THE DETERMINATION OF THICKNESSES OF THE CONTINENTAL LAYERS FROM THE TRAVEL TIMES OF SEISMIC WAVES.

A. W. Lee, M.Sc., D.I.C.

(Communicated by Dr. F. J. W. Whipple)

(Received 1931 November 27)

In connection with an investigation of the seismological data for the North Sea Earthquake of 1927 January 24 I have had occasion to consider the relation between the travel times of the waves and the thicknesses of the layers through which they are transmitted. Dr. Whipple, with whom I discussed the problem, has suggested that the connection would be most
clearly brought out if one wrote down the detailed expressions for travel of the waves. With data for the waves through the granitic and intermediate layers as well as the normal \( P \) and \( S \), we should have sufficient equations to determine the thickness of the layers, the depth of focus, and the time of origin. Application of this method is very straightforward, and it is easily adapted for comparing the conditions with the focus in different layers.

The time of arrival of the waves at a distance \( \Delta \) from the epicentre is usually expressed in the form \( \mathcal{C} + C_1 \Delta + C_2 \Delta^2 \), where \( \mathcal{C} \) is the apparent time of starting, and \( C_1 \) and \( C_2 \) are constants; for epicentral distances up to about \( 8^\circ \) the term in \( \Delta^2 \) is negligible and the equations are linear. The difference between \( \mathcal{C} \), the apparent time of starting of the waves, and \( T_0 \), the time of the shock at the focus, is termed the "delay of starting." The "delay of starting" for any wave may be expressed in the form

\[
\sum \frac{\text{(height interval)}(\cot \text{inclination})}{\text{characteristic velocity}},
\]

where the characteristic velocity is that of the wave during the horizontal part of its path. The waves may be referred to as characteristically compressional or characteristically distortional according to the nature of the motion during the horizontal part of the path. The velocities are known with tolerable accuracy, and the inclinations can be computed, since by the laws of reflection and refraction the sines of the angles of incidence are proportional to the velocities. The cotangent of inclination divided by characteristic velocity will be termed the "delay depth" coefficient. The summation

\[
\Sigma \text{(height interval)}(\text{delay depth coefficient})
\]

for any wave can be performed provided we know the path it follows in traversing the layers; on the other hand, if the path is not known, we can assume approximate thicknesses and determine which of the possible paths would be most consistent with the observed "delay of starting."

The occurrence of three layers over the ultrabasic is generally accepted, and recent studies by R. Stoneley and by the author indicate that there may be a fourth between the basaltic and the ultrabasic; the new layer will be termed the lower intermediate to distinguish it from the basaltic or upper intermediate. A convenient notation for the layers is 0 (ultrabasic), 1 (lower intermediate), 2 (upper intermediate), 3 (granitic), and 4 (sedimentary). Let \( u_0 \ldots u_4 \) and \( v_0 \ldots v_4 \) be the velocities of compressional and distortional waves in the layers. Inclinations of the seismic rays to the vertical are denoted by \( \alpha \), \( \beta \), \( \gamma \) with two suffixes; the first suffix indicates the characteristic layer for the wave, the second the layer in which the specified inclination occurs. \( \alpha \) angles refer to compressional waves, \( \beta \) to distortional, and \( \gamma \) to characteristically compressional waves when they are

* H. Jeffreys The Earth, 2nd edition, p. 97, equation (5).
transmitted as distortional waves through the layer indicated by the second suffix. The velocities, inclinations, and delay-depth coefficients are given in Table I. The lower half of the table (β angles) refers to characteristically distortional and the upper to characteristically compressional waves. The history of a derived compressional wave is followed by using the right side of the table (γ angles) while the wave is distortional, and the left side (α angles) when it is compressional. The velocities are those given by

