

Snowmelt Estimated from Energy Budget Studies

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The snowmelt at a point is estimated through a simplified energy budget. From the energy equation a degree-day method is derived. The rate of snowmelt is found as a constant multiplied by the temperature excess over an equilibrium temperature which depends on solar radiation.

The snowmelt events in Kiruna (3 years) and Luleå (4 years) are analysed utilizing energy balance computations and the degree-day method. First observed and calculated day when the ground was free of snow is for all seven years in good agreement. The rate of snowmelt calculated by the proposed degree-day method is almost identical to the rate found from energy balance computations. Snowmelt rates estimated by a simple degree-day method using a constant equilibrium temperature are too fast in the beginning and too slow at the end of the melting season.

Introduction

Snowmelt and sequential streamflow forms one of the most important phases of the hydrologic cycle in northern regions. Still hydrologists cannot accurately forecast the peak flow due to snowmelt or even the snowmelt at a point.

The snowmelt process depends on the net heat exchange between the snowpack and its environment. The only strictly correct way of computing the amount of melt is through an energy budget. However, since the various sources and processes influencing heat transfer to and from a snowpack are very difficult even to estimate, the rate of snowmelt is usually described by a *degree-day method*. The amount of snowmelt of one day should then be determined as a constant multiplied by the temperature surplus above a certain critical air temperature. Since air temperature is only one of several meteorological parameters influencing snowmelt, neither the constant nor the critical temperature can be truly constant, but must depend on solar radiation, cloudiness, wind speed, air humidity and effect of forest cover (canopy density).

Energy Balance of the Snow Pack

Summing all energy fluxes gives the heat balance of the snow pack

$$R + C + G + r \equiv L \cdot E + \Delta M + f \cdot Q \quad (1)$$

where

C = convective transfer of sensible heat from air (cal/cm², day)

G = conduction of heat from underlying ground (cal/cm², day)

r = heat transfer from rain (cal/cm², day)

E = evaporation rate from snow surface (or with negative sign condensation) (cm/day)

L = latent heat of sublimation (677 cal/cm³)

ΔM = change in heat stored in snow pack (cal/cm², day)

Q = snowmelt (cm/day)

f = latent heat of melting (80 cal/cm³)

R = radiation balance as

$$R = R_{Li} - R_{Lo} + (1 - \alpha) R_S \quad (2)$$

where

R_{Li} = incoming longwave (terrestrial) radiation which, since the longwave albedo of a snow surface is very small, is equal to the absorbed longwave radiation.

R_{Lo} = outgoing longwave radiation

α = albedo of the snow surface for shortwave radiation

R_S = incoming shortwave radiation.

The most simple and accurate measurement of net insolation (shortwave *solar radiation*) is by solarimeters or pyrhelimeters. Usually, however, only the incoming solar radiation is measured, so the albedo for the snow surface must be estimated. The albedo varies widely. When the snow is fresh it is commonly taken as 80 per cent, but as the snow becomes granular, the albedo can decrease to less than 40 per cent. The albedo decreases faster during the melting season than during the accumulation season. According to U.S. Army Corps of Engineers (1956) the time variation in the albedo of a snow surface is as shown in Fig. 1.

Measurements from Canada indicate that the albedo drops very fast down to 0.2 during the very last period of the snowmelt event (verbal communication Dr. D. Gray, Univ. of Saskatchewan, Canada).

The longwave radiation can be measured but the instruments are inaccurate and thus has not been used very much. Since the atmosphere does not radiate as a blackbody, Stefan's law can be used only after the emissivity of the atmosphere is known. Measurements made by U.S. Corps of Engineers (1956) indicate that for clear sky conditions the atmospheric emissivity, ϵ_a , for snow is

