The signatures of voids – II. In front of the last scattering surface

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ABSTRACT

We explore the signatures of various compensated, subhorizon-sized, quasi-linear voids in the matter-dominated Universe. We show that the temperature distortion functions (the energy that photons have relative to photons moving outside the void) for cold dark matter voids with positive energy are qualitatively the same and quantitatively similar regardless of the velocity profile. Photons are blueshifted on entering and leaving the void by
\[ \frac{\cos \theta}{cH} \delta R\frac{dRH}{3c}, \]
and are redshifted linearly with distance inside the void by a total amount of
\[ \frac{\cos \theta}{cH} \delta R\frac{dRH}{3c}, \]
where \( cH^{-1} \) is Hubble’s radius, \( R \) is the radius of the void and \( R \sin \theta \) is roughly the distance of closest approach. These effects are large since they are first-order in \( RH/c \). We also show that a positive-energy, quasi-linear, cold dark matter void will grow asymptotically (independently of its initial velocity profile) after 10 times the initial time, with relative expansion coefficient inside the void of \( \delta = 2\delta/9 \), where \( \delta \) is the underdensity of the void. If a quasi-linear, pressureless, positive-energy void is growing asymptotically when cosmic microwave background photons cross it, then it appears as a cold spot surrounded by a hot ring with temperature anisotropy
\[ \Delta T/T = \frac{2}{5} \delta R\frac{dRH}{3c} \cos \theta [1 - (5/3) \cos^2 \theta]. \]
However, if this same void is not asymptotically evolving when the photons cross it, then even though the temperature distortion functions are changed little, this void can appear as either a cold or a hot spot on the microwave background just by changing the initial velocity profile from unperturbed to perturbed. Thus positive energy, pressureless voids that have equivalent underdensities and sizes can have very similar temperature distortion functions (which ultimately determine the signature of a void on the last scattering surface), but can have very different signatures in front of the last scattering surface unless they are asymptotically evolving. This is due to cancellations to third-order in \( RH/c \). In addition, reduction of the energy of a void to zero completely reverses the temperature distortion functions; photons experience redshifts entering and leaving a zero-energy void, and a blueshift linear with distance crossing the inner void region. However, a zero-energy void can appear as a cold spot but with a signature that is 20 times larger than that for an asymptotically evolving cold dark matter void. We also find that voids with small amounts of pressure have very complicated temperature distortion functions because of the wall explosion and resultant inward and outward-travelling shocks. However, their signatures can be comparable to those of cold dark matter voids.

Key words: hydrodynamics – methods: analytical – methods: numerical – cosmic microwave background.

1 INTRODUCTION

A void is an underdense region characterized partially by its radius, \( R \), and underdensity,
\[ \delta = 1 - \rho_{\text{in}}/\rho_{\text{out}}, \]
where \( \rho_{\text{in}} \) and \( \rho_{\text{out}} \) are the energy densities inside and outside the void, respectively. The evolution of a cold dark matter void consists of three distinct phases. In the first phase when \( \delta \ll 1 \), the void evolves linearly as a small perturbation. Its amplitude and radius grow linearly with the scalefactor of the Universe, \( a(t) \). During the second phase when \( 0.1 \leq \delta \leq 0.9 \), the void grows quasi-linearly in a complicated manner – the void deepens quickly and its radius grows faster than \( a(t) \). In addition, a pronounced wall forms. In the third phase where \( 0.9 \leq \delta \leq 1 \), the void grows non-linearly, and its evolution can be described by a simple analytic similarity solution. In particular, the void deepens rapidly because the scale-factor grows as \( t \) inside the void, and the thin shell becomes thinner and
moves outward with radius $R(t) \propto t^{2/3 + \xi}$ ($\xi > 0$). If the void is compensated (uncompensated), $\xi = 2/15$ ($2/9$) when the compression is adiabatic (see Bertschinger 1985 for a review). The quasi-linear regime is interesting from a large-scale structure point of view because galaxy surveys have found that our Universe is filled with quasi-linear voids surrounded by galaxies, clusters of galaxies and great walls (Kirshner et al. 1981, 1987; de Lapparent, Geller & Huchra 1986, 1988; Da Costa et al. 1988; Geller and Huchra 1989; Vogele, Geller & Huchra 1991; Slezaek, de Lapparent & Bijaoui 1993; Geller et al. 1997). It is therefore useful to determine the sensitivity to initial conditions of the anisotropies imprinted on the cosmic microwave background radiation (CMBR) from isolated voids.

The signature of a subhorizon-sized linear perturbation in a flat Friedmann–Robertson–Walker (FRW) universe is much smaller than that of a non-linear distribution of mass of the same size, because higher order cancellations occur for the linear perturbations (Sachs & Wolfe 1967; Rees & Sciama 1968). The signature of a subhorizon-sized, compensated, pressureless, non-linear void embedded in a flat FRW universe was first obtained by Thompson & Vishniac (1987) via lengthy calculations. The same result was later derived using the simple and powerful potential approximation approach (Martinez-González, Sanz & Silk 1990, Martinez-González & Sanz 1990). The result is

$$\Delta T/T = \frac{1}{3} \cos \psi_e \left[ \frac{8}{9} \xi TV - \frac{16}{81} \cos^2 \psi_e \right]$$

and

$$\Delta T/T = \frac{2}{3} \left( R_0 H_0/c \right)^3 \cos \psi_e \left[ 1 - \frac{5}{3} \cos^2 \psi_e \right].$$

Here, the subscript ‘e’ denotes the value of the function when the photon leaves the void, $\xi TV = 2/15$, $H = \dot{a}/a$ is the Hubble constant outside the void, and $\eta_e = c^{-1} R_0 \dot{\psi}_e$. In addition, $\psi_e$ is the angle between the line bisecting the centre of the void and the location of the photon at $t_e$, and the line tracing the path of the photon at $t_e$. Setting $X_e = R_e \sin \psi_e$, photons are redshifted for $X_e/R_e < .63$, and are blueshifted otherwise – a non-linear void in front of the last scattering surface (LSS) appears as a cold spot surrounded by a hot ring. (By ‘in front of the LSS’, we mean that nearly all photons comprising the signature scatter behind the void, so that a negligible percentage of photons last scatter within or in front of the void.) For a central photon, then, $\Delta T/T = -\left(4/15\right) \left( R_0 H_0/c \right)^3$. This result is an order of magnitude smaller than that found by Mészáros (1994), who neglected the time-dilatation effect and obtained $\Delta T/T = -2 \left( R_0 H_0/c \right)^3$ for a central photon passing through a similar void but with constant wall thickness. Panek (1992) also studied the effect pressureless, compensated, quasi-linear voids $100 h^{-1}$Mpc away have on the CMBR using the Tolman–Bondi solutions with smooth boundaries between the void and background universe. For two realistic voids with $R_0 = 30 h^{-1}$Mpc, he found that the secondary temperature anisotropies for the central photons are $-3 \times 10^{-7}$ and $-2 \times 10^{-7}$ when $\delta_z = 1$ and $\delta_z = 0.7$, respectively, which are below the detection limits. That work also provided a numerical confirmation of equation (3). Baccigalupi, Amendola & Occhionero (1997) studied the effect primordial, pressureless, quasi-linear voids have on the CMBR. They looked at the signatures of voids on the LSS, including the first-order temperature distortion effects previously found by Vadas (1995), as well as other physical processes of the LSS not previously included. Their work also verified equation (3).

In this paper we study the signatures of quasi-linear, compensated voids in front of the LSS. This work improves and expands upon previous studies (Vadas 1995). These voids are embedded in an $\Omega = 1$ FRW matter-dominated universe with zero cosmological constant. In a companion paper (Vadas, in preparation, hereafter referred to as V3), we study the signatures of quasi-linear voids lying on the LSS, which produce secondary anisotropies from the integrated effect. (By lying on the LSS, I mean that a majority of the CMBR photons scatter for the last time somewhere within the void or void wall.) These signatures are purely gravitational, and do not include bremsstrahlung and Compton processes. We do include energy dissipation to prevent shell-crossing in the deepest void, as well as a small amount of non-homogeneous pressure in two other voids. For these voids, exact solutions are not available. (The exact Tolman–Bondi solutions describe the evolution of spherically symmetric, pressureless, zero-viscosity, subhorizon-sized or superhorizon-sized matter distributions (Tolman 1934 and Bondi, 1947).) Therefore, we integrate the full spherically symmetric, general relativistic fluid equations and geodesic equations. Although we are constrained computationally, we are able to obtain meaningful numerical signatures for voids with $RH_0/c = 0.2$ in this paper, and can easily extrapolate our results to smaller voids. Consequently, we are able to go beyond the previous work in exploring the signatures of voids in front of the LSS with a wide range of initial conditions.

The organization of this paper is as follows. In Section 2 we review the axisymmetric geodesic and photon energy equations. In Section 3, we analytically calculate the expansion rate of an ideal, asymptotically evolving, pressureless void as a function of its underdensity. In Section 4, we look analytically at the different contributions to the temperature anisotropy. In Section 5, we review the initial conditions and simulation parameters for our numerical studies. We then calculate numerically the temperature distortion functions of photons crossing evolving quasi-linear voids as a function of the initial conditions and show that our analytic approximations work well. In Section 6, we study the effects maturity and initial velocity conditions have on the signature of cold dark matter voids, and in Section 7, we study the corresponding effects for quasi-linear voids with pressure and for zero-energy quasi-linear voids. Finally, Section 8 contains a discussion of our results.