### Table I

**Inclinations of Seismic Rays and Corresponding Delay-Depth Coefficients**

<table>
<thead>
<tr>
<th>Characteristic Compressional Waves</th>
<th>Compressional Wave in Layer</th>
<th>Distortional Wave in Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Characteristically Compressional Waves</td>
<td>Characteristic Velocity (km./sec.)</td>
<td>0</td>
</tr>
<tr>
<td><strong>P</strong></td>
<td>7.80</td>
<td>90</td>
</tr>
<tr>
<td><strong>P_q</strong></td>
<td>7.00</td>
<td>...</td>
</tr>
<tr>
<td><em><em>P</em>, P</em>*</td>
<td>6.30</td>
<td>...</td>
</tr>
<tr>
<td><em><em>P</em>, P_q</em>*</td>
<td>5.50</td>
<td>...</td>
</tr>
<tr>
<td>Characteristically Distortional Waves</td>
<td>Characteristic Velocity (km./sec.)</td>
<td>0</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>4.35</td>
<td>90</td>
</tr>
<tr>
<td><strong>S_q</strong></td>
<td>3.95</td>
<td>...</td>
</tr>
<tr>
<td><em><em>S</em>, S</em>*</td>
<td>3.65</td>
<td>...</td>
</tr>
<tr>
<td><em><em>S</em>, S_q</em>*</td>
<td>3.35</td>
<td>...</td>
</tr>
</tbody>
</table>

E. Tillotson,* except those for **P_q** and **S_q** waves which have been obtained by the author; they are expressed to the nearest 0.05 km./sec. Velocities of 5.00 km./sec. and 3.30 km./sec. have been taken for the waves in the sedimentary layer. These velocities appear to be appropriate for the older sedimentary rocks; for newer rocks the velocity of transmission is much lower.

The analysis will be most easily followed if applied to a particular earthquake. The obvious choice, since we need an earthquake for which the travel of a number of phases is accurately known seems to be the shock near Imotski, Yugoslavia, on 1923 March 15, which has been studied by Tillotson (loc. cit.). Velocities and apparent times of starting are given for eight waves, **P, P*, P_q, P*, S, S*, S_q** and **S_q**. As the **P*, P_q, S, S*, S_q** and **S_q** waves may not be true body waves through the sedimentary layer, we shall only consider six of the waves given by Tillotson. The “delays of starting” of **P, P*, P_q, S** and **S**, relative to **S_q**, are 7.5, 5, 4, 3, 9 and 1.5 seconds respectively. Stoneley has determined the “delay of starting” of **P_q** from observations at three stations as 10 seconds.

The choice of layer in which to place the focus must be governed by the circumstances for the particular earthquake being investigated. The records for 1923 March 15 show that the $S_\nu$ waves were propagated, and consequently the focus must have been in the granitic or sedimentary layers. Both of these possibilities have been examined, but no consistent solution has been found for a focus in the sedimentary layer. For a focus in the granite there is good agreement between computed and observed “delays of starting,” and only this solution will be given.

Taking the paths of the waves as shown by H. Jeffreys,* the equations for “delay of starting” are:

\[
\begin{align*}
\tau_P - T_0 &= [z \cot \gamma_{02} + 2h_1 \cot \alpha_{01} + 2h_2 \cot \alpha_{02} + 2h_3 \cot \alpha_{03} + h_4 \cot \alpha_{04}] / u_0. \\
\tau_{P0} - T_0 &= [z \cot \gamma_{13} + 2h_2 \cot \alpha_{12} + 2h_3 \cot \alpha_{13} + h_4 \cot \alpha_{14}] / u_1. \\
\tau_{P*} - T_0 &= [z \cot \gamma_{23} + 2h_3 \cot \alpha_{23} + h_4 \cot \alpha_{24}] / u_2. \\
\tau_{P*} - T_0 &= [z \cot \gamma_{34} + h_4 \cot \alpha_{34}] / u_3. \\
\tau_S - T_0 &= [(2h_3 - z) \cot \beta_{03} + 2h_1 \cot \beta_{01} + 2h_2 \cot \beta_{02} + h_4 \cot \beta_{04}] / v_0. \\
\tau_{S*} - T_0 &= [(2h_3 - z) \cot \beta_{23} + h_4 \cot \beta_{24}] / v_2. \\
\tau_{S*} - T_0 &= [h_4 \cot \beta_{34}] / v_3.
\end{align*}
\]

On substitution of the values from Table I and the observed “delays of starting” we obtain the equations:

For $P$,

\[
\begin{align*}
0.26z + 0.12h_1 + 0.18h_2 + 0.26h_3 + 0.15h_4 &= t + 7.5, \\
0.26z + 0.14h_2 + 0.22h_3 + 0.14h_4 &= t + 10, \\
0.25z + 0.18h_3 + 0.12h_4 &= t + 5, \\
0.23z + 0.08h_4 &= t + 3, \\
-0.19z + 0.20h_1 + 0.30h_2 + 0.38h_3 + 0.20h_4 &= t + 9, \\
-0.12z + 0.24h_3 + 0.13h_4 &= t + 1.5, \\
0.05h_4 &= t.
\end{align*}
\]

$t$ denoting the “delay of starting” for the $S_\nu$ wave.