$$\epsilon_a = 0.757$$

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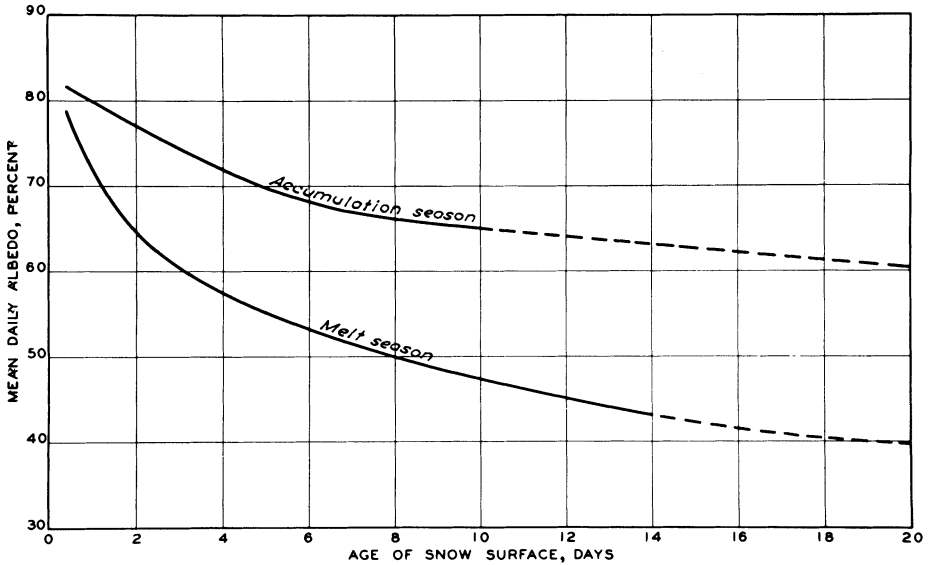


Fig. 1. Variation in albedo with time (Snow Hydrology, 1956).

if the emissivity is defined so that

$$R_{Li} = \epsilon_a \sigma (T+273) \tag{3}$$

where

σ = Stefan's constant = $1.17 \cdot 10^{-7}$ cal/cm², day, (°C)⁴

T = ground-level air temperature (°C)

Measurements for example by Raphael (1962) show that the emissivity of snow is almost identical to the emissivity of water when the vapour pressure is around 6 mb. It should therefore be possible to use Raphael's empirical relationship between the atmospheric emissivity of water and cloud cover which, when the vapour pressure is 6 mb, reads

$$\epsilon_a = 0.76 + 0.12 N \tag{4}$$

where N is part of the sky that is covered with clouds.

Raphael's relation is found from measurements over a water surface and thus effects of a forest cover are not included. By assuming that a cover of trees radiates as a blackbody at the ambient temperature the effective atmospheric emissivity, ϵ , is found to be

$$\epsilon = (1-F) \epsilon_a + F \tag{5}$$

where F = the canopy density.

Snow is an almost perfect blackbody with respect to longwave radiation and thus longwave radiation emitted by a snow surface can be estimated from Stefan's law. Assuming that the temperature of the snow surface is 0°C during the part of the day, t , when the air temperature is above freezing and equal to the air temperature during the part of the day, $1-t$, when the air temperature is below freezing, the outgoing longwave radiation is

$$R_{Lo} = \sigma \{ t \cdot 273^4 + (1-t) (273 + T_-)^4 \} \quad (6)$$

where T_- = mean temperature of the part of the day when the temperature is below freezing ($^{\circ}\text{C}$).

The *evaporation* from a snow cover is very small. Water balance computations for the Lapträsket representative basin in Sweden (Persson, 1975) show that evaporation over the entire winter period is negligible. Lemmelä and Kuusisto (1974) found that in southern Finland evaporation from snow did only take place during March and April and was about 8 mm/winter. Although the evaporation is negligible in water balance computations it may be of significant importance in energy balance computations since the heat needed to evaporate 8 mm water from the snow corresponds to about 6 cm water equivalents of snow melt.

A common method to estimate evaporation when the water supply is unlimited is the aerodynamic method:

$$E_P = k_e (1+bW) (e_o - e) \quad (7)$$

where

E_P = potential evaporation rate (cm/day)

e_o = saturation vapour pressure at the snow surface temperature (mb)

e = air vapour pressure (mb)

W = wind speed at 2 meter level (m/s)

and k_e and b are constants, which are reported by Kullus (1973) to be

$$k_e = 0.018 \text{ cm/mb, day} \quad b = 0.54 \text{ s/m} \quad (8)$$

Evaporation can take place at this rate only if the supply of energy is sufficient. The evaporation cannot exceed

$$E = \frac{1}{L} (R + C + G + r - \Delta M) \quad (9)$$

Obviously the vapour pressure of the snow surface may be in excess of the vapour pressure of the air so that E becomes negative and condensation takes place.

