Strong gravitational lensing by spiral galaxies

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ABSTRACT
We investigate gravitational lensing using a realistic model of disc galaxies. Most of the mass is contained in a large spherical isothermal dark matter halo, but the potential is modified significantly in the core by a gravitationally dominant exponential disc. The method used is adapted from a very general multilens ray-tracing technique developed by Möller. We investigate the effects of the disc-to-halo mass ratio, the disc scalelength, the disc inclination to the line of sight and the lens redshift on two strong-lensing cross-sections: the cross-section for multiple imaging and the cross-section for large magnifications, in excess of a factor of 10. We find that the multiple-imaging cross-section can be enhanced significantly by an almost edge-on Milky Way disc compared with a singular isothermal sphere (SIS) in individual cases; however, when averaged over all disc inclinations, the cross-section is only increased by about 50 per cent. These results are consistent with other recent work. The presence of a disc, however, increases the inclination-averaged high-magnification cross-section by an order of magnitude compared with a SIS. This result has important implications for magnification bias in future lens surveys, particularly those in the submillimetre waveband, where dust extinction in the lensing galaxy has no effect on the brightness of the images.

Key words: methods: numerical – galaxies: fundamental parameters – galaxies: spiral – cosmology: theory – gravitational lensing.

1 INTRODUCTION
There is ever-increasing interest in gravitational lensing by individual galaxies. Deep surveys in the radio waveband, for example the CLASS survey (Myers et al. 1995), have identified many galaxy-lensed systems. In many cases, models of the lensing mass distribution as spherical or elliptical isothermal haloes have been reasonably successful in reproducing the observed image positions and magnification ratios (for example, Rhee 1991). An increasing number of observed lens systems, however, have image configurations that are not consistent with this simple picture (for example Jaunsen & Hjorth 1997). These have recently led to the consideration of lens models that include a disc component within a dark halo (Bartelmann & Loeb 1998; Keeton & Kochanek 1998; Koopmans, de Bruin & Jackson 1998; Maller, Flores & Primack 1997; Wang & Turner 1997).

As a disc with mass comparable to that of the halo would be gravitationally unstable, the mass contained in the disc is not expected to exceed about 10 per cent of the halo mass. Even so, within a few kpc of the core of such a galaxy the projected surface mass density on the lens plane is dominated by the disc component and this enhancement should lead to an increase in the strong-lensing cross-section. The effect is expected to be particularly significant for a nearly edge-on disc.

Wang & Turner (1997) calculated the cross-sections for multiple imaging for lenses consisting of a singular isothermal sphere (SIS) and a thin uniform disc, and found a large increase at large disc inclinations; however, the model of a uniform galaxy disc is not very realistic. Maller et al. (1997) used an exponential disc mass distribution to fit the two-image lens system B 1600+434 (Jackson et al. 1995) with reasonable success. Keeton & Kochanek (1998) presented a model that combined three different elliptical mass components to produce an analytical approximation to an exponential disc potential with a flat rotation curve at large distances. This model involves many parameters, and the potential differs significantly from that of an exponential profile near the core of the galaxy. While we were preparing this paper, Bartelmann & Loeb (1998) discussed the statistical significance of lensing by spiral discs using a technique similar to ours to derive strong-lensing optical depths, including the effect of dust extinction along the lines of sight to different images. We consider these effects in detail in a subsequent paper (Blain, Möller & Maller, in preparation).

This work was motivated by interest in the effects of disc lensing on the population of lenses in the millimetre/submillimetre waveband, where, in some circumstances, magnification biases are expected to be very large and up to 5 to 10 per cent of sources detected in a survey could be lensed (Blain 1996, 1997, 1998). An enhanced strong-lensing cross-section caused by discs could increase this effect even further (Blain et al., in preparation), especially because dust extinction is unimportant in the submillimetre waveband.

We adapted an existing general ray-tracing method, which was
first developed to study weak lensing (Möller 1997), to solve the problem. The method and lens model are outlined in Section 2. We then investigate the effects of the surface density and scalelength of the disc on the cross-section for the formation of multiple images, which is essential for determining whether a source in a survey will be classified as a lens, in Section 3, and the cross-section for large magnifications, which determines the importance of magnification bias (Borgeest, V. Linde & Refsdal 1991), in Section 4. In Section 5 we compare our results with those of previous authors and briefly discuss their implications for lensing statistics and magnification biases. We assume that the density parameter \( \Omega_0 = 1 \) and Hubble’s constant \( H_0 = 60 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Angular diameter distances are calculated assuming a smooth distribution of matter.

