

## CHANGES IN WIDOWHOOD AND DIVORCE AND EXPECTED DURATIONS OF MARRIAGE

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*Abstract*—A new interpretation of mathematical formulas developed by Keyfitz illustrates how the concept of entropy ( $H$ ) can be applied to the analysis of marriage dissolution. The quantities  $H^{(\text{divorce})}$  and  $H^{(\text{widowhood})}$  indicate the changes in marriage duration which would result from small, constant changes in duration-specific divorce and widowhood rates, respectively. An examination of values for the United States, Nepal and Colombia illustrates the utility of  $H^{(i)}$  in assessing the impact of changes in widowhood and divorce and clarifies the relationship between  $H$  and changes in life expectancy.

Demographic inquiries are frequently concerned with assessing the impact of changing demographic behavior on vital rates. Broadly speaking, these inquiries are addressed either by a series of population projections or simulations, or by a mathematical analysis of the effects of changes in specific parameters on demographic functions.

The latter approach is sometimes referred to as "sensitivity analysis" because it is concerned with the sensitivity of estimates to changes in input variables. In the demographic literature, the relevant input variables have been quantities such as birth rates and death rates, and the functions of interest have been age distributions, rates of increase and life expectancy. For example, Keyfitz (1971) analyzes the effects of small changes in birth and death rates, and Preston (1974) analyzes the effects of small changes in death rates on the age composition and rate of growth of a stable population. Recently, Arthur (1981) has used functional calculus to examine a variety of demographic relationships in stable and nonstable population analysis.

In this paper, we focus on a specific sensitivity analysis in demography: the effect of a small change in the mortality

hazard function  $\mu(x)$  (i.e., instantaneous death rate) on life expectancy ( $e_0$ ). Keyfitz (1977a) has demonstrated that with a uniform proportional change in the death rate  $\mu(x)$  at all ages, the change in life expectancy can be expressed in terms of a quantity " $H$ " known as information or entropy in the physical sciences. Keyfitz (1977a) explores the theoretical relationship between the shape of the survivorship curve  $l(x)$  and  $H$ , the relationship between life expectancy and  $H$  for mortality in the United States, and the variation in the magnitude of  $H$  across single-decrement life tables for specific causes of death. The utility of  $H$  lies in its ability to predict the improvement in life expectancy from a small reduction in overall or cause-specific death rates at all ages. The usefulness of  $H$  for predicting the rise in  $e_0$  from the elimination of a cause of death is doubtful: Keyfitz (1977a) illustrates that for some causes of death  $H$  predicts the rise in  $e_0$  fairly well, but not for others. The goal of this paper is to reinterpret the quantity  $H$  to measure the effects of small changes in marriage dissolution rates—rates of divorce and widowhood—on expected durations of marriage. The changes over time in  $H$  values for marriage duration, the relative importance of changes in widowhood and di-

voice for marriage duration, and the circumstances under which  $H$  can predict the impact of eliminating a cause of marriage dissolution will be explored.

#### DERIVATION OF $H$

Keyfitz (1977a, pp. 62–72) has shown how the logarithm of the survivorship curve [ $\ln l(x)$ ] contains useful information about the effects of small changes in death rates on life expectancies. The same concept can be used or reinterpreted to measure the effects of small changes in dissolution rates on expected durations of marriage.

Consider the overall death rate at age  $x$  (or dissolution rate at duration  $x$ ),  $\mu(x)$ , and suppose that it changes at all ages (or durations) by  $100\delta$  percent. Then, the new death rate,  $\mu^*(x) = \mu(x)(1 + \delta)$  gives rise to a new survivorship curve,

$$l_x^* = e^{-\int_0^x \mu^*(a)(1 + \delta) da} = l_x^{1 + \delta} \quad (1)$$

and a new life expectancy,

$$e_0^* = \int_0^{\omega} l_a^{1 + \delta} da, \quad (2)$$

where  $\omega$  is the oldest age of life. To determine the effect of  $\delta$  on  $e_0^*$ , we consider the derivative of  $e_0^*$  with respect to  $\delta$ :

$$\frac{de_0^*}{d\delta} = \int_0^{\omega} (\ln l_a) l_a^{1 + \delta} da. \quad (3)$$