On March 14, 18, 19, 21 and April 18, 21, 22 1975 this writer carried out intense measurements of the wind-, temperature- and humidity profiles over a snow cover as described by Bengtsson (1975). For determining turbulent transfer the exchange-coefficient hypothesis was utilized. The influence of thermal stratification was taken into account by adding a linear term to the ordinary used logarithmic profile. The

humidity flux was determined to be directed downward (condensation) on March 14 and 21 and on April 22 but evaporation was never found to take place. The condensation rate during the three days mentioned was about 0.5 mm/day. The mean condensation rate for the seven days when measurements were carried out was thus 0.07 mm/day or 0.2 mm/month.

Eq. (7) used with the coefficients proposed in Eq. (8) shows that condensation/evaporation is significant only when the saturation deficit is more than 2 mb, which is in agreement with this writer's observations. Since such high saturation deficits are unlikely to occur during snowmelt events it is reasonable to neglect the evaporation term in the energy balance budget.

The transfer of sensible heat between air and snow cover depends on the temperature difference between air and snow surface and probably also on the wind speed. It is usually assumed that the convective heat transfer from air can be estimated from

$$C = k_e (1+aW) (T-T_0) \tag{10}$$

where

C = convective sensible heat transfer rate (cal/cm², day)

W = wind speed (m/s)

T = air temperature at 2 m level

T_0 = snow surface temperature

and k_e (cal/cm², day, °C) and a (s/m) are constants.

Assuming the thermal and vapor diffusivities to be equal, the Bowen ratio is

$$B = \frac{C}{-LE} = k_B p \frac{T-T_0}{e-e_0} \approx 0.65 \frac{T-T_0}{e-e_0} \tag{11}$$

where

k_B = a constant (appr. $6.4 \times 10^{-40} \text{C}^{-1}$)

p = atmospheric pressure (appr. 1013 mb)

e = vapour pressure of air (mb)

e_0 = saturation vapour pressure at surface temperature (mb)

Inserting Eq. (10) and Eq. (7) into Eq. (11) makes it possible to express the convective heat coefficients in terms of the evaporation coefficients.

$$k_e = 0.65 L k_e; \quad a = b \tag{12}$$

Assuming that the snow surface adjusts to the air temperature when below freezing and that the snow surface temperature is at the melting point when the air temperature is above freezing, the convective heat transfer from the air can be expressed as

$$C = k_e (1+aW) T_+ t \text{ (cal/cm}^2 \text{, day)} \tag{13}$$

where

T_+ = mean temperature of part t of the day ($^{\circ}\text{C}$)

t = part of day when the air temperature is above freezing.

The maximum temperature of the day must be closely related to T_+ .

From the profile measurements previously mentioned (Bengtsson, 1975) a correlation between air temperature and convective heat transfer was found:

$$C = 0.9 (T - 1.5) \text{ cal/cm}^2, \text{ h} \quad (14)$$

where

T = air temperature ($^{\circ}\text{C}$).

However, no correlation between sensible heat flux and wind speed was found.

The convective transfer of heat from air to snow was poorly correlated to the mean temperature of the day, but as seen in Fig. 2 the maximum temperature correlates quite well. The convective heat flux is expressed by the equation

$$C = A (T_{\text{max}} - B) \text{ cal/cm}^2, \text{ day} \quad (15)$$

where

A = 6 cal/cm², day, $^{\circ}\text{C}$

B = 1.5 $^{\circ}\text{C}$

T_{max} = maximum temperature ($^{\circ}\text{C}$)

