq. (33). This is in accordance with the physical conditions in the experi-

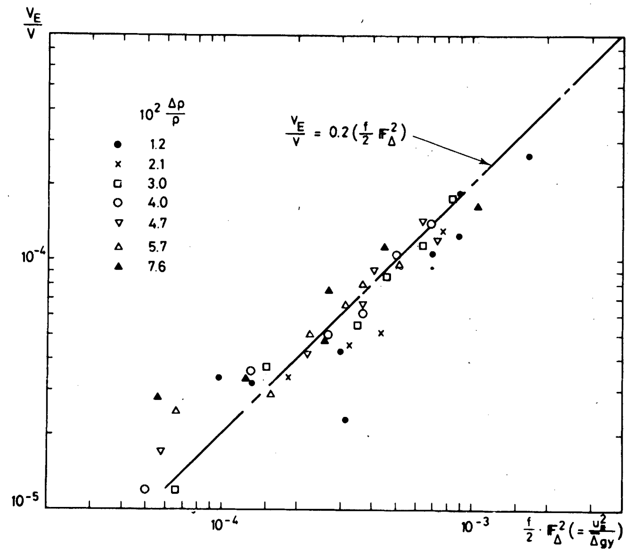

Fig. 10. Entrainment measured in a laboratory flume (Lofquist 1960).

ments. For a dense bottom current in the nature, one may expect the value of η to be at the lower limit stated, but the actual value depends to a high degree on the schematization used in the model. This knowledge of a rational formula for the entrainment (which has been confirmed by model test) makes it possible to apply the hydrodynamic model developed for the Belt Project, see DHI-Report (1977), or another advanced two-layers model on the dense bottom currents in the Baltic. However, if we only intend to make an order of magnitude analysis for the steady state conditions, we may simplify the dynamic and the geometry to a dense bottom current with natural depth in a wide, rectangular-shaped cross-section. Natural depth, i.e. the flow far away from upstream and downstream boundary influence, is in an entraining dense bottom current the depth corresponding to a constant densimetric Froude's number (or constant velocity). Hence a relation between the depth y_n and the discharge Q can be established, by use of the momentum equation in the longitudinal direction. If we neglect the curvature of the flow and the Coriolis effect in the longitudinal direction the momentum equation in longitudinal direction reads:

$$\frac{1}{B}\frac{d}{dx_1}(\rho VQ) = \bar{\Delta}\rho g y I_0 - \frac{d}{dx_1}(\alpha\bar{\Delta}\rho g y^2) - \tau \tag{34}$$

where the term on the left hand side is the inertia term, and the terms on the right hand side are the pressure at the bottom, the pressure on the vertical and the total shear stress. The pressure has been reduced by the hydrostatic pressure in the ambient fluid. For a subcritical flow with natural depth, it is easily shown, that the inertia term and the second term on the right hand side of Eq. (34) are an order of magnitude less than the other terms. Hence the momentum equation for natural depth conditions degenerates to

$$\tau \simeq \bar{\Delta}\rho g y I_0 = \rho\left(\frac{f}{2}\right) V^2 \tag{35}$$

just as for the non-entraining bottom current, and y_n can therefore be determined by Eq. (21).

If we insert Eq. (35) in Eq. (33), the continuity Eq. (23) yields

$$\frac{dQ}{dx_1} = \eta\, I_0 B V = \eta\, I_0 \frac{Q}{y_n} \tag{36}$$

The depth y_n is eliminated by introducing q (21), i.e.

$$\frac{dQ}{dx_1} = \eta\, I_0 \sqrt[3]{\frac{B^2 g I_0 (\bar{\Delta}Q)}{(f/2)}} \tag{37}$$

As the mass deficit flux $\bar{\Delta}Q$ is constant (see Eq. (24))we introduce the constant

$$\bar{\Delta}_R\, Q_R = \bar{\Delta}Q \tag{38}$$

in Eq. (37), which turns out to be

$$\frac{dQ}{dx_1} = \eta \sqrt[3]{\frac{g(\bar{\Delta}_R Q_R)}{(f/2)}} \sqrt[3]{B^2 I_0^4} \quad (39)$$