## 2 Geodesic Equations

Consider a spherically symmetric relativistic fluid distribution centred on the origin. The comoving metric we use to describe it is

$$ds^2 = -c^2 \Phi^2(t, r)dr^2 + \Lambda^2(t, r)d\theta^2 + d\zeta^2,$$

where $t$ is the time, $r$ is the radial coordinate, and $R$ is the physical radius. We define the time-ray-tracing begins to be $t_{GR}$, and the comoving-frame radius, $R_{CG}$, to be the physical radius at this time: $R_{CG}(r) = R(t_{GR}, r)$. Thus the location of a central photon as a function of time is described either by the physical radius $R(t, r)$ of a shell at that location, or by the comoving-frame radius $R_{CG}(t)$. A shell (labelled by $r$) therefore has a constant comoving-frame radius in an inhomogeneous fluid. In addition, we define $Z_{CF} = R_{CG} \cos \theta \cos \zeta$, $X_{CF} = R_{CG} \sin \theta \cos \zeta$, and $Y_{CF} = R_{CG} \sin \zeta$. Fig. 1 of Vadas 1998 (Paper I, this issue, hereafter referred to as V1) shows the orientation of the coordinate system. The comoving frame is useful because the expansion of the universe is factored out.

1 Pressure can be included only if it is spatially homogeneous.

2 By ‘central photon’, we refer to a photon which travels through the origin.
We take our equation of state to be that of a perfect fluid with a small amount of dissipation included in a few of the simulations (as artificial viscosity) to prevent shell-crossing from occurring. Note that for cold dark matter, the pressure is initially zero everywhere. For non-relativistic flows with a small amounts of pressure, the specific energy is $\epsilon(t, R) \ll c^2$. In addition, a void has zero energy if $\Gamma = 1$, and positive energy if $\Gamma > 1$, where $\Gamma = R'/\Lambda$. The interested reader can find pertinent references, the equations of motion and numerical methods used in the existing code in V1, Vadas (1994b) and Vadas (1993).

In V1, we show that the generalized geodesic equations for $\xi = 0$ can be rewritten as

$$\frac{dw}{dr} = -\frac{2}{R} \left[ \Phi U + R'z \right] + \frac{\Phi'}{\Phi} R' U' z^2 + \frac{2\Phi'}{\Phi} z + \frac{R U}{\Phi} w^2,$$

and

$$\frac{dz}{dt} = \left[ \frac{\Phi'}{\Phi} R' U' z^2 + \frac{2\Phi'}{\Phi} z + \frac{R U}{\Phi} w^2 \right] - \frac{\Phi' \Gamma^2}{R^2} \left[ \frac{R'}{R} \right]^2 \frac{\Gamma^2 - \Gamma' R'}{R^2} \left( \frac{R}{R'} \right)^2 + \left( \frac{R}{R'} \right)^2 \frac{R U}{\Phi} w^2,$$

where $\dot{z} = \partial z/\partial t$, $\dot{r} = \partial r/\partial t$, and $w = \partial w/\partial t$. In addition, the ‘velocity’ of a shell is $U = R\Phi$, and at time $t$, the energy of a non-radially-propagating photon is

$$E(t) = E(t_{\text{ray}}) \frac{\Phi(t)}{\Phi(t_{\text{ray}})} \frac{w(t_{\text{ray}})}{w(t)} \frac{[R(t_{\text{ray}})]^2}{[R(t)]^2},$$

where $E(t_{\text{ray}})$ is the energy at the start of ray-tracing. Because integrating through inhomogeneous distributions of pressure and past-shell-crossing is now possible, these four first-order ordinary differential equations generalize those previously to ray-trace through pressureless, Tolman–Bondi solutions (Raine & Thomas 1981; Panek 1992). V1 describes how these equations are solved numerically and how the errors are accessed. A non-uniform grid is used here for the ray-tracing simulations, with parameters given in Section 5.1.

### 3 Expansion Coefficient versus Underdensity

In Section 4, we will calculate the approximate first-order effects contributing to the temperature distortion function. Because these expressions are derived in terms of the expansion coefficient of a void (to be defined momentarily), it is useful to obtain an approximate expression that relates the expansion coefficient to the underdensity of a void, $\delta$. We will only consider pressureless, subhorizon-sized, isolated voids embedded in a flat, matter-dominated universe in deriving this expression.

We can parametrize the physical radius of a shell with comoving radius $r$ by the function $a(t, r)$ as

$$R(t, r) = R(t_i, r_i) \exp(\alpha(t)) = R(t_i, r) \exp(\alpha(t)),$$

where $\alpha = \ln(a'/a)$. Since the pressure is negligible, we set $\Phi = 1$ (V1). And because $\partial \alpha/\partial t = t^{-1} \dot{a}/a$, where $\partial_x = \partial/\partial x$, the velocity of this shell is

$$\dot{R} = \frac{1}{R} \left[ \frac{\Phi U}{R} \right] + \frac{\Phi'}{\Phi} R' U' z^2 + \frac{2\Phi'}{\Phi} z + \frac{R U}{\Phi} w^2,$$

where we have defined the expansion coefficient to be $\dot{\alpha} = \alpha + x \dot{\alpha}$. Thus $\dot{\alpha}$ equals $\alpha$ only if $\alpha$ is constant in time. The acceleration of this shell is then

$$\ddot{R} = r^{-2} \left( R \dot{\alpha} + \ddot{\alpha} R - \dot{\alpha} R \right).$$

Since the pressure is negligible, the mass $M$ contained within a shell is constant and the acceleration is (V1)

$$\ddot{R} = -GM/R^2.$$

Assuming that the matter within the inner void region is approximately homogeneous and isotropic (i.e. that it is a mini open universe embedded in the background space–time), we can easily calculate the mass within any shell with radius $R$ in the inner void region by integrating the equation $M = 4\pi c^2 R^2 \rho R^3$ (V1) out to radius $R$. We find that $M(R) = 4\pi c^2 R^3 \rho_{\text{void}}$, where $\rho_{\text{void}}$ is the approximately constant energy density within the inner void region. Because $H^2 = 8\pi c^2 G \rho_{\text{void}}/3$, $H$ is the Hubble ‘constant’,

$$GM = H^2 R^3 (1 - \delta)/2,$$

where we have used equation (1). If the background universe is flat and matter-dominated, then $H^2 = 4/(9\Omega^2)$ and equations (10)–(12) become

$$\partial_t \dot{a} + \dot{\alpha} \dot{a} - \ddot{a} = -2(1 - \delta)/(9).$$

Defining the relative expansion coefficient $\xi = \dot{\alpha} - 2\xi/3$, this yields

$$\partial_x \xi + \frac{1}{3} \xi + \xi^2 = \frac{2}{9} \delta.$$

We can solve this expression exactly for linearly evolving voids (i.e. when $\xi \ll 1$). In this case, because $\delta \approx r^3 \exp(2\delta/3)$ (Kolb & Turner 1990), we obtain

$$\xi(\delta) = \frac{2}{9} \delta, \quad \alpha(\delta) = \frac{2}{3} + \frac{2}{3} \delta.$$

Using equation (9) and $H = 2(3\Omega)$, we can relate the relative expansion coefficient of a void with the radius and velocity of an inner shell as

$$\dot{\xi} = \frac{1}{R} \left[ \frac{U}{R} \right] + \frac{2}{3} \left( \frac{U}{R} \right) \frac{1}{\dot{R}^3} - 1.$$

Equating equations (15) and (16), we can calculate the velocity of any shell within a void knowing only the underdensity of the void. Although equation (15) was derived for linear voids, we will show numerically in Section 6.2 that it also works well for quasi-linear voids. We will therefore apply it to several situations involving quasi-linear voids in this paper.

In addition, defining $\xi = \alpha - 2/3$,

$$\dot{\xi} = \dot{\alpha} - 2/3$$

In the linear regime then, using equation (15),

$$\alpha(t) = \frac{2}{3} + \dot{\xi}(t) = \frac{2}{3} + \frac{\delta(t - \delta_i)}{3 \ln(t/\delta_i)}.$$

### 4 Temperature Anisotropy Crossing a Void

As a photon enters, crosses, and exits a void, its energy relative to a

$$\delta = \alpha - 2/3; \quad \xi = \dot{\alpha} - 2\xi/3,$$

In Vadas (1994a), $\alpha$ should be replaced by $\ddot{\alpha}$ in the expression after equation (3.1) to give the correct expression $\ddot{\alpha} = 2/3 + 2\delta/9$.
photons moving outside this void [hereafter called the ‘temperature distortion function’], represented mathematically by \( \Delta T(t_{\text{ray}}, t) \) changes in a complicated manner. This complication is important for understanding the subtle details of the signature of a void when in front of the LSS, because its signature is the net temperature distortion, \( \Delta T(t_{\text{ray}}, t_0) \), where \( t_{\text{ray}} \) is the time ray-tracing began, and \( t_0 \) is the time today. It is also very important for understanding the signature of a void lying on the LSS.

Although the signature of a non-linear void in front of the LSS is a tiny, third-order effect [i.e. \( \Delta T/T \sim (\dot{R}/c^3) \)], the temperature distortion of a photon which crosses the void is large, first-order and location-dependent. The tiny, net signature is obtained because cancellations occur to first and second-order. This behaviour is displayed graphically by the temperature distortion changing sign several times (e.g. figs 3 and 6 in V1; Figs 1 and 5 in this paper).