We have therefore seven equations connecting six variables, and if the time measurements were more accurate should obtain a complete solution, leaving one equation to spare for checking. With the “delays of starting” only measured to half-seconds, and knowing that $h_4$ can only be a few kilometres, $t$ is less than the error of observation and must be neglected. Equations (3) and (6) are used for determination of $z$ and $h_3$ simultaneously on the assumptions that $t = 0$ and that $h_4 = 0$ or $h_4 = 1$. The solutions are:

\[
\begin{align*}
t &= 0, & h_4 &= 0, & z &= 11.4, & h_3 &= 12.0. \\
t &= 0, & h_4 &= 1, & z &= 11.3, & h_3 &= 11.4.
\end{align*}
\]

For larger values of $h_4$, $z > h_3$, consequently $h_4 = 1$, $z = 11.3$ and $h_3 = 11.5$ may be taken as the approximate solution.

Inconsistencies between equations (1), (2) and (5) are apparent on substitution of these values for $z$, $h_3$ and $h_4$. The equation for $P_0$ being based only on observations at three stations must be less accurate than those for the other waves, and an error in the “delay of starting” of one or two seconds is

* The Earth, 2nd edition, p. 100, fig. 6.
not unlikely. Defects in equations (1) and (5) are less obvious, but there is reason to think that the error lies in the equation for $P$; the possible source of error will be discussed later. Equation (5) gives:

$$20h_1 + 30h_2 = 6.6.$$  

(8)

The limiting conditions are $h_1 = 0$, $h_2 = 22$ and $h_1 = 33$, $h_2 = 0$, and the total thickness ($h_1 + h_2$) is between $22$ km. and $33$ km.

The coefficients of $h_1$ and $h_2$ in equations (1) and (5) are in the same ratio, so the delay in $P$ during its passage through these layers must be $4.0$ seconds.

The "delays of starting" which would be expected from waves travelling through layers of the given thicknesses are compared with the observational delays in Table II.

### Table II

<table>
<thead>
<tr>
<th>Wave</th>
<th>P</th>
<th>P0</th>
<th>P*</th>
<th>P0</th>
<th>S</th>
<th>S*</th>
<th>S0</th>
</tr>
</thead>
<tbody>
<tr>
<td>From observations (O)</td>
<td>7.5</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>9</td>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>From hypothesis (H)</td>
<td>10</td>
<td>5.6-8.7*</td>
<td>5.0</td>
<td>2.7</td>
<td>9.0</td>
<td>1.5</td>
<td>0.0</td>
</tr>
<tr>
<td>O-H</td>
<td>-2.5</td>
<td>...</td>
<td>0.0</td>
<td>+0.3</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

In five of the seven waves there is excellent agreement between the observational and hypothetical values. The discrepancy in the case of $P$ is too large to be attributed to casual errors of timing, and cannot be explained by assuming that the ray follows another path. Three alternative paths for generation of compressional waves from a focus in the granitic layer are shown in fig. 1. The track assumed for the analysis of the "delay of starting" is $sP$, which gives an hypothetical delay of 10 seconds. The other tracks are for a compressional wave from the focus (P, with delay of 5.8 seconds), and for a distortional wave travelling downwards to the ultrabasic and generating a compressional wave immediately below the $\sigma 0$ boundary. Such waves may be termed $s'P$, and would have a delay of $9.5$ seconds. Since the $P*$ observations are explicable on the assumption that the compressional wave is generated by reflection from the top of the granite, and incomprehensible for alternative paths, $sP$ is more likely to occur than $s'P$.

The results would be consistent with a small compressional wave, $P$, being followed by a larger movement, $sP$, for the genuine $P$ would be recognised on some of the seismograms and might be overlooked in others. The observational "delay of starting" (7.5 seconds) would then be intermediate between the values for $P$ and $sP$, which is actually the case. Suspicion of errors in identification would therefore rest upon movements tabulated as $iP$ rather than upon those given as $eP$. The occurrence of a $P$ wave followed by an $sP$ is confirmed from the Eskdalemuir record, part of which is reproduced in fig. 2. The movement starts slowly at about 5 h 44 m 27 s, * These values correspond to the limiting values of $h_2$. 