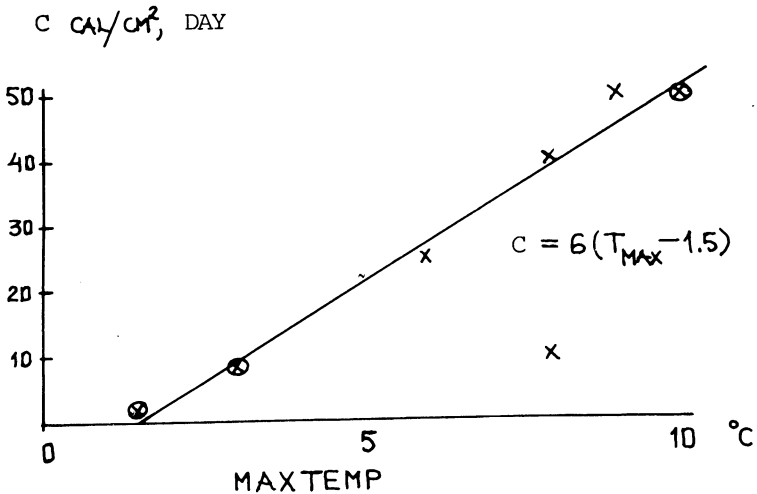


Fig. 2. Convective heat transfer from air to snow versus maximum temperature of the day.

Rain above 0°C falling on a snow surface transfers heat to the snow. This contribution is generally small and can be stated directly by the equation

$$r = \rho c T_r p = T_r p \text{ cal/cm}^2, \text{ day} \quad (16)$$

where

- r = heat transfer from rain
- ρ = density of water
- c = specific heat of water
- T_r = temperature of rain (°C)
- p = rain intensity (cm/day)

Even rain of 5.0 cm at 4°C gives only a heat input to the snow cover of 20 cal/cm² and can thus melt only 0.25 cm water equivalents of snow.

Melt produced by *heat conduction at the ground* is generally considered insignificant. A nominal value of 4 cal/cm², day is recommended by the U.S. Corps of Engineers.

Neglecting the heat from rain and ground and from condensation the energy balance can now be expressed as

$$\begin{aligned} (1-F)(1-\alpha)R_S + \epsilon\sigma 273^4 t - \sigma\{t \cdot 273^4 + (1-t)(273+T_-)^4\} + A(T_{max} - B) \\ = \Delta M + fQ \end{aligned} \quad (17)$$

or if the aerodynamic Eqs. (7), (13) and the relationship (12) are utilized.

$$\begin{aligned} (1-F)(1-\alpha)R_S + \epsilon\sigma 273^4 t - \sigma\{t \cdot 273^4 + (1-t)(273+T_-)^4\} \\ + Lk_e(1+bW)\{0.65(T-T_o) + e - e_o\} = \Delta M + fQ \end{aligned} \quad (18)$$

Degree-day Method

At a certain temperature, the equilibrium temperature, T_e , no net transfer of heat takes place between air and snow. The vapour pressure corresponding to this temperature is e_e . The energy balance at equilibrium can, neglecting heat transfer from rain and ground, be expressed as

$$\begin{aligned} (1-F)(1-\alpha)R_S + \epsilon\sigma(T_e + 273)^4 - \sigma(T_o + 273)^4 \\ + k_e(1+aW)(T_e - T_o) + k_e(1+bW)(e_e - e_o) = 0 \end{aligned} \quad (19)$$

For the actual air temperature T the energy balance is

$$(1-F)(1-\alpha)R_S + \epsilon\sigma(T+273)^4 - \sigma(T_O+273)^4 + k_e(1+aW)(T-T_O) + k_e(1+bW)(e-e_O) = \Delta M + Q \cdot f \quad (20)$$

Combining Eqs. (19) and (20) gives

$$\epsilon\Delta\{(T+273)^4 - (T_e+273)^4\} + k_c(1+aW)(T-T_e) + k_e(1+bW)(e-e_e) = \Delta M + Q \cdot f \quad (21)$$

The expression between the first brackets is approximately $4(T+273)^3(T-T_e)$. Furthermore

$$e - e_e = RH(e_{s,s} - e_{e,s}) = RH \frac{de}{dT}(T-T_e) \quad (22)$$

where

- e_s = saturation vapour pressure
- $e_{e,s}$ = saturation vapour pressure at equilibrium temperature
- RH = relative humidity
- de/dT = saturation vapour pressure - temperature gradient