For values of  $\delta$  near zero, we have

$$\frac{de_0^*}{d\delta} \approx \int_0^{\omega} (\ln l_a) l_a da \quad (4)$$

or

$$\frac{\Delta e_0}{e_0} \approx \left[ \frac{\int_0^{\omega} (\ln l_a) l_a da}{\int_0^{\omega} l_a da} \right] \delta. \quad (5)$$

Defining  $H$  as minus the expression in brackets (so as to make  $H$  positive), we have

$$\frac{\Delta e_0}{e_0} \approx -H\delta. \quad (6)$$

Hence, for a small proportional reduction  $\delta$  in the death rate at all ages, the proportional increase in life expectancy can be simply approximated as  $H$  times  $\delta$ . The value of  $H$  depends directly on the concavity of the survivorship curve: if all people die in a narrow range of ages before  $\omega$  ( $\omega - \epsilon$  to  $\omega$ ), then  $l(x)$  equals unity and  $\ln l(x)$  equals zero for all ages below  $\omega - \epsilon$ , and  $H$  is close to zero. Hence, a change in  $\mu(x)$  has almost no effect on life expectancy. At the other extreme, if the risk of dying is constant at all ages, the survivorship curve declines exponentially,  $H$  equals unity (as integration in equation (5) will show) and a change  $\delta$  in the death rates is reflected by an equal proportionate change in  $e_0$ .

Since actual life tables are associated with  $H$  values considerably below one, increases in life expectancy are usually much more modest than are the reductions in death rates. Keyfitz (1977a) shows that  $H$  values for U.S. life tables have decreased from about 0.37 for 1919–1921 (when  $e_0$  equalled 55 years) to about 0.18 for 1959–1961 (when  $e_0$  equalled 70 years). Calculations for 1981 indicate an  $H$  value of 0.16 and a life expectancy of about 74 years. In general,  $H$  values for females are about .03 to .04 lower than for males. Hence a further improvement in mortality, say a substantial one of about 10 percent at all ages, would increase  $e_0$  in the United States by only 1.6 percent according to equation (6), that is, by about one year.

A similar approach can be used to estimate effects of a change in only one cause of death (or in one type of marital disruption) on  $e_0$ . If we consider a change  $\delta$  in the  $i$ th cause of death,  $\mu_i^*(x) = \mu_i(x)(1 + \delta)$  then the proportionate change in  $e_0$  can be approximated by

$$\frac{\Delta e_0}{e_0} \approx -H^{(i)} \delta, \quad (7)$$

where  $H^{(i)}$  is defined as the weighted average of the logarithm of the associat-

ed single decrement life table for cause  $i$ :

$$H^{(i)} = \frac{-\int_0^\omega [\ln l_x^{(i)}] l_x dx}{\int_0^\omega l_x dx} \quad (8)$$

Note that  $l_x^{(i)}$  is the probability of surviving to age  $x$  in the presence of only cause  $i$ . By the multiplicative property of single-decrement life tables,  $l_x = \pi l_x^{(i)}$ , the cause-specific  $H$  values are additive,  $H = \Sigma H^{(i)}$ . Equations (7) and (8) have been used to study the effects of various causes of death on U.S. mortality over time (Keyfitz, 1977a).

Since all people eventually die and all marriages eventually terminate, the quantity  $H$  captures only the shape of the survivorship curve. However, as described in more detail below, the quantities  $H^{(i)}$  reflect not only the pattern of decrements for cause  $i$ , but also the level or magnitude of dissolution associated with cause  $i$ .

#### DATA AND METHODS

In order to estimate the range of  $H$  values which are associated with marriage duration life tables, explain changes in  $H$  and relationships between  $H$  and  $e_0$  over time, and assess the usefulness of  $H^{(i)}$  in predicting the impact of changes in widowhood and in divorce, we need to explore data from a variety of sources. Below, we analyze data for the United States for the past two decades as well as recent data for Nepal and for Colombia. The latter two data sets have been included to obtain a wider range of  $H$  values associated with widowhood and divorce: by American standards, Nepal has high mortality and a very low incidence of divorce and, in contrast to Nepal and the United States, Colombia's divorce rates are closer in magnitude to its widowhood rates.