2 METHOD

Even for a uniform disc, the analytical derivation of the form of caustics and critical lines is very involved. In order to study lensing by exponential discs, we adapted an existing general multi-lens-plane ray-tracing routine (Möller 1997), in which a regular grid of \( R_0 \times R_0 \) images is mapped on to the source plane by propagating light rays through a set of \( N \) lens planes. In the present computations, \( N = 1 \). The simulated source plane is finite, and so care was taken that no rays were deflected beyond its boundaries. In principle, extended sources can be treated, but all the results are derived here for point sources; the brightest regions observed in far-infrared-luminous galaxies at low redshifts are typically less than a few hundred pc across (Solomon et al. 1997), corresponding to an angular size of order \( 10^{-2} \) arcsec at redshift \( z = 2 \), and so this approximation is unlikely to affect our applications significantly. To generate maps of the total magnification on the source plane, we find all the images of a regular grid of \( R_0 \times R_0 \) point sources using the ‘grid search method’ described by Schneider, Ehlers & Falco (1992). Once all the images of each source are located, we find its total magnification by summing all the individual magnifications.

The accuracy of this procedure depends on the resolution of the grid on the source and image planes. If the image resolution is too coarse, then the grid search method may miss images near caustics, and so the total magnification is underestimated. This problem can be partially overcome by making \( R_0 \gg R_s \). There is a trade-off between the accuracy of magnifications near caustics, the coarseness of the resulting map and the computing time. The cross-sections for different magnifications are determined by adding together the areas of all pixels in the source plane that contain sources with that magnification. Hence, a small value of \( R_0 \) leads to an overestimate of the high-magnification cross-section, as the average magnification over the whole pixel area is not sampled adequately. To verify the accuracy of our procedure and to find optimum values for \( R_0 \) and \( R_s \), we tested its predictions against analytic results derived for an SIS lens. For resolution ratios \( R_0/R_s > 2 \), the results agreed with the analytical predictions to within about 5 per cent. In all computations, \( R_0 = 1000 \) and \( R_s = 400 \) were used.

The deflection angle \( \alpha \) is calculated by adding together the deflections resulting from the two different lens components, a SIS halo and a disc,

\[
\alpha = \alpha_{\text{SIS}} + \alpha_{\text{disc}}. \tag{1}
\]

The deflection resulting from the halo is

\[
\alpha_{\text{SIS}} = \frac{4GM_0}{rR_{\text{LIS}}^2} r. \tag{2}
\]

where \( M_0 \) is the mass contained within radius \( R_0 \) and \( r \) is the impact parameter. The projection of an inclined thin disc on to the lens plane produces an elliptical surface mass density profile. We evaluate \( \alpha_{\text{disc}} \) numerically using a useful elliptical surface potential formalism developed by Schramm (1990).

We consider a single model lens at redshift \( z_l \). The lens consists of a SIS dark matter halo with velocity dispersion \( \sigma_s \) and a disc with a surface mass profile \( \Sigma(r) = \Sigma_0 \exp(-r/R_s) \), in which \( r_s \) is the radial scalelength. For clarity, no bulge component is included, but our model can incorporate additional mass components without difficulty. The effect of including a bulge is discussed briefly in Section 4. In Fig. 1, four realistic flat rotation curves predicted by the mass models used in this paper are shown. Values of \( \sigma_s = 150 \text{ km s}^{-1} \), \( \Sigma_0 = 10^9 M_\odot \text{ pc}^{-2} \), and \( r_s = 3.2 \text{ kpc} \) produce a reasonable fit to the rotation curve of the Milky Way (Binney & Tremaine 1987).