Now Eq. (21) can be simplified to a degree-day expression,

$$K(T-T_e) = Q + \frac{\Delta M}{f} \quad (23)$$

where

$$K = \frac{1}{f} 4 \epsilon \Delta (T+273)^3 + k_c + k_e \frac{de}{dT} RH + (k_c a + k_e b \frac{de}{dT} RH) W \quad (24)$$

Since the temperature is close to 0°C de/dT is close to 0.47 mb/°C. Also the relative humidity is fairly constant ranging only maybe from 0.75 - 1.0 and this does not influence the value of K . The coefficients k_c and a are correlated to k_e and b as suggested by Eq. (12). The degree-day coefficient can therefore be simplified to

$$K = 0.12\epsilon + 1.06 k_e(1+bW) \text{ (cm/day)} \quad (25)$$

where the wind speed is given in m/s.

Inserting Eqs. (4), (5) for the emissivity makes it possible to determine K as a function of cloud cover (N), forest density (F) and wind speed (W):

$$K = (0.091 + 1.06K_e) - 0.029F + 0.014N(1-F) + 1.06k_e bW \text{ (cm/day)} \quad (26)$$

If the values of the constants given by Eq. (8) are inserted the following K -values are obtained:

$$\begin{aligned} K &= 0.11 + 0.010 W \quad (F = 0 \ N = 0) \\ K &= 0.12 + 0.010 W \quad (F = 0 \ N = 0) \\ K &= 0.14 \quad (F = 1 \ \text{wind speed negligible}) \end{aligned}$$

Since the coefficients used in Eqs. (7) and (10) have been given widely different values by different investigators and also since they depend on the energy supply and on the length of the part of the day when the temperature is above freezing, it is better to use arbitrary coefficients instead, hence

$$K = C_1 + C_2 F + C_3 W \quad (27)$$

As seen from the discussion above, K is fairly constant and should be about 0.10 - 0.15 cm/day. As stated earlier this writer did not find any wind dependence which means that for a certain area K should be a true constant.

Inserting Eq. (24) into Eq. (19) the equilibrium temperature is found to be

$$T_e \equiv T_O + \frac{1}{Kf} \{ (1-\epsilon) \sigma (T_O + 273)^4 + (1-RH) k_e (1-bW) e_O - (1-F) (1-\alpha) R_S \} \quad (28)$$

The term $(1-RH) k_e (1-bW) e_O$ is very small compared to the other two terms. Since the emissivity can be determined from Eqs. (4), (5) and the snow surface temperature can be assumed not to differ much from 0°C, the equilibrium temperature can be expressed as

$$T_e = T_O + \frac{1-F}{fK} \{ 156 (1-0.5N) - (1-\alpha) R_S \} \quad (29)$$

or using arbitrary coefficients

$$T_e = T_O + (1-F) \{ C_4 (1-C_5 N) - C_6 (1-\alpha) R_S \} \quad (30)$$

It has been shown that the degree-day method is a method based on the full heat balance equation. For a certain area the degree-day coefficient can be considered to be constant. The equilibrium temperature, however, varies depending on cloud cover and solar radiation.

Analysis of Snowmelt Processes

Energy balance computations

The snowmelt events in Kiruna ($\varphi = 67^{\circ}51'N, \lambda = 20^{\circ}15'E$) 1961, 62, 63 and in Luleå ($\varphi = 65^{\circ}87'N, \lambda = 22^{\circ}06'E$) have been analysed utilizing energy balance computations according to Eq. (17). The available daily meteorological observations were solar radiation, R_s , cloud cover, N , air vapour pressure, e , temperature, T , wind speed, W , and precipitation, p . The energy balance of each day was estimated. The albedo was determined from Fig. 1 and the longwave radiation from Eqs. (3), (4) with a canopy density of 10%. The convective heat transfer was also determined by Eq. (13) and the transfer of latent heat was estimated by Eq. (7) using the coefficients of Eqs. (8) and

(12). The energy balance thus found from Eq. (18) was in agreement with the simpler Eq. (17).