In order to estimate  $H$ , and the two components  $H^{(\text{divorce})}$  and  $H^{(\text{widowhood})}$ , we require information on  $M_d^D$ , the di-

vorice rate at duration  $d$  (the interval between  $d$  and  $d + 1$  years of marriage) and  $M_d^S$ , the rate of widowhood due to the death of either spouse at duration  $d$ . Then, the proportion of marriages that survive to duration  $y$  can be estimated as

$$\hat{\lambda}_y = \exp\left[-\sum_{d=0}^{y-1} (M_d^D + M_d^S)\right] \quad (9)$$

(Preston, 1975; Weed, 1980). The resulting life table values should be viewed as "synthetic" probabilities since they describe the expected lengths of marriage implied by mortality rates and divorce rates observed during a specified time period.

Determination of marriage duration in the absence of one of the two sources of disruption is just as straightforward. For example, the proportion of marriages that survive to duration  $y$  in the absence of divorce can be estimated as

$$\hat{\lambda}_y = \exp\left(-\sum_{d=0}^{y-1} M_d^S\right) \quad (10)$$

In order to calculate  $M_d^S$ , the death rate of either spouse at duration  $d$ , we assume for simplicity that all men and women marry at the mean ages at marriage  $\bar{x}$  and  $\bar{y}$  of groom and bride respectively. Then,  $M_d^S$  equals  $M^m(\bar{x} + d) + M^f(\bar{y} + d)$  where  $M^m(x)$  and  $M^f(y)$  are male and female death rates at ages  $x$  and  $y$  respectively.<sup>1</sup> Because of limited data, we also assume in all of our calculations that the divorce rate  $M_d^D$  for durations over 30 is equal to the reported rate for durations 25 to 29.

Divorce rates by duration of marriage for the United States are taken from the Bureau of the Census (1971, p. 20) for 1960-1966; Preston (1975, p. 440) for 1969; and Weed (1980, p. 16, 20) for 1975 and for 1976-1977. With the exception of rates for 1960-1966 which are based on only first marriages, the remaining data pertain to all marriages.<sup>2</sup> Mortality rates and mean ages at marriage for 1963, 1969, 1975 and 1977 are taken from vital

statistics reports for the appropriate years.

Divorce rates by duration of marriage for Nepal and for Colombia are derived from data in the 1976 Nepal Fertility Survey and the 1976 Colombia National Fertility Survey. The two surveys are part of the World Fertility Survey and have sample sizes of about 5,000 women each. The calculation of divorce rates is based on reported dates of "permanent" separation for the five-year period prior to each survey. Hence, for these two sets of calculations, "divorce" actually refers to permanent separation. Because of the relatively small sample sizes in each duration interval, these rates were smoothed prior to calculation of  $l_x$  values. Mean ages at marriage of husband and wife are also derived from the survey data. Mortality rates for Nepal and for Colombia for the approximately corresponding periods are taken from United Nations (1980) and the U.N. Demographic Yearbook (1979), respectively. The reported rates were smoothed on the basis of Coale and Demeny (1966) model life tables.

## RESULTS

### *Relationship among $H$ , $l_x$ , and $e_0$*

The proportions of intact marriages by successive durations for the United States during 1960–1966, 1969 and 1976–1977 are shown in Figure 1. Estimates of  $H$ , derived by numerical approximation<sup>3</sup> of (5), are shown alongside the curves, together with survivorship curves with extreme  $H$  values of 0 and 1.

Figure 1 shows the drastic reduction in marriage duration in the United States over the relatively short period from the early 1960s to the mid-1970s. For example, in the span of only 14 years, the median duration of marriage dropped in half, from about 40 to 20 years. The associated  $H$  values indirectly highlight this rapid change in the shape of the  $l_x$  curve. In the early 1960s, a change in marital dissolution rates of 10 percent at

all durations would have increased marriage durations by only about 4 percent ( $.4 \times 10$ ) because most of the dissolution was due to the death of a spouse at the higher durations. On the other hand, in recent years, a 10 percent reduction would increase average marriage duration by about 7 percent because the major cause of disruption is divorce, much of which occurs at low durations. Current divorce rates are such that the  $l_x$  curve is close to a "maximum entropy" situation. The value of  $H$  would equal one if dissolution rates were constant across marriage durations, a situation which could occur if a monotonically decreasing divorce rate (by marriage duration) exactly counteracted monotonically increasing mortality rates.

Theoretically, values of  $H$  need not bear any particular relationship to values of  $e_0$ . For example,  $H$  would equal one if dissolution rates were constant across duration, regardless of the magnitude of this constant; yet,  $e_0$  would depend directly on the size of the dissolution rate. Nevertheless, Figure 2 indicates that the association between  $H$  and  $e_0$  is very strong. In fact, it is almost linear, although the left-hand tails of the curves suggest that the linearity may break down for high values of  $H$  (and hence low values of  $e_0$ ).