Consider a compensated, pressureless, non-empty void with positive energy in a flat, \( FRW \) universe. (A positive energy void is ‘relatively increasing’, because its comoving-frame radius and underdensity increase with time.) Here, the void and void wall expand outward faster than the expansion rate of the universe. There are essentially three large, first-order effects that contribute to the temperature distortion of a photon as it enters, crosses and exits this void. The first effect is that the photon is Doppler blueshifted upon entering the void, because the wall is moving outward faster than the Hubble expansion. The second effect is that the photon is linearly redshifted as it crosses the void. This results because the photon is crossing an underdense, approximately homogeneous region (i.e. a mini \( \Omega < 1 \) \( FRW \) universe) which expands faster than the Hubble expansion rate outside the void. This redshift is a combination of a Dopplerian blueshift, gravitational redshift and a blueshift of expansion (Rees & Sciama 1968; Mészáros 1994). Finally, the photon is Doppler blueshifted upon leaving the void, because again the wall is moving outward faster than the Hubble expansion rate. There are also smaller, second-order effects present; entering and leaving the void results in a smaller gravitational redshift and blueshift, respectively.

Note that because the observers inside the void are comoving, they are accelerated. Therefore, a photon will always be redshifted on crossing the inner region of an expanding void with our set of observers, even if the void is empty. This property is in contrast to that considered by Thompson and Vishniac (1987). Because they examined only empty voids, they conveniently chose Minkowski observers inside the void, so that the energy of a photon is constant across the inner void region in this coordinate system. Because we are interested in non-empty, thick-walled voids, comoving observers are our natural choice for observers. However, although the contribution to \( \Delta T/T \) due to the Doppler shift and the gravitational potential depends on the gauge chosen (Padmanabhan 1993), the temperature anisotropy as measured by a specific observer does not, as long as one is consistent.

We can express the net temperature anisotropy of a photon which passes through a void in terms of three separate, approximately calculable contributions. Consider a photon with initial energy \( E(t_{\text{ray}}) \). At time \( t_{\text{ray}} \), the photon is moving through the homogeneous fluid outside the void region. At time \( t_1 \), the photon passes through the void wall and enters the homogeneous region inside the void. At time \( t_2 \), the photon reaches the end of the approximately homogeneous region and enters the second void wall. At time \( t_1 \), the photon has passed through the second void wall and is completely beyond the void region. We compare the final temperature \( E(t_1) \) with that of a photon that travelled outside the void region from \( t_{\text{ray}} \) to \( t_1 \). (These two photons do not move the same distance in general.)

The net temperature anisotropy of the former photon at time \( t_1 \) then is

\[
\frac{\Delta T}{T}(t_{\text{ray}}, t_1) = \frac{E(t_1)\alpha(t_1)}{E(t_{\text{ray}})\alpha(t_{\text{ray}})} - 1
\]

\[
= \left[ \frac{\Delta T}{T}(t_{\text{ray}}, t_1) + 1 \right] \frac{\Delta T}{T}(t_1, t_2) + 1
\times \left( \frac{\Delta T}{T}(t_2, t_1) + 1 \right) - 1
\]

\[
= \frac{\Delta T}{T}(t_{\text{ray}}, t_1) + \frac{\Delta T}{T}(t_1, t_2)
+ \frac{\Delta T}{T}(t_2, t_1),
\]

where we define the temperature distortion function between any times \( t_2 \) and \( t_1 \) to be

\[
\frac{\Delta T(t_1, t)}{T} = \frac{E(t_1) - E(t)}{E(t)} = \frac{E(t_1)\alpha(t)}{E(t)\alpha(t)} - 1,
\]

where \( E(t) \) is the relative energy of a photon propagating outside the void at the same time, and where \( \alpha(t) \) is the scale-factor of the background Universe. To obtain equation (21) from (20), it was assumed that \( \Delta T/T \ll 1 \), a good assumption if the void is subhorizon-sized. Equation (21) then shows that the three largest terms contributing to the temperature distortion function are additive. We will show in this section and in Section 5.2 that to first-order, the first and third terms in equation (21) are the Doppler blueshifts entering and leaving the void, and the second term is the redshift of expansion from crossing the inner void region.

Although equation (19) looks simple as written, the temperature anisotropy actually contains three distinct contributions:

\[
\frac{\Delta T}{T} = \frac{1}{3}(\phi_0 - \phi_0) + 2 \int_{t_0}^{\infty} \frac{d\rho}{dt} + n_0 \cdot \mathbf{v}_0/c,
\]

where the terms (in order) are the Sachs–Wolfe effect (Sachs & Wolfe 1967), the integrated Sachs–Wolfe (or Rees Sciama) effect, and the Doppler effect (see Martínez-González & Sanz 1990 and references therein). Here, \( \phi = GM/c^2 R^2 \) is the Newtonian potential, \( t_0 \) is the time emission, \( n_0 \) is the unit vector in the direction of the photon, and \( \mathbf{v} \) is the fluid velocity.

Although we are not able to determine the temperature anisotropy analytically for a thick-walled, non-empty void, we can calculate the approximate first-order contributions to the three temperature distortion terms in equation (21) (subsections 4.1 and 4.2 below). This calculation is important for checking our numerical results as well as providing a framework for analytically calculating the signatures of voids lying on the LSS (V3). Subsections 4.3 and 4.4 below look at higher order effects.

### 4.1 ‘FRW’ redshift

We first calculate the contribution to the temperature distortion a photon attains crossing the inner region of a homogeneous, pressureless, relatively increasing void embedded in a flat matter-dominated \( FRW \) universe. We can do this because the (pressureless) matter within any sphere of comoving radius \( R_{\text{CF}} \) evolves in a manner independent of the dynamics outside of this region (Birkhoff’s Theorem). Let \( t_1 \) be a time just before the photon reaches the void, and let \( t_1 \) be the time as the photon enters the inner void region after crossing through the first void wall. Because the inner region is nearly homogeneous, the energy of a photon as it crosses the inner region...
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4.2 Doppler effect

We now calculate the first-order contribution to the Doppler shift for a photon entering and leaving a void. If the wall expands outward at velocity $v$ relative to the background expansion, then the change in frequency upon entering or leaving the void is $(\sqrt{1 + v^2/c^2}/\sqrt{1 - v^2/c^2} - 1 = v^2/c$ when $|v^2/c| \ll 1$, where $v_2$ is the component of the velocity of the void wall in the $+Z$-direction. Using equation (9), the relative velocity in the $Z$-direction is

$$v_Z = \cos \theta_e [U - (2/3)R_V] = (\cos \theta_e) \xi \eta R_V/t_e.$$  

Thus, the resulting blueshift is

$$\frac{\Delta T}{T} = (\cos \theta_e) \xi \eta$$

for a subhorizon-sized void. Comparing this with equation (30), it is clear that the sum of each Doppler shift entering and leaving the void cancels the redshift attained crossing the inner void region to lowest order, as is well known.

4.3 Gravitational potential (Sachs–Wolfe) effect

Using the quasi-Newtonian approximation, we can estimate the gravitational potential effect. Let the void have radius $R_V$ and wall thickness $\Delta R_V$, and assume that $\Delta R_V \ll R_V$. Because the Newtonian potential is $\phi = GM/c^2$, upon entering or leaving the void the change in $\phi$ is $\Delta \phi = G\Delta M/(c^2 R) = 4\pi G c^{-2} \rho_{\text{void}} R^3/3$. Upon entering and leaving the void, then, at times $t_1$ and $t_2$ respectively, a photon is redshifted and blueshifted, respectively, by the amounts

$$\frac{\Delta \phi(t_1)}{3} = -\frac{\delta(t_1)}{6} [R(t_1)H(t_1)c]^2,$$

$$\frac{\Delta \phi(t_2)}{3} = \frac{\delta(t_2)}{6} [R(t_2)H(t_2)c]^2,$$

which are both second-order effects. (This expression is only a rough estimate, because the actual prefactors depend on the details of the distribution of matter.) Thus, if a photon last scatters within the void, its signature will have a second-order contribution from the change in gravitational potential, as is well known. In addition, if a photon last scatters before entering the void, the gravitational potential contribution entering and leaving the void will cancel to third-order for subhorizon-sized voids.