Since only observations of snow depth and not of the water equivalent of snow were available the observed and calculated snowmelt could only be compared approximatively. The total snowmelt of the entire snowmelt period could, however, be estimated by adding all observed snow precipitation after a snow cover was formed. The observed precipitation values were increased by 10%, since snow precipitation measured by ordinary gauges ordinarily is underestimated. The observed and calculated first day of snowfree ground in Kiruna 1961, 62, 63 and in Luleå 1962, 63, 64, 65 are shown in Table 1.

Table 1 - First observed and by energy budget calculated day when the ground was free of snow.

Place	Year	Observed	Calculated
Kiruna	1961	14 may	14 may
Kiruna	1962	21 may	21 may
Kiruna	1963	7 may	7 may
Luleå	1962	27 april	27 april
Luleå	1963	23 april	23 april
Luleå	1964	8 may	8 may
Luleå	1965	7 may	7 may

Note that for all seven calculations the last snow is estimated to disappear exactly on the day the ground was observed to be free of snow. Still, the energy balance equation used does not have to describe the true energy balance very accurately since the measured precipitation is somewhat arbitrarily increased 10%.

Degree-day method accounting for global radiation

The snowmelt events analysed by the energy balance method was analysed also by the degree-day method. The degree-day »constant«, K , was assumed to be constant. The equilibrium temperature was found from Eq. (30) assuming that the cloud cover was a function of the solar radiation so that

$$T_e = T_o + C_7 + C_8 (1-\alpha) R_S \quad (31)$$

For Kiruna 1961 and 62 the total observed and estimated snowmelt agreed, if the degree-day coefficient was taken as 0.10 cm/day and thus

$$Q = 0.10 (T - T_e) \text{ cm/day} \quad (32)$$

where T is the mean air temperature of the day and the equilibrium temperature is

$$T_e = 1.2 - 0.054 (1 - AL) R_S \quad (33)$$

The corresponding equations were used for Kiruna 1963 and for Luleå 1962, 63, 64 and 65. The first observed and calculated day when the ground was free of snow are compared in Table 2.

Table 2 - First observed and by degree-day method calculated day when the ground was free of snow.

Place	Year	Observed	Calculated
Kiruna	1961	14 may	14 may
Kiruna	1962	21 may	21 may
Kiruna	1963	7 may	7 may
Luleå	1962	27 april	29 april
Luleå	1963	23 april	23 april
Luleå	1964	8 may	8 may
Luleå	1965	7 may	7 may

The correspondence is very good even for Luleå although the degree-day coefficient and the coefficients of the equilibrium temperature equation have been determined independent of the observations of Luleå.

Simple degree-day method

The degree-day method is of course very much simplified if the equilibrium temperature is taken as constant. Using data for Kiruna from 1961 and 62, different equilibrium temperatures were tried (-4°, -3°, -2°, -1° and 0°C). The best correspondence to the more sophisticated degree-day method was obtained for five-days periods if the equilibrium temperature was chosen to be -3°C. The snowmelt is then described by

$$Q = 0.14 (T + 3) \text{ cm/day}$$

Using the corresponding equation the snowmelt events were analysed for Kiruna 1963 and for Luleå 1962, 63, 64 and 65. The comparison between observed and calculated snowmelt is shown in Table 3.

Table 3 - First observed and by the simple degree-day method calculated day when the ground was free of snow.

Place	Year	Observed	Calculated
Kiruna	1961	14 may	13 may
Kiruna	1962	21 may	21 may
Kiruna	1963	7 may	8 may
Luleå	1962	27 april	26 april
Luleå	1963	23 april	23 april
Luleå	1964	8 may	6 may
Luleå	1965	7 may	9 may

Also by using the simple degree-day method good correspondence is found between calculated and observed total snowmelt. However, a day-by-day comparison shows that the time distribution of the snowmelt calculated by the simple degree-day method, deviates from the distributions calculated by the other two methods. The distributions of the snowmelt calculated by the three methods are compared in Figs. 3-7.