The Bottom Current from Darss Sill to Bornholm Deep

In Fig. 11 is reproduced one of the seventeen measurements performed by Petrén a.o. (1975) in the Bornholm Strait during a one and a half years period. The figure is typical and shows clearly the existance of a significant bottom current into the Bornholm Basin. From the figures of the time average inflow (Petrén a.o. (1975) Figs. 3:1 to 3:4) the average salinity of the inflowing water can be estimated to about S_{BS}=*14* ‰, which is 6‰ higher than the surface water salinity S_0. Hence the average relative mass deficit in the Bornholm Strait $\bar{\Delta}_{BS}$ is

$$\bar{\Delta}_{BS} = (S_{BS} - S_0)\, 0.75 = 4.5 \cdot 10^{-3} \quad (40)$$

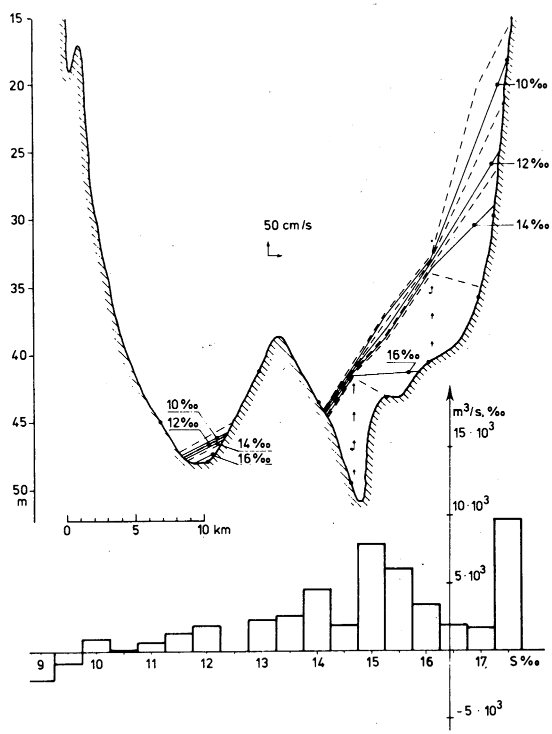

Fig. 11. Salinity and velocity measurements in the Bornholm Strait (Petrén a.o. (1976), Fig. 2:17).

From the previous chapter we know, that the mass-deficit flux ($\bar{\Delta}\cdot Q$) in an entraining current is a constant, the order of magnitude of which was evaluated for the bottom outflow from the Bornholm Basin:

$$\bar{\Delta}_R Q_R = 0.75(S_i - S_0)Q_i = 82\ \mathrm{m^3/s} \tag{41}$$

Hence, the volume fluxes of dense bottom water Q_B and salt SF_B through the acutal cross-section are:

$$Q_B = \frac{\bar{\Delta}_R Q_R}{\bar{\Delta}_{BS}} = 18\cdot 10^3\ \mathrm{m^3/s} \tag{42}$$

and

$$SF_B = S_{BS} Q_B = 255\ t/\mathrm{s} \tag{43}$$

respectively. These numbers are only 10% higher than the upper estimate made by Petrén a.o. (1975) based on direct measurements. Besides of having related the measurements of Petrén a.o. (1975) directly with those of Rydberg (1976) the calculations performed show that a significant entrainment Q_{EB} occurs between the actual cross-section in the Bornholm Strait and the Stolpe Sill cross-section:

$$Q_{EB} = Q_i - Q_B = 16\cdot 10^3\ \mathrm{m^3/s} \tag{44}$$

This extremely high rate of entrainment takes place in a restricted area just north of Bornholm, where the dense bottom current (see Figs. 1 and 11) spreads out. An order of magnitude analysis (not shown) indicates an extension of this high entraining area as indicated in Fig. 1. As approximately half of the total supply to the deep water of the Baltic Proper seams to originate from this area, and as a detailed calculation of this very complicated flow pattern probably is impossible for many years, a field investigation of this special area ought to be performed.

The dense bottom current from Darss Sill to Bornholm Strait can be calculated by a two-layer hydrodynamic model, which takes account of the entrainment, the friction, the Coriolis force, the bottom topography etc., but let us restrict ourselves to a very crude order of magnitude calculation.

From the salinity observations at the mouth of the Baltic (i.e. at Gedser Rev Light-vessel) we know the average salinity $S_{17} \simeq 17‰$ at the sill depth (17 m) in the 12 years period, see the Belt Project (1976, Fig. 2.2.1.1.2). This salinity is 9‰ greater than the average surface salinity S_0 in the southern part of the Baltic. Hence the mass deficit at Gedser $\bar{\Delta}_G$ amounts to

$$\bar{\Delta}_G = 0.75(S_{17} - S_0) = 6.75\cdot 10^{-3} \tag{45}$$

By applying the constant mass deficit flux $\bar{\Delta}_R Q_R$(=82 m³/s) we obtain for the discharge Q_{DS} of the deep water flow at Darss Sill

$$Q_{DS} = \frac{\bar{\Delta}_R Q_R}{\bar{\Delta}_G} = 12\cdot 10^3\ \mathrm{m^3/s} \tag{46}$$

which means, that the total entrainment $Q_{E,DB}$ from Darss Sill to Bornholm Strait amounts to