* These values correspond to the limiting values of $h_2$. 

G 2
and there is a sharper impulse at $5^h 44^m 31^s$; thus the difference between the times of arrival agrees with the computed difference between the "delays of starting" of $P$ and $sP$ waves. The earlier movement was tabulated as $P$ at Eskdalemuir, but the latter was chosen by Mr. Tillotson.

**GENERATION OF COMPRESSIONAL WAVES FROM A FOCUS IN THE GRANITIC LAYER.**

![Diagram of wave propagation through different layers](image)

**EARLY PHASES FROM IMOTSKI EARTHQUAKE OF MARCH 15, 1923**

*(Recorded at Eskdalemuir)*

**N-S. Component.**

**Z. Component.**

$5^h 46^m$  $5^h 45^m$  $5^h 44^m$

A third phase, arriving at Eskdalemuir at $5^h 44^m 36\frac{1}{2}^s$, is more conspicuous on the horizontal than on the vertical component. The origin of this movement is doubtful, but it is interesting to consider the possibility.
that it may be a distortional wave derived from the $sP$ in the ultrabasic layer. For convenience it is noted on fig. 2 as $sPs$; the paths of $P$, $sP$ and $sPs$ waves on arrival are shown in fig. 3. The "delay of starting" for an $sPs$ wave relative to that of the wave from which it is derived is

$$1.16h_1 + 1.15h_2 + 1.13h_3 + 1.12h_4.$$ 

This must be equal to the difference between the times of arrival of the phases, and on substitution of the values previously determined for $h_3$ and $h_4$:

$$1.16h_1 + 1.15h_2 = 3.9.$$ \hspace{1cm} (9)

The solution of equations (8) and (9) is $h_1 = 10$ km., $h_2 = 15$ km., and the result looks reasonable. Thus the approximate solution is:

- Thickness of sedimentary layer, 1 km.
- Thickness of granitic layer, 11.5 km.
- Thickness of upper intermediate layer, 15 km.*
- Thickness of lower intermediate layer, 10 km.*
- Depth of focus, 11.3 km. below the top of the granite.

These determinations have been incorporated in figs. 1 and 3, but in the former case the focus is shown above the actual position to give a clearer representation of the paths followed by the waves. The values do not differ seriously from those obtained by Tillotson (sedimentary, 4 km.; granitic, 13 km.; intermediate, 25.3 km.; and depth of focus, 12 km.). The method he employed was essentially an approximation to the present one, but it is assumed that the delay depth coefficients for the compressional or distortional waves in a given layer are equal; reference to Table I shows that such is by no means the case.

* These values are very doubtful.
Data for two further examples are available from a paper by Jeffreys "On Two British Earthquakes."* The earthquakes occurred in Jersey on 1926 July 30, and in Herefordshire on 1926 August 15. The "delays of starting" relative to $S_*$ are:

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$P*$</th>
<th>$P_*$</th>
<th>$S$</th>
<th>$S*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jersey</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>4 sec.</td>
</tr>
<tr>
<td>Herefordshire</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>9</td>
<td>3</td>
</tr>
</tbody>
</table>

There is good evidence that in each case the focus was in the granitic layer, consequently the equations for "delay of starting" are the same as those for the earthquake near Imotski; since no $P_Q$ waves were observed terms in $h_1$ do not appear. If $h_4$ and $t$ be negligible the equations become:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Jersey</td>
<td>$P_262 + 18h_3 + 26h_4 = 9$</td>
<td>$P_252 + 18h_3 = 5$</td>
<td>$P_232 = 3$</td>
<td>$S_292 + 30h_2 + 38h_3 = 8$</td>
<td>$S_272 + 12h_2 + 24h_3 = 4$</td>
</tr>
<tr>
<td>Herefordshire</td>
<td>$P_252 + 18h_3 = 5$</td>
<td>$P_232 = 3$</td>
<td>$S_272 + 30h_2 + 38h_3 = 8$</td>
<td>$S_272 + 12h_2 + 24h_3 = 4$</td>
<td></td>
</tr>
</tbody>
</table>

The solution I think most reliable is that from the equations for $P$, $P^*$ and $S$. There are two reasons for this selection: (1) the use of the $P_*$ equation risks concentrating a comparatively large error in $z$; and (2) the $S^*$ equation is probably the least reliable since the $S^*$ wave is the most difficult to observe. The solutions obtained are:

|        | $z = 10$ km. | $h_2 = 15$ km. | $h_3 = 14$ km. |
| Jersey | $h_2 = 15$ km. | $h_3 = 14$ km. |
| Herefordshire | $h_2 = 16$ km. | $h_3 = 14$ km. |

The "delays of starting" computed from these solutions are compared with those given by the observations in Table III.