The distribution calculated by the sophisticated degree-day method (Eqs. (32), (33)), is in very good agreement with the distribution obtained from energy balance computations. It is only for Luleå 1962 and 63 that the two methods do not agree. The simple degree-day method generally shows a faster snowmelt rate at the beginning of the melting season and a slower melting at the end of the melting season than the other two methods do. This is due to the increase of daylight which occurs during the melting season and this causes an increase of solar radiation at the same time as the snow gets granular and its albedo is decreased. The result will be a decrease of the equilibrium temperature.

Using mean values for Kiruna and Luleå the equilibrium temperatures of different months can be estimated to be for January + 1°C, February + 1°C, March -1°C, April -2°C, and May -9°C.

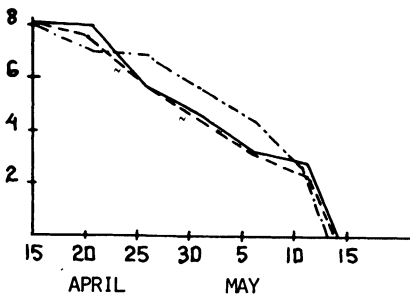


Fig. 3. Snowmelt in Kiruna 1961 estimated by energy balance computations (unbroken line) by degree-day method using variable equilibrium temperature (- - -) and by simple degree-day method (-.-.).

Snowmelt Estimated from Energy Budget Studies

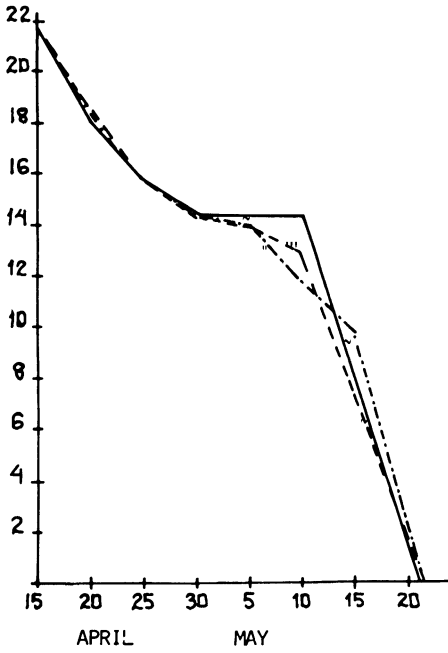


Fig. 4. Snowmelt in Kiruna 1962 estimated by energy balance computations (unbroken line) by degree-day method using variable equilibrium temperature (- - -) and by simple degree-day method (-.-.).

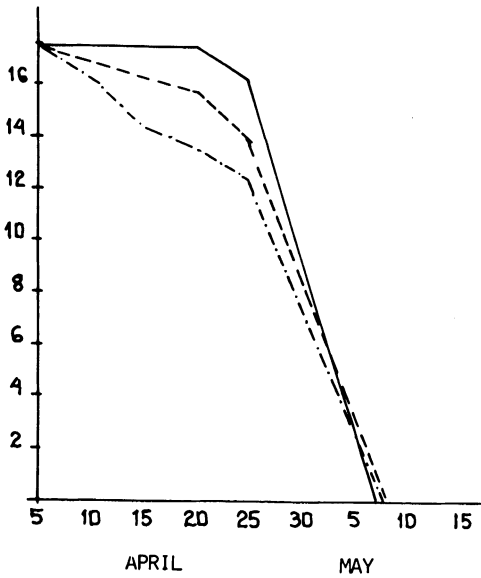


Fig. 5. Snowmelt in Kiruna 1963 estimated by energy balance computations (unbroken line) by degree-day method using variable equilibrium temperature (- - -) and by simple degree-day method (-.-.).

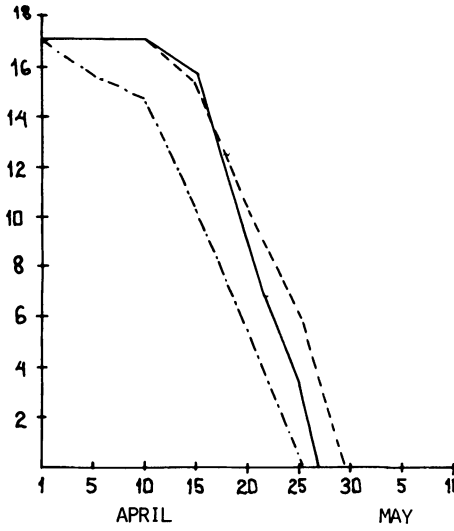


Fig. 6. Snowmelt in Luleå 1962 estimated by energy balance computations (unbroken line) by degree-day method using variable equilibrium temperature (- - -) and by simple degree-day method (-.-.).