Note from Figure 2 that the rapid changes in  $H$  values over time for marriage duration are in marked contrast to the slow changes in  $H$  values associated with (mortality) life tables. For example, a change in  $H$  from about 0.4 to 0.2 for United States (mortality) life tables occurred over the 40-year period from 1920 to 1960, and a change in  $H$  from 0.7 to 0.4 for Swedish (mortality) life tables took place over the period from 1780 to 1905 (Keyfitz, 1977a, 1977b). In addition, the trend over time in  $H$  values for mortality is opposite to that for marriage duration in the United States (and in most developed countries). As mortality rates have declined over time, divorce rates have increased, life expectancy has

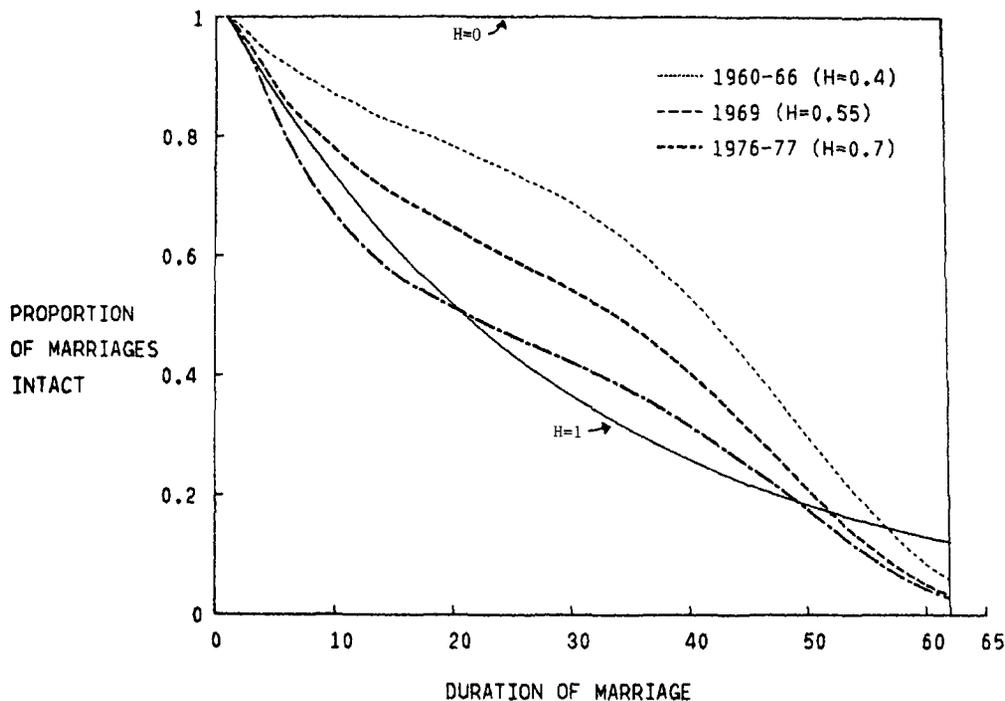


Figure 1.—Proportion of Marriages Intact by Successive Durations, According to Divorce and Mortality Rates for the United States, 1960–1966, 1969 and 1976–1977.

risen and although widowhood rates have consequently declined, the increase in divorce has more than compensated for this; the overall impact has been a large decline in average marriage duration. In the United States today, the  $H$  values for mortality are very low (about 0.16 for 1977), whereas for marriage dissolution,  $H$  values have been increasing by about 0.2 per decade reaching a value of 0.7 for 1976–1977.

#### Cause-specific Decrement

Estimates of  $H$  for the separate components of divorce ( $H^{\text{divorce}}$ ) and of widowhood ( $H^{\text{widowhood}}$ ) for the United States, Nepal and Colombia are given in Table 1. The data for the United States indicate a change in the relative importance of widowhood and divorce over the past two decades. For example, during the period of 1960–1966, a 1 percent reduction in divorce rates would have

lengthened marriage by only 70 percent (0.17/0.24) as much as would a 1 percent reduction in widowhood rates (i.e., a 1 percent reduction in mortality for each sex). Only twelve years later (1975), a small percentage reduction in divorce would have 2.3 (0.48/0.21) times the impact of a small percentage reduction in mortality. Over the short period from 1960–1966 to 1976–1977, the magnitude of  $H^{\text{divorce}}$  has increased almost threefold while  $H^{\text{widowhood}}$  has barely changed. As discussed in Goldman and Lord (1983), mortality improvements in the United States since 1960 have been concentrated at older ages and hence have had only a modest impact on marriage duration.