4.4 Integrated Sachs–Wolfe effect

We now estimate the magnitude of the integrated Sachs–Wolfe effect crossing the void. This effect comes from the change in the gravitational potential as the photon crosses the void. If we let $t_c$ be the time-scale for change for the Newtonian potential and $R_V$ be the radius of the void, then

$$\frac{\Delta T}{T} = \frac{\Delta \phi(t_e)}{t_c} = \frac{2\cos \theta_e G(\Delta M)}{c^3 t_c},$$

where the time to cross the void is $\Delta t = 2(\cos \theta_e) R_V/c$, and $\Delta \phi = G(\Delta M)/(c^2 R_V)$. But because $G(\Delta M) = 4\pi \delta G p R_V ^3/3$, 

$$\frac{\Delta T}{T} = \delta \cos \theta_e H R_V^3/c^3 t_c.$$  

Panek (1992) suggests that $t_c$ should be of order the cosmic timescale, $H^{-1}$. This argument is true for non-linear voids. However, the time-scale is longer for linear and quasi-linear voids. We estimate

\[ E(t) = E(t_1) \frac{a(t)}{a(t_1)} (24) \]

where $a(t)$ is the expansion rate inside the void. From equation (8), $a(t) = a(t_1) \beta(t_1)(t/t_1)^{y_1}$, where we have suppressed the dependence of $\alpha$ on $r$ because of the homogeneity within the inner void region. At the same time, a photon with identical energy at $t_1$ travelling outside the void has energy $E_{\text{out}}(t) = E(t_1) a(t_1) a(t)$ at time $t$, where $a(t) = a(t_1) (t/t_1)^{y_1}$. Then, the temperature distortion from $t_1$ to $t_2$ is

\[ \Delta T = E(t) - E_{\text{out}}(t) = \frac{a(t_1) a(t)}{a(t_1) a(t)} - 1 = a(t_1) a(t) - 1 = \left( t/t_1 \right)^{y_1} \left( t/t_1 \right)^{2 \beta} - 1 \]

This expression can be rearranged to give

\[ \Delta T = \left( \frac{t}{t_1} \right)^{y_1 + \beta} - 1. \]

We now define $\tilde{e} = \ln(t_1 - 1)$. In addition, $\xi(t)$ can be Taylor-expanded to give $\xi(t) = \xi(t_1) + (t - t_1) \xi_x \xi = \tilde{e} \xi_x \xi$ to lowest order, where $x = \ln(t_1)$. Because the void is subhorizon-sized, $\tilde{e} \ll 1$ so that equation (26) becomes

\[ \Delta T = \left( 1 - \tilde{e} \xi_x \right) \left( 1 + x_1 \right) \tilde{e} \xi_x \xi = \left( 1 - \tilde{e} \xi_x \right) \left( 1 - \tilde{e} \xi_x \xi \right) \]

\[ = \left( \tilde{e} \xi_x \xi \right)^2 \left( 1 - \tilde{e} \xi_x \xi \right) \]

\[ = 2 \tilde{e} \xi_x \xi \left( 1 - \tilde{e} \xi_x \xi \right) \]

\[ = -2 \tilde{e} \xi_x \xi \left( 1 - \tilde{e} \xi_x \xi \right) \]

where we have neglected the second term in brackets to lowest order.

From the metric, when the axes are chosen so that the photon moves in the $X-Z$ plane (i.e. $\xi = 0$) with $X_C$ constant, one finds that $\tilde{e} = e^{-1} [Z_{\text{CF}}(t) - Z_{\text{CF}}(t_1)]/t_1$, which is just the Minkowski result applicable when small distances are travelled. Then the temperature distortion at time $t$ relative to $t_1$ is approximately

\[ \Delta T = \left( \frac{t}{t_1} \right)^{y_1 + \beta} \left[ Z_{\text{CF}}(t) - Z_{\text{CF}}(t_1) \right] H(t_1). \]

Therefore, as expected, the total temperature distortion redshifts linearly with distance within a subhorizon-sized void. For a photon traversing the entire inner region under these assumptions, $Z_{\text{CF}}(t_2) = Z_{\text{CF}}(t_1)$, $c_\text{C} = \cos \theta_e R_C(t_c)$, where $c_\text{C} = \cos \theta_e R_C(t_c)$, and the subscript 'c' denotes the value of the function when the photon leaves the void. (For example, a radial or central photon has exit angle $\theta_e = 0$.) Then the total temperature distortion in the inner void region is

\[ \Delta T = -3 \cos \theta_e \xi \epsilon R_c H/c = -2 \cos \theta_e \xi \epsilon \eta e, \]

where $\eta = R/(ct) = (3/2) R H/c$. 

the time-scale \( t_c \) by noting that it should be comparable with the time-scale for which the ratio \( \rho_{\text{in}}/\rho_{\text{out}} \) changes appreciably, which is of the order of that for the radius of a shell within the void to change appreciably compared with its radius if the void were not there. Neglecting pressure, the energy density within the void divided by that outside the void is proportional to \( \tilde{t}^2/\tilde{r}^{3\tilde{t}} = \tilde{t}^2/\tilde{r}^{3\tilde{t}} \exp(-3\tilde{t}) \). Then, the time-scale for change of this ratio is \( t_c = [\beta/\rho]^{-1} = t(3\tilde{t}) \), where we have used equation (17), or

\[
\begin{align*}
  t_c &= \frac{2H^{-1}}{9\xi}. \\
  \tilde{t} &= \frac{2H^{-1}}{9\xi}. 
\end{align*}
\]

For linear voids, this time-scale is much longer than the Hubble time from equation (15) since \( \xi = 28/9 < 1 \), while for non-linear voids the time-scale is roughly the Hubble time. (Note that \( t_c \) should not be confused with the time-scale for the density or shell radius to change in time, which is of order a Hubble time.) Using equations (15) and (35), the secondary temperature anisotropy then is of order

\[
\frac{\Delta T}{T} = \delta^6 \cos \theta_i (R_c H/c)^3. 
\]

For linear voids, \( p = 2 \), and for clusters, \( p = 1.5 \) (Nottale 1984, Panek 1992). The integrated Sachs–Wolfe effect is therefore third-order in \( R_c H/c \), as is well known.

## 5.1 Initial conditions and simulation parameters

Here we summarize the initial conditions used by the code, as well as tabulating the initial conditions used for the particular simulations performed here. See V1 for more details. We choose all voids to be compensated initially, with ‘mass–energy’ profile

\[
M(t_i, R) = 5\epsilon^{-2}4\pi\rho_{\text{out}}(t_i)(1 + \tan x)
\]

\[
+ \beta(1 - \tan x) R^3/3, 
\]

where \( x = (R - R_v)/\Delta R_v, R_v \) is the void wall radius, \( \Delta R_v \) is the thickness of the void wall initially, and \( \rho_{\text{out}} \) is the energy density outside the void. In addition, \( \beta \) is a constant, and is roughly \( \rho_{\text{in}}(t_i)/\rho_{\text{out}}(t_i) = 1 - \delta(t_i) \), where \( \rho_{\text{in}} \) is the energy density in the inner void region. Note that the excess mass–energy density in the void wall compensates for that missing from the void, because \( M \) reaches the FRW value outside the void. This profile is a compensated void with a distinct, smooth wall and with \( \rho \) constant in the inner void region as long as \( \Delta R_v \ll R_v \).

In addition, the initial velocity for the positive-energy voids is parametrized by

\[
U(t_i, R) = \tilde{a}(t_i, R) R l^i. 
\]

We consider two types of positive-energy voids here. The first type, called the ‘unperturbed’ case, is \( \tilde{a}(t_i, R) = 2/3 \). Here, each shell within the void and void wall is initially moving outwards at the background Hubble expansion rate. In this case, the velocity within the void is smaller than its asymptotic value. The second type, called the ‘perturbed’ case, is

\[
\tilde{a}(t_i, R) = 2/3 + [-1 + 3\sqrt{1 - 4GM(R)c^2/R^3}]^6. 
\]

In this case, the velocity of each shell within the void is not only greater than the background Hubble expansion rate, but is also larger than the asymptotic expansion rate. We arrive at equation (40) heuristically as follows. We start with \( R \propto \rho^{3/2} \) from equation (8), and assume that the inner void region is nearly homogeneous and isotropic. Then, the scale-factor within the inner region is \( a_i \propto \rho^{3/2} \). Now, we incorrectly assume that \( a(t) \) is constant in time during the photon-crossing time. Then because the acceleration of a shell within the void is \( \ddot{R} \propto -GM(R)c^2/R^3 \) for pressureless voids (equation (10)), we calculate \( \ddot{R}/\dot{R} = a\dot{R} \) and \( \ddot{R}/\dot{R} = a/(1 - \gamma) c^2 \). Note that terms of order \( \alpha \) and \( \tilde{a} \) have been neglected. Solving for \( \ddot{a}(= \dot{a} \alpha \) as is assumed to be constant here), we arrive at equation (40).

Equation (40) gives a value of the initial expansion coefficient within the void which is too large. In the inner void region, the mass is \( GM = H^2 R^3 (1 - \gamma)/2 \) from equation (12). Using this and the fact that \( H^2 = 4/(9X^2) \) in a matter-dominated, flat universe, equation (40) becomes

\[
\tilde{a} = 2/3 + [-1 + 1 + 8\delta^6]^{1/6}. 
\]

In the linear regime \( (\delta \ll 1) \), \( \xi = \tilde{a} - 2/3 = 25/3 \), and in the non-linear regime \( (\delta = 1) \), \( \alpha = 1 \). Comparing this with the correct linear result given by equation (15), we see that the relative expansion coefficient \( \xi \) within the inner void region is initially a factor of 3 too large in the linear regime because of the neglect of crucial time-varying terms. Thus, equation (40) is a useful initial velocity expression, because the inner void region expands more quickly than asymptotically initially.

The third type of initial velocity distribution we consider here creates a ‘zero-energy’ void. Here, we set \( I(t_i) = 1 \). Using equation (8) of V1, we then calculate the initial velocity to be

\[
U(t_i, R) = \sqrt{2GM(R_i)/R}. 
\]

Using the same notation from V1 for all of the ray-tracing simulations in this paper (i.e. voids 1–9), the Courant number is \( C = 0.3 \), the speed of light is \( c = 1 \), Newton’s constant is \( G = 1 \), the adiabatic index for a monatomic gas is \( \gamma = 5/3 \), the initial simulation time is \( t_i \), the specific energy at the outer boundary is \( e_R(t_i) \), the number of grid points is \( j_R \), the initial viscosity is \( Q(t_i) = 0 \) and the initial energy density outside the void is \( 4\pi\rho_{\text{out}}(t_i) = 3c^2/8(\tilde{t}_i)^3 \). Thus the initial Hubble radius is \( H_{\text{out}}(t_i) = 3t_i/2 \). In addition, the wall thickness is \( \Delta R_v(t_i) = R_w(t_i)/7 \), the grid spacing outside the void is \( \Delta R_v(t_i) = 3R_v(t_i)/3 \), the fluid time-step is limited by \( \tilde{t}_i = 0.0025 \), the grid spacing cannot change more quickly than \( \Delta t = 0.025 \), the photon time-steps are limited by \( \tilde{t}_s = 0.1 \) and \( \tilde{t}_s = 0.05 \), and the outer radius is located at \( R_{\text{out}}(t_i) = 3.8R_v(t_i) \).