3 MULTIPLE-IMAGING CROSS-SECTION

For an SIS lens, the cross-section for multiple imaging is given by

\[
\sigma_{\text{m}}^{\text{SIS}} = \left[ 8 \frac{\sigma_s^2}{c^2 D_{\text{LS}}} \right]^2 \pi \tag{3}
\]

(Schneider et al. 1992); \( D_{\text{LS}} \) is the lens–source angular diameter distance. If the lens redshift \( z_l = 0.2 \), the source redshift \( z_s = 1.5 \) and \( \sigma_s = 150 \text{ km s}^{-1} \), then \( \sigma_{\text{SIS}} = 15.8 \text{ kpc}^{-2} \). Including a disc component increases \( \sigma_{\text{m}} \) if the inclination angle of the disc \( \theta \) exceeds a critical angle \( \theta_{\text{c}} \) (equation 10 in Maller et al. 1997), \( \theta = 0 \) and \( 90^\circ \) corresponding to a face-on and edge-on disc respectively. Note that we add the disc to a halo with a fixed velocity dispersion and consequently, our pure SIS model does not take into account the disc mass. We use this approach because we aim to demonstrate how a disc can increase the lensing cross-section without requiring a more massive halo. For a Milky Way type galaxy, the disc-to-halo mass ratio within a 10-kpc radius is about 1:2, and including the disc mass in a pure SIS model would increase the velocity dispersion of the halo above the observed value.

The variation of \( \sigma_{\text{m}} \) as a function of inclination is shown in Fig. 2 for various disc parameters. In all cases, \( \sigma_{\text{m}} \) increases monotonically with increasing inclination, and the effect is considerably more significant for more massive discs. Although the parameters are chosen so that the upper curves in Figs 2(a) and (b) are derived for discs of the same total mass, the predicted cross-sections differ.
about 40–50 per cent of the total multiple-image cross-section being provided by each. Only about 10 per cent of multiply imaged sources will be split into four images. The cross-section for a source being lensed into five images is zero, within the numerical error. Note that these results do not take magnification bias into account.

4 HIGH-MAGNIFICATION CROSS-SECTION

For an SIS lens, the cross-section for magnifications $\mu > A$ is given by

$$
\sigma_\mu^{\text{SIS}}(A) = \begin{cases} 
4\pi m^2 A^{-2}, & \text{for } A > 2, \\
\sigma_\mu^{\text{SIS}}(A - 1)^{-2}, & \text{for } 1 < A \leq 2,
\end{cases}
$$

as can easily be obtained from the magnification curve (Schneider et al. 1992). For large magnifications, however, we predict that $\sigma_\mu$ is much larger for a SIS plus disc lens, compared with an SIS alone. Magnification maps derived at inclinations of 60, 70 and 85° for a lens similar to the Milky Way are presented in Fig. 3, and clearly show the distinctive shape of the caustic, with an elliptical component resulting from the halo and a diamond-shaped component, the ‘astroid’, resulting from the disc. For large inclinations, the size of the astroid increases significantly, leading to the increase of the multiple-imaging cross-section $\sigma_m$ discussed above.

The cross-section ratio $\sigma_m / \sigma_\mu^{\text{SIS}}$ for magnifications greater than $A$ for a galaxy lens similar to the Milky Way are presented in Fig. 4. The cross-sections are derived by adding up the area of all pixels on the source plane with magnifications greater than $A$. The calculated ratios become less smooth at large magnifications because the number of high-magnification pixels on the source plane is relatively small; only about 20 pixels in the source plane have magnifications larger than $A = 80$. The magnitude of the fluctuations in the ratios can be used as an estimate of the uncertainty in the results of these computations. A large increase in the cross-section ratio is predicted for inclinations that exceed 60°, at magnifications greater than a certain threshold. This threshold magnification is a strong function of inclination, and is reduced as the inclination increases. The cross-section ratios shown in Figs 4 and 5 also have a characteristic peak beyond this threshold magnification. Hence, although the largest cross-section ratios are predicted for almost edge-on discs at magnifications greater than 50, at magnifications of about 10 a disc at $\theta = 70°$ is expected to produce a larger cross-section than a disc at $90°$. Hence, nearly edge-on discs are not necessarily more effective in producing large magnifications, compared with discs at smaller inclinations. This is because points interior to the astroid caustic lie closer to caustic lines and experience larger average magnifications when the caustic is smaller, as it is at moderate inclinations. This effect is clearly demonstrated in Fig. 3: although the area of the astroid caustic is larger in the lower panels, the mean magnification in the enclosed region is smaller.

The inclination-averaged high-magnification cross-section $\bar{\sigma}_m$ shown in Fig. 4 was derived by evaluating $\sigma_m$ at 5° intervals from 0 to 90° and approximating the integral (equation 4) by a sum. The result is very significant: $\bar{\sigma}_m$ is greatly increased by a randomly oriented galaxy similar to the Milky Way, compared with the value for an SIS lens, and $\bar{\sigma}_m / \sigma_\mu^{\text{SIS}} \approx 10$ at magnifications greater than about 50. This increase could have a very significant effect on magnification bias in galaxy surveys.