The values for Nepal are in marked contrast to those for the United States: a reduction in divorce would have almost no impact on the length of marriage, since divorce is so low, whereas a cur-

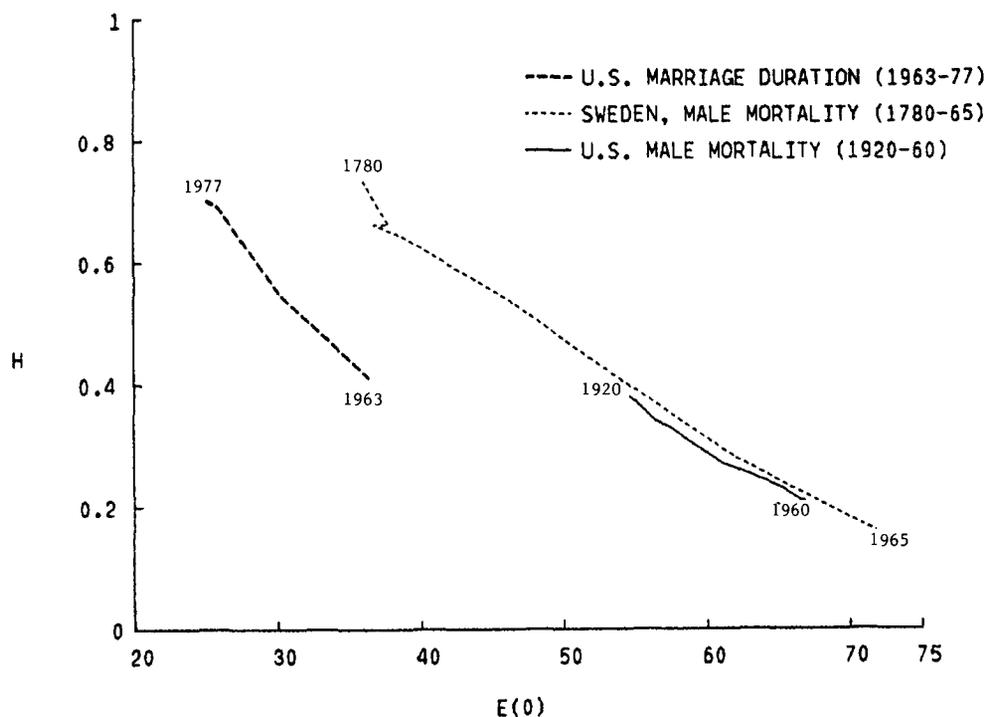


Figure 2.—A Comparison of  $H$  Values and  $e_0$  for Mortality, United States Males (1920–1960) and Swedish Males (1780–1965), and for Marriage Duration, United States (1963–1977).

rent reduction in mortality would have twice the impact as in the United States. In Colombia, unlike in both the United States and Nepal, small percentage changes in mortality rates and in divorce rates would have almost identical impacts on marriage duration.

This latter result is somewhat surprising because duration-specific dissolution rates and hence single-decrement survi-

vorship curves for divorce and for widowhood are very different from one another. Figure 3 shows the proportions of intact marriages in Colombia for the single decrements of divorce and of mortality. As one would expect, most divorces occur at early durations of marriage, whereas widowhood occurs with greater frequency at higher durations. In spite of the very different shapes of these curves,

Table 1.— $H$  Values for Divorce, Mortality of Either Spouse and All Causes of Marital Disruption.

