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A NONLINEAR ANALYSIS OF THE WHIRLING MOTIONS OF SLENDER BEAMS UNDER VARIOUS RESONANT EXCITATIONS

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EXTENDED ABSTRACT

The response of a slender, elastic, cantilevered beam to a simple harmonic excitation is investigated. The nonlinear equations governing the motion of the beam are essentially those derived earlier by Crespo da Silva and Glynn (1978). In the current study, the governing equations are extended to include static deflection. These equations not only include nonlinear curvature, nonlinear inertia, inextensionality and static deflection, but also include torsional displacement. In all cases, the torsional displacement is written in terms of lateral displacements and eliminated from the governing equations. Previous derivations of equations of motion contain only the linear and cubic terms without consideration of the static displacement produced by the weight of the beam. As a result of this static deflection, there are quadratic terms in the governing equations, which introduce the possibility of superharmonic and subharmonic resonances of order two.

The partial-differential equations of motion are converted into a system of coupled ordinary-differential equations in time by the application of Galerkin's procedure. Approximate solutions of the temporal equations are determined by the method of multiple scales. The analysis reveals that only the in-plane modes directly excited by a primary or secondary resonance and the out-of-plane modes excited by an internal resonance are involved in the first approximation of the response. The amplitudes of the other modes decay. Under some circumstances, there is no steady-state (constant amplitude, constant phase) response. Instead, the amplitude and phase are slowly modulated. Under some circumstances, the modulations are harmonic and produce discrete side bands around the fundamental frequency. For other circumstances, the modulations are chaotic.

Both stable and unstable whirling motions are found in every resonance when the principal moments of inertia of the cross-section are approximately equal. The longer the beam is, the more prominent the whirling motion becomes. The accuracy of some of the approximate solutions is verified by numerical integration. The analysis reveals some interesting possibilities: For example, in a subharmonic resonance of order two, it is possible for the out of plane motion to have a frequency that is exactly one half that of the in-plane motion, which has a frequency equal to that of the excitation. It is also possible for the frequency of the in-plane motion to be equal to that of the out-of-plane motion, which is the one half frequency of the excitation.

References

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