For each simulation, 11 photons were integrated through the evolving void with \( X_{\text{CF}}(t_{\text{ray}})/R_{\text{CF}}(t_{\text{ray}}) = 0.01, 0.15, 0.3, 0.45, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.5 \), where \( t_{\text{ray}} = 1 \) is the time when ray-tracing begins. They initially start at the same value of \( Z_{\text{CF}}(t_{\text{ray}}) \) behind the void, and move in a direction parallel to the \( Z \) axis; in particular, from V1 (equations A2 and A4),

\[
\begin{align*}
  z(t_{\text{ray}}) &= -c\Phi/\sqrt{1 + (\Gamma \tan \theta)^2}, \\
  w(t_{\text{ray}}) &= -(\tan \theta)z(t_{\text{ray}}) \dot{R}/R. 
\end{align*}
\]

The typical time at which the nearly central photon completely

---

5 In the Swiss cheese model of cluster formation (where a cluster is a compensated lump in an empty hole embedded in a FRW universe), the temperature anisotropy for a central photon is \( \Delta T/T = -2(RH/c)^3 \Delta T \), for a non-linearly evolving clump, where \( \Delta T > 1 \) is the ratio of energy densities of the clump to the background matter, and \( H \) is the background Hubble constant. We obtain this by noting that for a collapsing non-linear cluster, Nottale (1984) obtains \( \Delta T/T = -2c^2GM/R^3 \). Using the fact that \( c^2/GM = 5HRH/c \), we obtain \( \Delta T/T = -1.8(RH/c)^3 \Delta T \).

6 The Z-axis is in a direct line to the observer.
passes out of each void is \( t = 6 \). Table 1 gives a list of the other parameters used for the different simulations.\(^7\) Also given in Table 1 are the shell-crossing times for pressureless, positive-energy voids, labelled by \( t_{SC} \). For times greater than the shell-crossing times, the Tolman–Bondi metric is no longer appropriate because the density becomes infinite. Because the density becomes extremely large near the void wall as the photon exits void 6, accurate ray-tracing values would not have been obtained without artificial viscosity in that simulation. However, in order to obtain results that are as clean as possible, we only used artificial viscosity when absolutely necessary – void 6 is the only ray-tracing simulation to use it. Thus, nearly all of the simulation results are completely independent of the artificial viscosity.

### 5.2 Numerical results

In Section 4, we found that the dominant contributions to the temperature distortion are the Doppler blueshifts entering and leaving the void, and the linear-with-distance expansion redshift crossing the inner void region. Because these effects are proportional to \( RH/c \), they are first-order and large for subhorizon-sized voids.

We can demonstrate that all three terms in equation (21) are first-order numerically in the quasi-linear case. Place a compensated cross the inner void region. Because these effects are proportional to \( RH/c \), they are first-order and large for subhorizon-sized voids.

In Fig. 1, the solid lines depict the temperature distortion functions of five photons as they pass through this evolving void. (Note that the error bars are negligible in this figure.) These photons are different distances from the \( Z_{CF} \)-axis at \( t_{ray} \) = 1, the time ray-tracing begins: \( X_{CF}(t_{ray})/R_{CF}(t_{ray}) = 0.01, 0.45, 0.7, 0.9 \) and 1.0. (Because the void is subhorizon-sized, the distance of closest approach is approximately \( X_{CF}(t_{ray}) \), where \( X_{CF}(t_{ray}) = X_{CF}(t_{ray}) = \sin \theta \cdot R_{CF}(t_{ray}) \)). The numerical data were obtained from simulation 4. Exit parameters for the nearly central photons with \( X_{CF}(t_{ray})/R_{CF}(t_{ray}) = 0.01 \) are given in the first two columns of Table 2. (We define the exit location to be the space–time point in the path of the photon where \( \rho(t')/\rho_{0} \) = 1 is the maximum in the second void wall.) For void 4, then, its radius is about a fifth of a Hubble radius and \( \delta_{e} = 0.403 \) when the nearly central photon exits the void.

Time goes from left to right in Fig. 1, with the largest positive and negative temperature distortions occurring for \( X_{CF}(t_{ray})/R_{CF}(t_{ray}) = 0.01 \). As predicted in Sections 4.1 and 4.2, the energy of a photon is blueshifted upon entering and leaving the void, and is redshifted linearly with distance while crossing the void. In addition, the final temperature anisotropies [i.e. the temperature distortion functions for \( Z_{CF}(t_{ray})/R_{CF}(t_{ray}) > 1.5 \)] are much smaller than the distortions obtained en route. As the distance of closest approach [given approximately by \( X_{CF}(t_{ray})/R_{CF}(t_{ray}) = \sin \theta \cdot R_{CF}(t_{ray}) \)] increases, (i) the maximum positive and negative temperature distortions corre-

### Table 1. Simulation parameters.

<table>
<thead>
<tr>
<th>Number</th>
<th>( R_{V}(t_{i}) )</th>
<th>( \varepsilon_{O}(t_{i}) )</th>
<th>( t_{i} )</th>
<th>( \beta )</th>
<th>( k^{2} )</th>
<th>( \Delta_{o} )</th>
<th>( J_{in} )</th>
<th>( \Gamma )</th>
<th>( t_{SC} )</th>
<th>Void description at ( t = 1 )</th>
</tr>
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<td>.1</td>
<td>.6</td>
<td>1155</td>
<td>Pert perturbed void with pressure</td>
</tr>
</tbody>
</table>

\(^7\) \( R_{V}(t_{i}) \) is the initial void radius, \( \varepsilon_{O}(t_{i}) \) is the specific energy at the outer boundary, \( t_{i} \) is the initial time, \( 3t_{i}/2 \) is the initial Hubble radius, \( \beta \) is approximately \( 1 - \delta(t_{i}) \), \( k^{2} \) is the amplitude of the artificial viscosity, and \( \Delta_{o} \) is defined by equation (12) in V1, and ensures that the number density changes sufficiently slowly from one grid point to the next. In addition, \( J_{in} \) sets the grid spacing inside the void relative to that outside the void, \( R_{1} \) and \( R_{2} \) are the radii at which the grid spacing begins to decrease and increase, respectively, and \( J_{h} \) is the total number of grid points. Moreover, the \( \Gamma \) column denotes the initial velocity. When equal to ‘1’ (a zero-energy void), \( U(t_{i}, R) = \sqrt{2GM(t_{i}, R)/R^{3}} \). Otherwise, for initial velocity distribution \( U(t_{i}, R) = \delta(t_{i}, R)/R(t_{i}) \), perturbed voids (denoted by ‘Pert’) are given by equation (40), and the unperturbed voids (denoted by ‘Unpert’) are given by \( \delta(t_{i}, R) = 2/3 \). In addition, \( t_{SC} \) denotes the shell-crossing times for pressureless, positive-energy voids. Finally, the last column contains a short description of the void at \( t = 1 \), when ray-tracing commences. AE stands for ‘asymptotically evolving’, which is a void which evolves independently of its initial velocity conditions, and CDM stands for ‘cold dark matter’ which is a pressureless void.

**Figure 1.** Temperature distortion functions of photons as they pass through three evolving CDM voids from negative to positive values of \( Z_{CF} \). (\( Z_{CF} = 0 \) is the centre of each void). The five solid lines depict photons passing through void 4, which is a positive-energy, asymmetrically evolving void. The dot–dashed (void 1) and dashed (void 2) lines are for the nearly central photons passing through non-asymmetrically evolving perturbed and unperturbed voids, respectively. Note that the photon passing through void 1 (void 2) has a larger (smaller) temperature distortion upon entering the void than that for the central photon in void 4. This result is because the wall is moving outward more quickly (slowly) than asymptotic, causing a much larger (smaller) blueshift. Qualitatively, however, the temperature distortion functions are all very similar.

spondingly decrease, and (ii) the net amount of redshifting in the inner void region decreases. The former occurs because the component of the velocity of the wall parallel to the line of direction of the photon decreases as the distance of closest approach increases, resulting in smaller Doppler blueshifts (see equation 32). The latter occurs because the photon has less distance to travel between the walls as the distance of closest approach increases (see equation 30).

The third and fourth columns of Table 2 are the numerically obtained maximum and minimum values of the temperature distortion function for the nearly central photons passing through each void. Because the temperature distortion function is a minimum or maximum near the outer edge of the inner void region, column 4 minus column 3 is approximately the numerically obtained ‘FRW’ redshift crossing the inner void region. For the asymptotically evolving CDM voids, we can compare these values with those predicted analytically. Using equations (30) and (15), the temperature distortion obtained crossing the inner void region is

$$\frac{\Delta T(t_1, t_2)}{T} = -\frac{46}{9}\eta_c \cos \theta_c.$$  (43)

For a central photon, $\theta_c = 0$, and from Table 2, $\delta_c = 0.403$ and $c^{-1}R_e(t_c)/H_c = 0.326$, but because the wall thickness is about 20 per cent of the total radius, we estimate $\eta_c = (0.80 \pm 0.04)(0.326) = 0.26 \pm 0.01$. Then using equation (43), we estimate $\Delta T(t_1, t_2)/T = -0.047 \pm 0.002$. This result is very close to the numerically obtained value of $-0.047$, the difference between columns 3 and 4 in Table 2.