Variable	United States			Nepal	Colombia	
	1960 - 1966	1969	1975	1976 - 1977	1972 - 1976	
Divorce	0.17	0.31	0.48	0.49	0.03	0.28
Mortality of either spouse	0.24	0.24	0.21	0.21	0.41	0.29
All causes	0.41	0.55	0.69	0.70	0.44	0.57

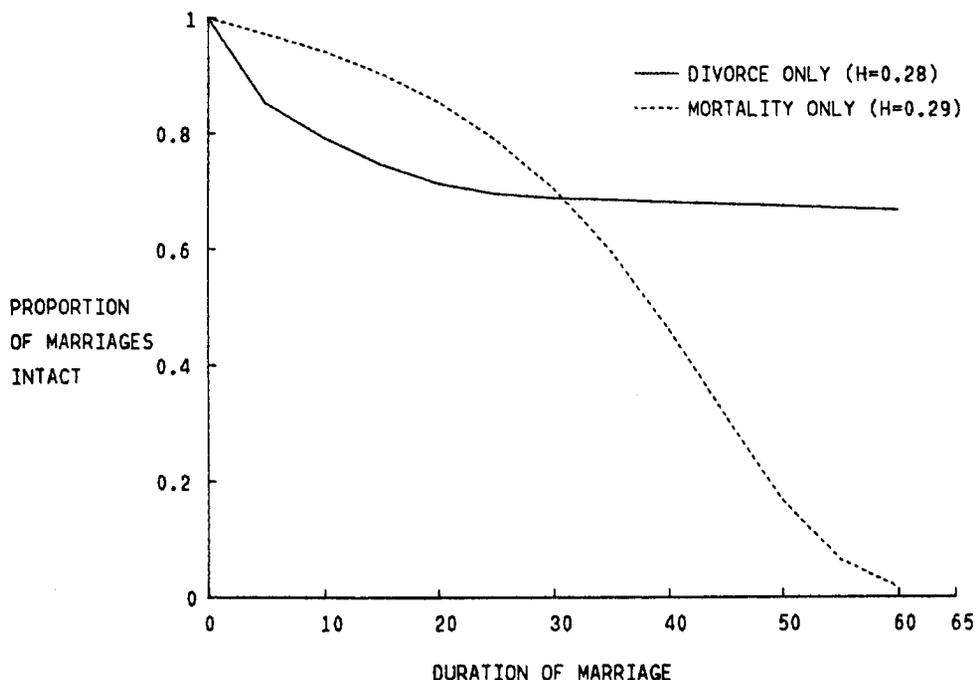


Figure 3.—Proportion of Marriages Intact by Successive Durations, for Single Decrements of Divorce and of Mortality, Colombia (1972–1976).

the  $H^{(i)}$  values are almost identical: a 1 percent reduction in either widowhood or divorce at all durations would raise the average duration of marriage by 0.3 percent from its actual value of 27 years. This result emphasizes the nature of the quantity  $H^{(i)}$ :  $H^{(i)}$  is affected not only by the proportion of decrements (in a multiple-decrement calculation) which are the result of cause  $i$ , but by the durations at which these decrements occur. An example from cause-specific mortality tables also illustrates this finding. Multiple-decrement U.S. life tables for males in 1964 (Preston, Keyfitz, and Schoen, 1972) indicate that about one-third more deaths are the result of respiratory diseases (influenza, pneumonia and bronchitis) than of motor vehicle accidents. Yet, calculations by Keyfitz (1977a) show that the  $H^{(i)}$  value for motor vehicle accidents (.013) is 65 percent greater than for respiratory diseases (.008). The fact that motor vehicle fatalities occur at

a younger age, on average, than do deaths from respiratory diseases is borne out by the finding that a small reduction in mortality from motor vehicle accidents would have the larger impact on life expectancy.

Since  $H^{(i)}$  describes the effects of small changes in  $\mu^{(i)}$  on  $e_0$ , one might expect  $H^{(i)}$  to contain information about the effect on  $e_0$  of eliminating cause  $i$ . For example, if equation (7) were valid for large values of  $\delta$ , the effect of eliminating cause  $i$  ( $\delta = -1$ ) would be an increase in  $e_0$  of  $H^{(i)}$  percent. As noted by Keyfitz (1977a), the relationship is not so clear-cut because the approximation in equation (7) is derived on the assumption of a small value of  $\delta$ , but not necessarily for a reduction of  $\mu^{(i)}$  to zero.