<table>
<thead>
<tr>
<th></th>
<th>$P$</th>
<th>$P^*$</th>
<th>$P_*$</th>
<th>$S$</th>
<th>$S^*$</th>
<th>$S_*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jersey</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>From observations (O)</td>
<td>9</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>From hypothesis (H)</td>
<td>8'9</td>
<td>5'0</td>
<td>2'3</td>
<td>7'9</td>
<td>2'2</td>
<td>0</td>
</tr>
<tr>
<td>$O-H$</td>
<td>+0'1</td>
<td>0'0</td>
<td>+0'7</td>
<td>+0'1</td>
<td>+1'8</td>
<td>0</td>
</tr>
<tr>
<td>Herefordshire</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>9</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>From observations (O)</td>
<td>8'1</td>
<td>4'0</td>
<td>1'4</td>
<td>9'0</td>
<td>2'7</td>
<td>0</td>
</tr>
<tr>
<td>From hypothesis (H)</td>
<td>8'1</td>
<td>4'0</td>
<td>1'4</td>
<td>9'0</td>
<td>2'7</td>
<td>0</td>
</tr>
<tr>
<td>$O-H$</td>
<td>-0'1</td>
<td>0'0</td>
<td>+0'6</td>
<td>0'0</td>
<td>+0'3</td>
<td>0</td>
</tr>
</tbody>
</table>

The largest discrepancy between the "delays of starting" is in the case of the $S^*$ wave for the Jersey earthquake. A similar difference was obtained by Jeffreys, who attributes it to the uncertainties in the measurements of $S^*$. The thicknesses obtained by Jeffreys were 10 km. of granite and

21 km. of basalt; the depths of focus were in the neighbourhood of 10 km. and 8 km.

The solutions obtained for the Jersey and Hereford earthquakes both indicate that the thickness of intermediate rock is only slightly greater than that of granite. The total thickness (30 km.) is reasonably close to that obtained for the Imotski shock if \( h_1 \) and \( h_4 \) are neglected (34 km.), but in the latter case the thickness of granite is about half that of intermediate rock. The difference between the relative thicknesses of the granitic and lower layers in the two localities may be due to folding and distortion of the layers in the more mountainous region around south-eastern Europe.

These detailed examples have shown that the method is sound, but it can only be applied in the study of earthquakes for which accurate values of the relative "delays of starting" are available. Accuracy to 0.1 second in the "delays of starting" should be attainable if data were only included from seismographs with an open time-scale (at least 15 mm./min.), from which the times of arrival of the pulses can be determined to 0.5 second.

**Summary**

A novel method is given for analysis of the connection between time of origin of an earthquake, the depth of focus, apparent times of starting of the seismic waves, and the thicknesses of the layers through which they travel.

Application of the method to the available data for an earthquake near Imotski, Yugoslavia, on 1923 March 15, shows that the focus was near the bottom of the granitic layer. The approximate thicknesses of the layers are determined as 1 km. of sedimentary material, 11.5 km. of granite, and between 22 and 33 km. of intermediate rock.

The travel times of the waves from the shocks in Jersey on 1926 July 30, and in Herefordshire on 1926 August 15, indicate that the thicknesses of the granitic and basaltic layers were 14 km. and 15 km., and that the foci were 10 km. and 6 km. respectively below the top of the granite.

**THE NORTH SEA EARTHQUAKE OF 1927 JANUARY 24.**

A. W. Lee, M.Sc., D.I.C.

(Communicated by Dr. F. J. W. Whipple)

(Received 1931 November 27)

1. *Introduction.*—A small earthquake which occurred early on 1927 January 24 was felt throughout southern Norway, in Denmark and over eastern Britain from the Shetlands to Norfolk. The disturbance was recorded at practically every seismological observatory in Europe, but the identification of phases on the seismograms was difficult owing to large microseismic disturbance; over forty stations have contributed readings to the *International Seismological Summary.* Shortly after the earthquake its