6 SIGNATURES OF COLD DARK MATTER VOIDS

As discussed in Section 5.1 and in V1, in our numerical simulations we send 11 parallel propagating photons into an evolving void. Initially, they are located at the same comoving distance $Z_C$ behind the void. Because non-radial photons are deflected away from the symmetry axis (Thompson and Vishniac 1987), however, these photons are defocused upon exiting the void. In addition, time dilation causes photons to cross the void more quickly than if the void were absent. For these reasons, the signature of the void (as measured by a single observer at one instant in time) will not consist of this particular set of photons. The cost of calculating these corrections for each photon and rerunning the simulations with the exact initial conditions, however, is prohibitive. Also, integrating backwards in time cannot be done in the presence of artificial viscosity because the increase in entropy from dissipative heating at shocks is irreversible in time. [Another possible solution not available to this author, due to the lack of disk space, is to integrate the fluid equations forward in time, saving the necessary variables at each time-step. Then, one would integrate the CMBR photons backwards in time, rereading the needed variables at each time-step from disk. This approach has been used successfully by others (Amminos et al. 1991; Tuluie & Laguna 1995; Tuluie, Laguna & Amminos 1995).] Fortunately, both the deflection angle and the amount of time dilation are small, second-order effects for subhorizon-sized voids (Thompson and Vishniac 1987), and lead to a higher order correction in the signature of a void. Therefore, following the example of Dyer (1976), we can neglect these effects for the voids considered here, and determine the signature of a void to third-order via this particular set of photons.

6.1 Dependence of the temperature distortion function on initial conditions

The net temperature anisotropy for void 4 (solid lines in Fig. 1) is plotted as diamonds with error bars in Fig. 2. Note that its magnitude at the centre is $\Delta T/T = -3.5 \times 10^{-4}$, which is a little more than 10 times larger than the CMBR on the 1/3 degree scale, the angular size this void would have if it were close to the LSS. This follows from the fact that because the Universe at redshift $z_e \gg 1$ was approximately flat so that $\Omega_0 = 1$ (eg. Kolb & Turner 1990), if the void had relative radius $R_eH/c$ at emission time, the total angular size of the void in degrees in the small-angle limit (see V3, equation A.7) is

$$\Theta = \frac{180\sqrt{\Omega_0 R_eH_c}}{\pi\sqrt{z_e}}.$$  (44)

Using $\Omega_0 = 1$, $z_e = 1100$ and $R_eH/c = 0.22$, we find that $\Theta = 0.38$ degrees. We will show in a moment that the temperature anisotropy for quasi-linear, asymptotically evolving voids is proportional to $\delta_c^{-3/2} \eta_c$. Therefore, if we decrease the underdensity of this void to $\delta_c \leq 0.11$ or decrease its size to $R_eH/c \leq 0.084\sqrt{\Omega_0}$, its temperature anisotropy will be reduced to less than $2 \times 10^{-5}$.

In Fig. 2, photons passing through void 4 with $X_{CF}(t_{ray})/R_e(t_{ray}) \leq 0.65$ appear cool with respect to the microwave background, while photons with $0.65 \leq X_{CF}(t_{ray})/R_e(t_{ray}) \leq 1.1$ appear warm; as with non-linear voids, then this void appears as a cold spot surrounded by a warm ring. However, this is not always the case. Voids 1 (squares) and 2 (triangles), which have the same exit parameters as 4, are also plotted in Fig. 2. Note that their error

Table 2. Exit parameters for the photon nearest to the origin.

<table>
<thead>
<tr>
<th>Number</th>
<th>$\eta_c = 1.5R_e/(cH^{-1})$</th>
<th>$\delta_c$</th>
<th>max($\Delta T/T$)</th>
<th>min($\Delta T/T$)</th>
</tr>
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<td>.400</td>
<td>.015</td>
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</table>
The signatures of voids – II  293

bars are smaller than the symbols. Although their initial radii and underdensities were chosen so that these three voids would have nearly identical exit parameters, their signatures are in fact quite different; void 1 has a warm, roughly constant signature, while void 2 has a cold, tapered signature. These qualitative and quantitative differences can be traced to the fact that voids 1 and 2 are not asymptotically evolving yet. Here, ‘asymptotically evolving’ denotes a void evolving long before ray-tracing begins (i.e. long before the photons comprising the signature cross the void), as is the case with void 4. We call it asymptotically evolving because the relative expansion coefficient of the inner regions, $\xi$, depends only on $\delta$ after a finite amount of time, regardless of the initial conditions, as will be seen in the next subsection.

The best example of the dependence of the signature on the ‘maturity’ of a void is found with voids 2 and 4. Although they both started with initially unperturbed velocities and with positive energy $[\delta(t_i) = 2/3$ with equation (39)], the signature of void 2 is a factor of 3 colder than that of void 4, and does not contain the outer warm ring. This is because void 4 started evolving long before ray-tracing began (i.e. $t_i = 0.1$, whereas void 2 started evolving just as ray-tracing began (i.e. $t_i = 1.0$). Consider the dashed line in Fig. 1, which is the nearly central photon of void 2. Because the initially unperturbed velocity of the void is lower than the asymptotic velocity, the blueshift entering the first void wall, the redshift crossing the inner void region, and the blueshift leaving the second void wall are all relatively smaller than those of void 4. However, because the void shell velocities are accelerating very quickly in order to catch up to their asymptotic velocities, it is thought that the extra amount of redshifting that occurs in the inner void region is enough to overcompensate for the increase in blue-shift in the second void wall, resulting in the redshift of all photons.

On the other hand, void 1 was started at the same time ray-tracing began with initially ‘perturbed’ velocity and with positive energy given by equation (40). This perturbed profile, described in Section 5.1, was chosen so that the void shells move faster than asymptotic initially. The dash–dotted line in Fig. 1 shows the temperature distortion of the nearly central ray as it crosses this void. Because the initial velocity is larger than the ‘asymptotic’ amount, the sequences of blueshifting, redshifting then blueshifting are all larger than asymptotic. Note that the velocity of the void accelerates very slowly initially, so that $\alpha$ is very nearly constant as the photon crosses the void. In addition, the blueshift obtained leaving the second void wall is only slightly larger than that entering the first void wall. The net effect is that all photons are blueshifted.

We have therefore found that the temperature distortion functions of photons passing through voids with different initial conditions (but with similar exit parameters) are fairly similar qualitatively and quantitatively. However, the small differences that do occur in the temperature distortion functions lead to large changes in the resultant signature, due to the large Doppler and gravitational potential cancellations which occur. Therefore, the final signature is very dependent on the precise history of a void. This suggests that it might be difficult to identify particular features in the microwave background as voids, because there is no general fingerprint signature for voids in front of the LSS, unless the void is asymptotically evolving, as will be shown in the next two subsections.

The fact that the signature of a spherically symmetric void depends sensitively on its velocity distribution is actually not very surprising. For the Swiss cheese model of cluster formation, Dyer (1976) found that the central region can appear hot or cold depending on the ratio of the velocities of the central lump to the hole. Indeed, if this ratio is less than $k(1 + \ln k)$, where $k < 1$ is the ratio of the lump radius to the hole radius, then the central region is cold. Because the outer region is always warm, a ring structure results in this case. (It is interesting to note that the Swiss cheese ring structure can be very similar to the ring structure from an asymptotically evolving void.) This leads one to wonder if the signature of a colliding, asymmetrically accreting void in the real universe is given by a simple formulae such as equation (45), or is very complicated and condition-dependent, as Fig. 2 suggests. Although an isolated, evolving void becomes more spherical in time (Fujimoto 1983; Centrella & Mellott 1983; Icke 1984; Bertshinger 1985; Blaes, Goldreich & Villumsen 1990), a non-isolated void becomes more spherical in the centre only, while the shape of its boundary is influenced to a large extent by the structures surrounding it (van de Weygaert & van Kampen 1993; Dubinski et al. 1993; van Kampen 1994). In addition, mini voids (van Kampen 1994) or mini pancakes and filaments (Sahni, Sathyaprakash & Shandarin 1994) may develop within the void interiors, although these substructures diminish with time (Sahni et al. 1994). These effects will most likely affect the signature of a quasi-linear void through changes in the velocity profiles.

### 6.2 Expansion coefficients of asymptotically evolving quasi-linear voids

To support the claim that unperturbed and perturbed voids eventually reach the same asymptotically evolving state, we ran many simulations of voids with initially perturbed and unperturbed velocity profiles and for different initial underdensities. For the unperturbed voids, we chose $\delta(t_i) = 1 - \beta = 10^{-3}, 10^{-2}, 0.05, 0.1$ and 0.25. Each simulation was run to a late evolutionary stage ($\delta > 0.965$) in order to capture as much of the non-linearity as possible. In order to obtain the same relative expansion coefficients $\xi(t)$ as a function of the underdensity $\delta(t)$ at late times, the initial underdensities for the perturbed voids needed to be somewhat smaller. For the perturbed voids then, we chose $\delta(t_i) = 1 - \beta = 10^{-3}/3, 10^{-2}/3, 0.017, 0.038$ and 0.11.