$$Q_{E,DB} = Q_B - Q_{DS} \doteq 6 \cdot 10^3 \ m^3/s \quad (47)$$

As the bottom current due to the Cariolis force flows along the southern bank of the Arkona Basin, can the average bottom slope I_0 of the schematized channel be calculated as the difference in the sill depths (28 m) divided by the distance along the bank ($140 \cdot 10^3$m), i.e. $I_0 \simeq 2 \cdot 10^{-4}$. If the constant η in Eq. (33) is taken as $\eta=0.1$ in accordance with the discussion in the preceeding chapter, and $f/2$ is estimated to about $3 \cdot 10^{-3}$, then Eq. (39) gives a width of the channel $B \simeq 14 \cdot 10^3$m which in turn implies, that the depth is $y_n = 7.3$ m (Eq. 21). If we compare these results with the measurements by Petrén a.o. (1976) (see Fig. 11), we may conclude, that the size of the schematized channel has the correct order of magnitude.

The measurements in the Bornholm Strait performed by Petrén a.o. (1976) indicate that the bottom current can change significant within some days. If we define the time scale for the bottom current T_{BC} as the active volume divided by the average, max discharge, we obtain

$$T_{BC} \approx 2 \text{ weeks} \quad (48)$$

which shows, that the observed very rapid changes are due to a pendling in- and outflow through the total cross-section and seiches in the Bornholm and the Arkona Basins, although the reservoir effect is weak compared with the reservoir effect in the Bornholm Basin.

Conclusion

The answers to some of the questions about the renewal of the Baltic Proper Deep water can be found in the Bornholm Basin. A statistical calculation of the conditions in the Bornholm Basin combined with a simplified hydrodynamic model for a dense bottom current has given the time average and the standard deviation of the bottom current discharge Q_i through the Stolpe Channel ($23 \cdot 10^3 \leq Q_i \leq 54 \cdot 10^3 m^3/s$), and furthermore the average fluxes of salt ($SF_i \simeq 375$ t/s) and oxygen ($O_2 F_i \simeq 7.3 \cdot 10^6 t/Y$). The average (space and time) upwards salt flux through the halocline in the Baltic Proper has been estimated to $SF_E \simeq 2.65 \cdot 10^{-6} kg/s/m^2$ by entrainment and $1.1 \cdot 10^{-6} kg/s/m^2$ by diffusion. The net oxygen flux is accordingly estimated to nil. It has been demonstrated, that the greatest inflow ever observed (1951) only corresponds to three to four months of continous inflow.

It has been made probable, that nearly half of the dense deep water discharge to the Baltic Proper stems from entrainment in a restricted area just north of Bornholm. An outlined entrainment function has made it possible in the future to calculate the bottom currents in the Baltic (Darss Sill- Bornholm Strait, Stolpe Channel etc.) by an advanced numerical model. In the present article only an order of magnitude analysis has been performed.

List of Symbols

B	: width of the dense bottom current
D	: total water depth
f_{index}	: friction factor (b=bottom; i=interface)
F_Δ	: densimetric Froude's number
g	: acceleration of gravity
h	: sill depth
I_0	: bottom slope
L	: length
O_2F_i	: oxygen flux in the dense bottom current at Stolpe Sill
PROD	: the rate of production of turbulent energy
Q_{index}	: discharge (*B*=bottom current at Bornholm; *DS*=bottom current at Darss Sill; *E*=entrainment; *F*=fresh water; *i*=bottom current at Stolpe Sill;0=surface water at Stolpe Sill)
R_f^T	: bulk flux Richardson number
S_{index}	: salinity (*BD*=Bornholm Deep; *BPD*=Baltic Proper Deep; *BS*=bottom current of Bornholm Strait; *i*=bottom current at Stolpe Sill; *PH*=primary halocline; 17=Gedser at 17 m depth; 0 =surface)
SF_{index}	: salt flux (*B,i*=in the dense bottom current at Bornholm Strait, Stolpe Sill; *D,E*=through the interface in Baltic Proper by diffusion, entrainment)
T_{index}	: Time scale (*BB*=active reservoir in Bornholm Basin; *BC*=bottom current from Darss Sill to Bornholm Strait)
V	: mean velocity
V_E	: entrainment velocity
W	: width
x_i	: cartesian coordinates
Y_{index}	: depth of bottom current (*c*=critical; *n*=natural)
$\alpha\ \beta\ \phi\ \xi\ \eta$	: numbers
Δ_{index}	: relative mass difference (*BS*=Bornholm Strait; *G*=Gedser)
$\Delta\rho$	: mass difference between the upper and the lower layers
$\Delta_R Q_R$	: mass deficit flux (constant)
Ω	: control volume
ρ	: mass
τ_{index}	: shear stress (*b*=bottom; *i*=interface).

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