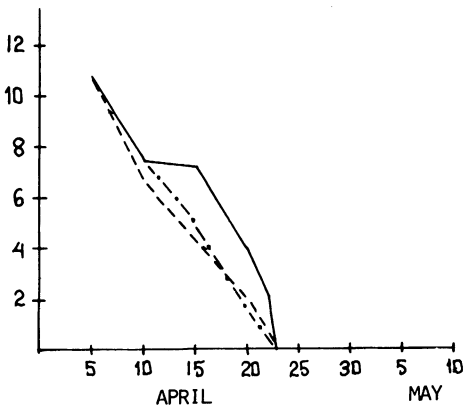


Fig. 7. Snowmelt in Luleå 1963 estimated by energy balance computations (unbroken line) by degree-day method using variable equilibrium temperature (- - -) and by simple degree-day method (-.-.).

Snowmelt Estimated from Energy Budget Studies

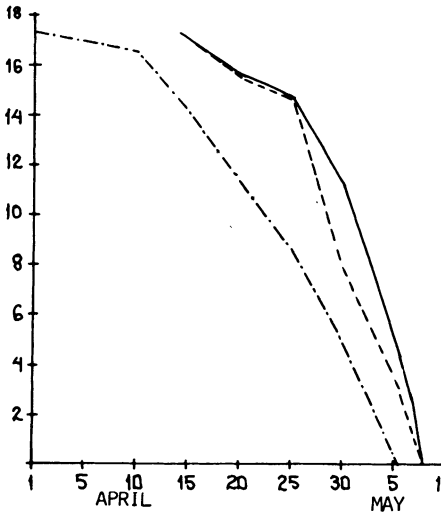


Fig. 8. Snowmelt in Luleå 1964 estimated by energy balance computations (unbroken line) by degree-day method using variable equilibrium temperature (- - -) and by simple degree-day method (-.-).

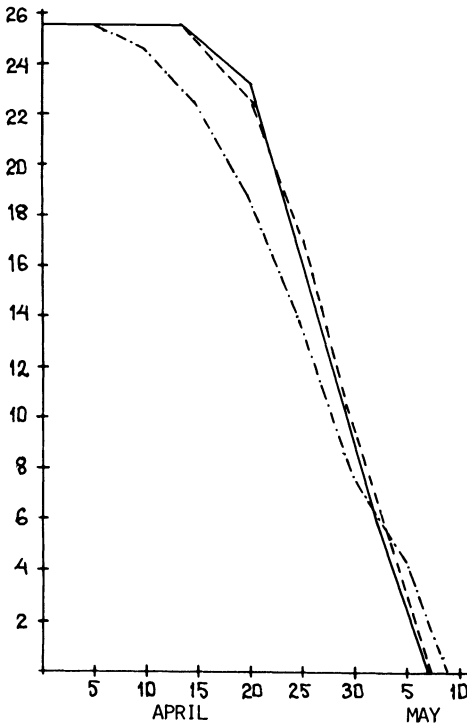


Fig. 9. Snowmelt in Luleå 1965 estimated by energy balance computations (unbroken line) by degree-day method using variable equilibrium temperature (- - -) and by simple degree-day method (-.-).

Conclusions

The degree-day method for calculating snowmelt at a point is theoretically based on the heat balance equation. It should be possible to use a degree-day method to forecast snowmelt events provided the rate of snowmelt is determined by the equation

$$Q = K (T - T_e) \text{ cm/day.}$$

where the equilibrium temperature depends on cloud cover, solar radiation and forest cover, and the degree-day coefficient, K (cm/day, °C), depends on forest cover only.

Further analysis on observations from three specific areas (a grass field, a dense forest, a swamp) are now carried out at the University of Luleå in order to improve the degree-day method.

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