We show the results of these simulations in Fig. 3. In Fig. 3(a), we plot $9\xi(t/28)$ versus $t$, with the solid (dashed) lines showing the initially unperturbed (perturbed) void simulations. In addition, a dotted line guides the eye for the linear solution $\xi(t/28\delta)$ = 1. It is clear that each unperturbed–perturbed pair of voids evolves asymptotically after $t \approx 10t_i$ (i.e. independent of their initial conditions), even if the evolution at that time is quasi-linear. In addition, note that the slopes of the lines at late times (i.e. in the quasi-linear phase) are roughly equal and are greater than 1. In Fig. 3(b), we plot $\xi$ versus $28\delta$ in order to more closely follow the relation between $\xi$ and $\delta$ in the quasi-linear regime. Again, the dotted line shows the linear solution $\xi = 28\delta$. It is seen that once the asymptotically evolving state has been reached, $\xi$ is only a function of $\delta$ which departs weakly from linearity when $\delta \approx 0.45$. In addition, note that the relation $\xi = 28\delta$ is approximately true for the entire quasi-linear regime. Therefore, we conclude that for pressureless, positive-energy voids, after roughly a factor of 10 in time (and somewhat larger for quasi-linear voids), the expansion rate in the inner void region of a quasi-linear void is determined solely by the underdensity of that void for all practical purposes.

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For all voids shown in Figs 3(a)–(b), $R_0 = 100$, $\Delta R(t_i) = R(t_i)/7$, $e_0(t_i) = 0$, $t_i = 1$, $k^2 = 8$, $C = 3$, $c_1 = 1$, $G = 1$, $\gamma = 5/3$, $\bar{f} = 0.0025$, $R_{\rho}(t_i) = 2.3 \times R(t_i)$ and $Q(t_i) = 0$ and $\Delta\rho \approx e^{-\gamma\delta}(10/3)e^{-\gamma\delta}$. Thus the initial Hubble radius is $cH_{\rho}(t_i) = 3t_i/2$. In addition, the grid is equally spaced with $\Delta R(t_i) = \Delta R/8$ and $\Delta t = 130$. 

with $p = 2$], we can conclude that the temperature anisotropy increases as the square of the underdensity (or slightly larger) for quasi-linear voids with $\delta_0 \approx 0.6$. We note that all of these voids appear as cold spots (for $X_{CF}(t_{Ray})/R_{V}(t_{Ray}) \leq 0.63$) surrounded by hot rings. We also plot the signature of the non-linear void (with zero wall thickness) as the solid line in this figure (equation 2). Note in Fig. 4 that the net temperature distortion for photons with $X_{CF}(t_{Ray})/R_{V}(t_{Ray}) = 1.0$ in the quasi-linear case is not zero as it is in the non-linear case because of the finite wall thickness of the quasi-linear voids. It is clear that the qualitative behaviour of the signatures is very similar, even though the quasi-linear voids have randomly chosen wall shapes (given by the mass function) and thicknesses (20 per cent was chosen as a reasonable thickness). In addition, it is seen that the quasi-linear results are asymptoting to the non-linear result through the $\delta_0^{2.2}$ dependence. Thus, the secondary temperature anisotropy for quasi-linear, compensated, pressureless, subhorizon-sized, asymptotically evolving voids can be written empirically as

$$\Delta T = \frac{2}{5} \delta_0^{2.2} \left( \frac{R_e c}{H_0} \right)^3 \cos \theta_e \left( 1 - \frac{5}{3} \cos^2 \theta_e \right).$$ (45)

Because the wall shape and thickness are generic and were randomly chosen, we believe that equation (45) is roughly independent of the wall thickness of a void (as long as it is not thicker than 50 per cent of the radius of the void), wall shape (i.e. initial mass profile, as long as the void is compensated and $\rho_0$ is constant), and initial velocity profile (as long as the void has positive energy).

We can use this new expression to calculate the signature of a very large void observed in our Universe, the Boötes void (Kirshner et al. 1981, 1987). This void is $150 h^{-1}$Mpc from us and has radius $R = 30 h^{-1}$Mpc. Therefore, its total angular size is $2 \pi (30/150) = 23'$. Because its underdensity is thought to be $\delta = 0.7 - 0.8$ in observable mass, there will be a decrease of 50 per cent in the maximum $|\Delta T/T|$ detected, so that the maximum temperature anisotropy is $\Delta T/T = -(4/15) \delta_0^{2.2} \left( H_0 R_e c \right)^3 = -(1.2 - 1.6) \times 10^{-7}$. This distortion is too small to be detected with present CMBR experiments. (However, note that Fullana et al. 1996 found that Boötes-type voids can have signatures an order of magnitude larger in an open, $\Omega_0 = 0.2$ universe for redshifts between 1 and 10). Also, if cold dark matter is the main constituent of matter in the Universe and galaxies are biased, $\delta$ will be even smaller, since galaxies would then clump more readily than CDM. This would lead to an even smaller temperature anisotropy, as was noted by Panek (1992). Martínez-González & Sanz (1990) suggested that a nearby supervoid with radius $100 h^{-1}$Mpc might be detectable, since the maximum temperature anisotropy then is approximately $10^{-5}$. However, if it is quasi-linear with $\delta_0 < 0.7$ and is asymptotically evolving, its signature would still be too small to be detected at present. In addition, Tuluie et al. (1995) suggest that the Rees–Sciama signal levels off to $\Delta T/T \sim 10^{-6}$ for voids larger than $60 h^{-1}$Mpc generated from an initial Harrison–Zel’dovich fluctuation spectrum. Baccigalupi et al. (1997) found that a primordial, bubbly distribution of voids can be compatible with the CMBR and the galaxy power spectrum for voids with radii between $30 h^{-1}$Mpc and $130 h^{-1}$Mpc – however, this study includes the contribution of voids on the LSS, first-order temperature distortion effects. It is also important to note that a supervoid with radius $100 h^{-1}$Mpc was predicted to be very rare; Blumenthal et al. (1992) found that typical voids should have diameters $D < 80 h^{-1}$Mpc, and

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The scaling $\delta_0^{2.2}$ was chosen because it gives the ‘best’ fit visually for this numerical data.

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Note that the values of $\theta_e$ and $\phi_e$ differ insignificantly for subhorizon-sized voids.
that there should be at most one void with diameter $D > 130 \ h^{-1} \mathrm{Mpc}$ in the entire Hubble volume for $\Omega_0 = 1$.

6.4 Averaged signatures of non-linear voids

Because the temperature anisotropies for asymptotically evolving quasi-linear voids are a cold core surrounded by a hot ring, the question arises as to whether it is possible for these voids to be detected in experiments with window functions larger than their size on the microwave sky. For a non-linear void, it turns out that the signature is averaged out to fourth-order, leading to the possibility that this is also true for asymptotically evolving quasi-linear voids.

The average temperature anisotropy is

$$\frac{\Delta T}{T} = \left( \pi R_0^2 \right)^{-1} \int_0^{R_0} \frac{\Delta T}{t} \frac{1}{\tau} \frac{X_t}{X_e} dX_e,$$

(46)

where $X_t$ is the perpendicular distance to the Z-axis when the photon leaves the void in a direction parallel to the Z-axis. In addition, the subscript ‘0’ denotes the exit value of the function where the central leaves the void in a direction parallel to the central axis when the photon leaves the void in a direction parallel to the Z-axis. Thus, the temperature anisotropy cancels to fourth-order because of the particular hot/cold nature of the ring structure.

Because our numerical results are only valid to third-order, as noted above, we are not able to calculate the average signals of the voids in our simulations. However, because equation (45) is approximately true for quasi-linear voids, and the exit values $\delta_e$ will vary to the next higher order ($\delta_e$ decreases slightly as $X_e$ increases), it is perhaps likely that the average signal will be fourth-order for quasi-linear, asymptotically evolving voids also.

7 SIGNATURES OF VOIDS WITH PRESSURE AND ZERO-ENERGY VOIDS

7.1 Dependence of temperature distortion functions on the initial conditions

We also present results for unusual voids – voids that do not have the standard blueshifts for photons entering and leaving the void wall and redshifts crossing the inner void region. In Fig. 5, we show the temperature distortion functions of nearly central photons with $X_{\text{C}}(t_{\text{in}})/R_0(t_{\text{in}}) = 0.01$ for voids 7–9. These voids have exit parameters of $\delta_e = 0.40$ and $\eta_e = 0.33$. The solid line shows the temperature distortion functions obtained crossing void 7. Because there is no extra energy within the void [i.e. $\Gamma(t_e, R) = 1$], we call it a zero-energy void – the void eventually flattens out and disappears because there is nothing to fuel its development. This development is why the initial underdensity is quite large, i.e. $\delta(t_i) = 0.86$ (see Table 1), even though the exit value is $\delta_e = 0.40$. From the figure, we see that the nearly central photon is redshifted upon entering and exiting the void wall, and is blueshifted upon crossing the inner void region. This result is because the void wall and inner region are expanding more slowly than the rate outside the void. Therefore, as a zero-energy void flattens out and disappears, its temperature distortion behaves oppositely to that of positive-energy voids.