The dependence of $\sigma_m$ on the scalelength and surface density of the disc is shown in Fig. 5. As in the case of the multiple-imaging cross-section, the more massive discs produce a more significant effect. Again, if the central surface mass density is increased, then the relative significance of discs with smaller inclinations increases.
Adding a central bulge does not have a significant effect on the high-magnification cross-section; for a spherical bulge with an $r^{1/4}$ law density profile, extending out to one kpc from the centre and containing a total of 10 per cent of the disc mass, we find that the high-magnification cross-section is increased by no more than 50 per cent. The projected surface mass density profile for an elliptical halo is similar to that for an inclined disc. Thus, we expect that a non-spherical halo which is flattened along the disc is likely to enhance the lensing effect of an inclined disc. However, this effect is not expected to be significant.

The high-magnification cross-section does not depend strongly on the redshift of the lens. This suggests that the high-magnification cross-sections in both the disc plus SIS halo model and the pure SIS halo model scale with the angular diameter distance in a similar way. Thus, we do not expect $\partial_j/\partial S_{\text{SIS}}$ to be strongly dependent on the value of the density parameter $\Omega_0$ or the comological constant $\lambda$.

5 DISCUSSION

We have studied the effects of lensing by an exponential disc inside a SIS on two important parameters: the multiple-imaging cross-section $\sigma_m$, and the high-magnification cross-section $\sigma_j(A)$. The
lens model we use has a density profile similar to that of the Milky Way. Both $\sigma_m$ and $\sigma_m(A)$ are sensitive to the lens parameters, in particular to the inclination angle and disc-to-halo mass ratio.

The results indicate that discs, and especially almost edge-on discs, have a significant effect on the image configurations produced by individual lenses, in agreement with the results of previous work by Maller et al. (1997) and Wang & Turner (1997), but that the statistical effect on $\sigma_m$ averaged over all disc inclinations is unlikely to be very important. A more significant result is that including an exponential disc at an inclination greater than about $60^\circ$ increases $\sigma_m$ at $A > 10$ by an order of magnitude compared with an SIS lens, and the inclination-averaged cross-section $\bar{\sigma}_m$ is increased by a similar factor. Hence, discs could have a large effect on the predicted abundances of strongly magnified galaxy lenses. This implies that in surveys for which dust extinction is unimportant, such as those in the radio and submillimetre bands.

![Figure 3. continued.](image1)

![Figure 4. High-magnification cross-sections for a galaxy with an SIS with $\sigma_v = 150\, \text{km s}^{-1}$ and a Milky Way disc. To indicate the uncertainties involved, the curves are not smoothed.](image2)

![Figure 5. High-magnification cross-sections for galaxy models with different disc parameters from those assumed in Fig. 4. In (a) and (b) the mass of the disc is 2.5 and 3 times larger, respectively, than for the model in Fig. 4.](image3)
wavebands, the number of detected strongly lensed sources will increase (Blain 1996, 1997), and that they will tend to be preferentially associated with edge-on spiral galaxies (Blain et al., in preparation). The preliminary results of the CLASS survey suggest that this is indeed the case (Browne et al. 1997). A recent preprint by Bartelmann & Loeb (1998) discussed the statistical significance of lensing by spiral galaxies using a technique similar to ours and included the effect of dust extinction in the lensing galaxy. They did not discuss the effects of different scalelengths or disc masses on the high-magnification cross-sections, and so it is difficult to compare our results in detail, but they appear to be in broad agreement, bearing in mind that they assume $\Omega_0 = 0.3$ and $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ and that their disc model deviates from our exponential profile.

6 CONCLUSIONS

(i) The configuration of images in an individual lens can be modified significantly by the presence of a disc component at a large inclination, but when averaged over all inclinations the multiple-imaging cross-section of the lens is increased by only about 50 per cent.

(ii) The large-magnification cross-section is increased considerably by including a disc component in the lens model. For a Milky Way disc the effect is very significant at inclinations of $60^\circ$, and so the inclination-averaged cross-section is also expected to increase significantly, by an order of magnitude at magnifications of the order of 10 compared with the value for a singular isothermal sphere lens.

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