Table 2 shows the effect on marriage duration of eliminating divorce and (more unrealistically) of eliminating mortality for the United States, Nepal and Colombia. These values have been cal-

Table 2.—Expected Duration of Marriage ( $e_0$ ) and Increment in  $e_0$  ( $\Delta e_0$ ) Due to the Elimination of Divorce or Widowhood, as Calculated from the Survivorship Curve and as Approximated by ( $H^{(i)} \cdot e_0$ )

Variables	United States			Nepal	Colombia
	1960 - 1966	1969	1976 - 1977	1972 - 1976	1972 - 1976
Average marriage duration ( $e_0$ )	36.4	30.1	25.2	31.2	27.4
$\Delta e_0$ due to elimination of divorce	6.7	11.2	17.2	1.0	9.0
$H^{(\text{divorce})} \times e_0$	6.2	9.3	12.3	0.9	7.7
$\Delta e_0$ due to elimination of mortality	16.7	13.0	9.4	28.4	17.8
$H^{(\text{widowhood})} \times e_0$	8.7	7.2	5.3	12.8	7.9

culated directly from the single decrement  $l_x$  values described for one case in equation (10) and are compared with the approximation  $H^{(i)} \cdot e_0$ . By and large, the approximation performs poorly. For example, in Colombia the value of  $H^{(i)} \cdot e_0$  (equal to about 7.7 for each decrement) is about a year less than the increase in the absence of divorce (9.0) and as much as ten years lower than the increase in the absence of mortality (17.8). The approximation appears to perform the worst (in proportionate terms) for the larger values of  $\Delta e_0$ . Why the discrepancy between the calculated value of  $H^{(i)} \cdot e_0$  and the effect on  $e_0$  of eliminating cause  $i$ ?

Suppose we consider average marriage duration in the presence of widowhood (cause 1) and divorce (cause 2), and want to determine the change in duration ( $\Delta e_0$ ) as a result of the elimination of divorce. Then,

$$\Delta e_0 = \int_0^{\omega} e^{-\int_0^x \mu(a) da} dx$$

$$= \int_0^{\omega} e^{-\int_0^x [\mu(a) + \mu_2(a)] da} dx \quad (11)$$

$$= \int_0^{\omega} e^{-\int_0^x \mu(a) da} \cdot [e^{\int_0^x \mu_2(a) da} - 1] dx.$$

If we expand  $e^{\int_0^x \mu_2(a) da}$  in a Taylor series, and note that  $\int_0^x \mu_2(a) da$  equals  $-\ln l_x^{(2)}$ , (where  $l_x^{(2)}$  equals the associated single-decrement life table for cause 2), we have

$$\Delta e_0 = \int_0^{\omega} l_x \left\{ 1 - \ln l_x^{(2)} + \frac{[-\ln l_x^{(2)}]^2}{2!} + \frac{[-\ln l_x^{(2)}]^3}{3!} + \dots - 1 \right\} dx \quad (12)$$

$$= [H^{(2)} + 1/2 H_2^{(2)} + 1/6 H_3^{(2)} + \dots] e_0$$

$$\text{where } H_i^{(2)} = - \left| \frac{\int_0^\omega [lnl_x^{(2)}]^i l_x dx}{\int_0^\omega l_x dx} \right|$$

Hence, if  $[lnl_x^{(2)}]^2$  is close to zero for most of the age range,  $H^{(2)} \cdot e_0$  will closely approximate the change in  $e_0$  due to the elimination of cause 2. In general, however, this will only be true if cause 2 has little impact on mortality—that is, if  $l_x^{(2)}$  is close to unity. A further examination of Figure 3 shows that because of very low divorce rates at high durations, and the consequential flattening of  $l_x^{(2)}$ , the weighted average of  $[lnl_x^{(1)}]^2$  is much higher than that of  $[lnl_x^{(2)}]^2$ . Hence, elimination of widowhood has the greater impact on marriage duration. Calculation of  $H_2^{(i)}$  values for Colombia shows that the approximation

$$\Delta e_0 \approx [H^{(i)} + \frac{1}{2} H_2^{(i)}] e_0 \quad (13)$$

yields an estimate for  $\Delta e_0$  of 8.8 years for the elimination of divorce versus an actual  $\Delta e_0$  of 9.0 years. On the other hand, (13) yields an estimate of 11.9 (versus an actual  $\Delta e_0$  of 17.8 years) for the elimination of widowhood. To estimate  $\Delta e_0$  in the latter case, six terms are needed (i.e., summation through the term  $\frac{1}{720} H_6^{(1)}$ ) for the estimate of  $\Delta e_0$  to be within 0.5 of a year of the actual value; that is, convergence of the Taylor series in (12) is slow. In summary, the approximation  $H^{(i)} e_0$  for the effect on  $e_0$  of eliminating cause  $i$  performs best when cause  $i$  has a small impact on  $e_0$  and when the impact is greatest at short durations.