We have also included a small amount of pressure initially for simulations 8 and 9 in order to see how the temperature distortion functions and signatures change because of the presence of a ‘hot’ component. For the two simulations shown here, the pressure is 1 per cent of the density initially. Note that the non-pressure analogues to simulations 8 and 9 are simulations 2 and 1, respectively. Because the presence of pressure, the void wall ‘explodes’ somewhat as the wall fluid tries to escape the high-pressure region – fluid moves outward and inward from the wall region, as can be seen in fig. 2 of V1.

Void 8 has initially unperturbed velocity. Its temperature distortion is the dashed line in Fig. 5. It is seen that the temperature distortion crossing the void is a complicated pattern because of the wall explosion. In particular, the nearly central photon is blueshifted upon entering the wall, is redshifted crossing the void wall (because of the explosion), is blueshifted upon entering the inner void region because of the inward-travelling shock, and is redshifted linearly with distance crossing the small, inner void region. A similar behavior occurs in reverse when this photon exits the void. For reference, the temperature distortion functions of all 11 photons are shown in fig. 6 of V1.

In contrast, the initial velocity of void 9 is perturbed. Its temperature distortion is shown in Fig. 5 by the dash-dotted line. Note that in contrast to void 8 (unpert, $p \neq 0$), the temperature distortion only changes sign here when the photon passes near the centre of the void. However, the general features of the temperature distortion function are similar to that of the unperturbed void with pressure (void 8).
7.2 Dependence of signatures on initial conditions

In Fig. 6, the signature of a zero-energy void (void 7) is plotted with diamonds; $\Delta T/T$ is obtained by multiplying the value of the diamonds by a factor of 5. Not only does this void appear exclusively as a cold spot on the microwave sky, but its signature is qualitatively very similar to that of the signature of an unper- turbed, non-asymptotically evolving CDM void (i.e. triangles in Fig. 2). Quantitatively however, its signature is almost an order of magnitude larger. In fact, the signature of the zero-energy void is much larger than the signature of the other simulated voids with the same exit parameters, e.g. voids 1, 2 and 4; in particular, its signature is 21 times larger than the signature of the asymptotically evolving CDM void (i.e. diamonds in Fig. 2).

We also plot the signature of the unperturbed void with pressure (void 8) with triangles in Fig. 6. As with the asymptotically evolving CDM void (diamonds in Fig. 2), this void projects as a cold spot surrounded by a warm ring. Thus, the two signatures are similar, even though they result from quite different dynamical processes and resultant temperature distortion functions. Note that because the pressure redshifts away faster than the density (Vadas 1993), the pressure will eventually become negligible so this void would become an asymptotically evolving void with a potentially lumpy interior.

Finally, we plot the signature of the perturbed void with pressure (void 9) as squares in Fig. 6. This signature is quite unusual; the net temperature anisotropy is very small near the centre of this void, and is surrounded by a fairly hot ring.

We thus conclude that the signature of a quasi-linear void in front of the LSS depends on its maturity, its initial energy or velocity profile, and its amount of pressure. If the void is not evolving asymptotically, the velocity and pressure profiles dramatically change the resulting temperature distortion and signature – the signature is not given by a simple, global expression. If it is pressureless and is evolving asymptotically, the signature is approximately given by the non-linear result with an additional suppression factor of $\delta^2$. 

8 DISCUSSION

We showed that a positive-energy, cold dark matter void will grow asymptotically (i.e. evolve independently of its initial velocity profile) after roughly 10 times the initial time. The relative expansion coefficient of the inner void region is $\xi = (2/3)(H^{-1}U_{\delta}/R_{\delta} - 1)$, where the subscript 'in' represents a shell value in the interior of the void, and where the velocity and radius of a shell are $U$ and $R$, respectively. In particular, we found that after a void starts asymptotically evolving, its relative-expansion coefficient can be approximately represented by the expression $\xi = 2\delta^2$ for asymptotically evolving voids in the quasi-linear regime, where $\delta$ is the underdensity of the void.

In addition, we find that the approximate, empirical signature of an asymptotically evolving quasi-linear, pressureless, positive-energy void is the well-known nonlinear result (Thompson and Vishniac 1987) multiplied by a function of the underdensity, $\delta^{2.2}$, i.e. $\Delta T/T = (2/5)\delta^{2.2}(RH/c^3)\cos\theta[1 - (5/3)\cos^2\theta]$, making the thick-walled void appear as a cold spot surrounded by a hot ring on the cosmic microwave background radiation (CMBR). Because this formula was obtained for voids with generic wall shape and random thickness (approximately 20 per cent), we believe that it holds roughly independent of the wall thickness and shape as long as the thickness is less than half or so of the radius of the void. Therefore, the net secondary anisotropy for a quasi-linear void is smaller than that for a non-linear void, making a large quasi-linear void harder to detect. We also argued that if an asymptotically evolving quasi-linear void is smaller than the angular window size of an experiment, its signature is likely averaged out to fourth-order, making it even harder to detect.

For the case of asymptotically evolving voids, we plot the net temperature distortion of central photons as a function of the underdensity of the void, $\delta_\epsilon$, and radius, $R_\epsilon(\epsilon)$, as the photons exit the voids. We use equation (45), and plot solid contour lines for $\Delta T(t_{\text{exit}}, t)/T = 5 \times 10^{-5}, 2 \times 10^{-5}, 1 \times 10^{-5}$ and $2 \times 10^{-6}$ in Fig. 7. Note that although a nearby fully non-linear void with radius $100 \, h^{-1}\text{Mpc}$ has a signal which is within CMBR limits, a nearby, quasi-linear void with $\delta_\epsilon = 0.4$ can have a radius twice that in order to have the same acceptable signal.


Figure 6. Secondary temperature anisotropies for the voids from Fig. 5. The zero-energy void is shown as diamonds, the initially unperturbed void with pressure is shown as triangles, and the initially perturbed void with pressure is shown as squares. The signature of the zero-energy void is obtained by multiplying the value of the diamonds by a factor of 5.

Figure 7. Signatures of asymptotically evolving voids. We plot the net temperature distortion $\Delta T/T = 5 \times 10^{-5}, 2 \times 10^{-5}, 1 \times 10^{-5}$ and $2 \times 10^{-6}$ for the central photons passing through asymptotically evolving voids with walls approximately 20 per cent of the radius of the void. Note that although a nearby, fully non-linear void with radius $100 \, h^{-1}\text{Mpc}$ has a signal which is within CMBR limits, a nearby, quasi-linear void with $\delta_\epsilon = 0.4$ can have a radius twice that in order to have the same acceptable signal.
the largest allowed supervoids would be too small (eg. Blumenthal et al. 1992).

Otherwise, because the signature of a void is very sensitive to its exact initial conditions (because of cancellations of the much larger Doppler and Sachs–Wolfe effects), the signature of a quasi-linear void can be hot or cold and with or without a ring structure. For example, perturbed and unperturbed CDM voids with the same exit parameters (i.e. the same exit radii and underdensities) can appear on the CMBR as cold and hot spots, respectively. Also, a zero-energy void with the same exit parameters as an asymptotically evolving CDM void can have a signature 20 times larger. In addition, an unperturbed void with a small amount of pressure can have a signature twice as strong but is qualitatively similar to that of a CDM void with the same exit parameters (i.e. is a cold spot surrounded by a hot ring), even though its temperature distortion functions do not follow the usual blueshift, redshift, blueshift pattern as the photons cross this void. (The temperature distortion functions do not follow the usual blueshift, redshift, blueshift that of a CDM void with the same exit parameters (i.e. is a cold spot surrounded by a hot ring), even though its temperature distortion functions do not follow the usual blueshift, redshift, blueshift pattern as the photons cross this void. (The temperature distortion function of a photon, \(\Delta T(t_{\text{ray}}, t)/T\), is its energy relative to that of a photon moving outside the void). Instead, the temperature distortion functions of this void follow a complicated, intricate pattern because of the wall-explosion of the void and resultant inwards-travelling shock. Thus the appearance of a cold spot surrounded by a hot ring does not necessarily imply the presence of an asymptotically evolving CDM void. However, unusual signatures can also occur for quasi-linear voids with pressure; a perturbed void with a small amount of pressure has a hot, outer ring, but has a comparatively negligible signature inside the ring. We therefore conclude that it is difficult, if not impossible, to generalize the functional form for the signatures of voids that are not asymptotically evolving (i.e. that depend on initial velocity conditions). This holds for voids with and without pressure, and for voids with positive energy or zero energy.

We also examined in detail the temperature distortion functions of photons passing through evolving quasi-linear voids. These functions are essential for understanding what signatures these voids will have when lying on the LSS (detailed in V3). We found that the temperature distortion functions for positive-energy, pressureless voids are qualitatively the same and quantitatively similar, regardless of whether or not the voids are asymptotically evolving CDM void. However, unusual signatures can also occur for quasi-linear voids with pressure; a perturbed void with a small amount of pressure has a hot, outer ring, but has a comparatively negligible signature inside the ring. We therefore conclude that it is difficult, if not impossible, to generalize the functional form for the signatures of voids that are not asymptotically evolving (i.e. that depend on initial velocity conditions). This holds for voids with and without pressure, and for voids with positive energy or zero energy.

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REFERENCES

Bertschinger E., 1985, ApJS, 58, 1
Kolb E. W., Turner M. S., 1990, The Early Universe. Addison-Wesley, Redwood City, CA
Rees M. J., Sciama D. W., 1968, Nat, 217, 511
Vadas S. L., 1994a, in Franco J., Lizano S., Aguilar L., Daltabuit E., eds,

van Kampen E., 1994, PhD thesis, Leiden Observatory

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