Of more importance, values of  $H^{(i)}$  can reveal a different picture of the relative importance of cause-specific decrements than do the values of  $\Delta e_0$ . A comparison of Tables 1 and 2 indicates that the ranking of countries (and time periods) with respect to the significance of either divorce or widowhood is the same whether  $H^{(i)}$  or  $\Delta e_0$  is chosen as the

criterion for ranking. However, the relative importance of the decrements appears quite differently in the two cases. For example, for Colombia, increases in  $e_0$  due to the elimination of divorce or widowhood indicate that mortality is about twice as important as divorce; on the other hand,  $H^{(i)}$  values indicate that mortality and divorce are of equal significance in Colombia. Similarly, according to  $H^{(i)}$  values, divorce appears to be the more important form of dissolution in the United States in 1969, but according to values of  $\Delta e_0$ , the elimination of widowhood would have the greater impact on marriage duration. Since we are almost always more concerned with the effect of small changes in decrement rates than with the elimination of a decrement, the criterion for assessing the relative importance of different causes of death or dissolution should probably be  $H^{(i)}$ .

### CONCLUSIONS

A new interpretation of mathematical formulas developed by Keyfitz (1977a) has shown how the concept of entropy can be applied to the analysis of marriage duration. Specifically,  $H$  can be viewed as the proportionate change in average marriage duration which results from small constant changes in duration-specific divorce and widowhood rates. An examination of  $H$  values for the past two decades for the United States indicates that the unprecedented increase in divorce has drastically altered the concavity of the  $l_x$  curve for marriage duration so that a current reduction of dissolution rates of about 10 percent at all durations would increase the length of marriage by as much as 7 percent. This  $H$  value of 0.7 for 1976–1977 is almost double that for 1960–1966 and is in marked contrast to an  $H$  value for mortality which indicates that a reduction of death rates in the United States by 10 percent at all ages would increase life expectancy by only 1.6 percent.

The mathematical derivations shown in equations (11) and (12) indicate why

$H^{(i)}$  values often do a poor job of approximating the increase in  $e_0$  due to the elimination of cause  $i$ : higher order terms of the form  $H_j^{(i)}$  are often needed to assess the impact on  $e_0$ . This finding, however, does not diminish the utility of  $H^{(i)}$ . In fact, the comparison shown in Table 2 suggests that  $H^{(i)}$  values might provide a much more useful index than  $\Delta e_0$  values for assessing the relative, as well as absolute, importance of decrements, be they causes of marriage dissolution or causes of death. Both  $H^{(i)}$  and  $\Delta e_0$  values take into account the number of deaths or dissolutions due to a specific type of decrement as well as the ages or durations at which the decrements occur. However,  $H^{(i)}$  values can be used to assess the effects of small changes in decrements rather than of the unrealistic assumption of a total elimination of a decrement.

An examination of  $H^{(\text{divorce})}$  and  $H^{(\text{widowhood})}$  values for the United States, Nepal and Colombia suggests the following results: (a) over a 14-year period, changes in mortality in the United States have had almost no impact on marriage duration, whereas the impact of divorce has increased by threefold; (b) the current impact of divorce in the United States is more than 15 times as great as in Nepal and almost double that in Colombia; (c) the current impact of widowhood in the United States is only half that in Nepal and about two-thirds that in Colombia; (d) in Nepal, the impact of widowhood is 14 times as great as that of divorce; (e) in Colombia, the impacts of divorce and of widowhood on marriage duration are for all practical purposes identical.

#### NOTES

<sup>1</sup> We could refine this calculation so that widowhood rates  $M_d^S$  incorporate the actual distribution of age of bride by age of groom (Goldman and Lord, 1983). In addition, we could base the rates  $M^m(x)$  and  $M^f(x)$  on only the married population. Marital-status-specific mortality rates are not available for the United States subsequent to 1960, but

calculations for 1950 indicate that average marriage duration is about two years higher when based on death rates of the married population than when based on death rates of the entire population (Goldman and Lord, 1983).

<sup>2</sup> Since the vast majority of marriages during the early 1960s were first marriages, the effect should be quite small.

<sup>3</sup> The numerical approximation is based on the extended trapezoidal rule (for example, Abramowitz and Stegun, 1965) applied to single-year values of  $l_x$  for the United States and five-year values ( $l_0, l_5, \dots$ ) for Nepal and Colombia.

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