The Isosinglet Meson Series
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Recent experiments indicate the existences of iso-singlet $\pi^+\pi^-$ resonance $f$ and $K\bar{K}$ resonance $\varphi$, whose masses and reasonably assigned quantum numbers are listed in Table I. This seems to establish the $T=Y=0$ series of mesons

$$n, \omega, \varphi, f,$$

which exhibits the following remarkable regularities.

<table>
<thead>
<tr>
<th>$T=Y=0$ mesons</th>
<th>mass (Mev)</th>
<th>$J^P$</th>
<th>$n$</th>
<th>baryon-antibaryon state</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>548</td>
<td>0--</td>
<td>1</td>
<td>$^1S_0$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>782</td>
<td>1--</td>
<td>2</td>
<td>$^3S_1$</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>1019</td>
<td>1--</td>
<td>3</td>
<td>$^3D_1$</td>
</tr>
<tr>
<td>$f$</td>
<td>1250</td>
<td>2++</td>
<td>4</td>
<td>$^3P_2$</td>
</tr>
</tbody>
</table>

(a) The mass spectrum of (1) obeys a complete equal-interval rule such that

$$m(n) = m(0) + n \cdot \kappa,$$

where $n$ is the ordering number ranging from 1 to 4 for each successive level in the series (1), and

$$\kappa = 234 \text{ MeV},$$

$$m(0) = m_\pi - \kappa = 2m_\pi - m_\omega = 314 \text{ MeV}.$$  

Equation (2) reproduces the observed masses of those four mesons within a few MeV, and moreover suggests that the possible level corresponding to $n=0$ with the mass value (4) may be identified with the ABC-particle.\(^0\)

(b) The charge-parity $C$ for any state of (1), which is identical with the $G$-parity in this case, is connected with the spin $J$ by

$$C = (-1)^J.$$  

Since the relation (5) holds for photons also, we conjecture that it represents a general connection valid for any possible self-conjugate bosons. Equation (5) may be extended to include $T=1$ states $\pi$ and $\rho$ mesons such that\(^0\)

$$G = (-1)^{J-1}.$$  

Equation (2) is essentially a linear oscillator formula with the energy quantum $\kappa$ and the "principal quantum number" $n$. This $n$ is related with $J^{PQ}$ but is not uniquely determined by the latter. Indeed the doubling of $1^-$ mesons $\omega$ and $\varphi$ represents a novel situation where all the usual quantum numbers are common but masses are distinct. Our $n$ just serves to play the role of the new quantum number to distinguish between $\omega$ and $\varphi$, pointing to the presence of a certain internal structure for the states. A direct interpretation of the formula (2) in terms of an oscillator model, however, would raise difficulty because of the particular correlation of $n$ with $J^{PQ}$.

A simple model which allows correct $J^{PQ}$ assignments and accords with (2) for all the states of (1) is obtained by assuming those mesons to be tightly bound states of baryon and anti-baryon pair, following Sakata.\(^5\) It is well known\(^6\) that by identifying $\eta$ and $\omega$ with $^1S_0$ and $^3S_1$ states of $NN$ and/or $AA$ system, respectively, their quantum numbers are correctly reproduced. We now introduce an additional assumption that the $f$-meson is the $^3P_2$ state of the same system and the masses are given for those three states by the formula

$$G = (-1)^{J-2}.$$  

\(^0\) It is to be noted that (6) is analogous to the rule\(^0\) $P = (-1)^{J-1}$, which restricted possible non-strange baryon states, i.e. nucleon and its resonances.
m = m_+ + \frac{1}{2} \mathcal{E} J(J+1), \text{ i.e. } n-1 = \frac{1}{2} J^2. \tag{7}

Evidently it assures \( J^{pa} = 2^{++} \) for \( f \) and yields the relation

\[ m_f - m_+ = 4(m_+ - m_+) = 3\mathcal{E}, \tag{8} \]

which is exactly consistent with (2). Equation (7) means that the composite system behaves like a rigid rotator of which energy is determined by the total angular momentum only.

To include the \( \varphi \)-level in this picture we identify it as the \( ^3D_1 \) state of the system. Then the mass formula (7) needs to be extended to

\[ n-1 = \frac{1}{2} (J(J+1) + L(L-1)), \tag{9} \]

or more simply, modified to

\[ n-1 = J(1+L/2), \tag{9'} \]

each of which is equivalent to (2) as to all the four states of (1).

The present model predicts in the \( T=Y = 0 \) series a pseudovector meson \( 1^{--} \) corresponding to \( ^1P_1 \)-state whose mass is near to that of \( \omega \) or \( \varphi \), and the second \( 2^{++} \) level at about \( m_+ + 5\mathcal{E} = 1720 \text{ Mev} \), corresponding to \( ^3F_2 \)-state. One more possible \( pv \) meson \( 1^{++} \) corresponding to \( ^3P_1 \) will be excluded, provided that the restriction (5) be imposed.

Finally it should be remarked that a simple regularity of the meson series (1), could not be envisaged by the Regge trajectory hypothesis\(^7\) nor by the general unitary-symmetry argument.\(^8\) Indeed, it may well be that those striking regularities should bear quite an essential significance pointing to a certain new theoretical foundation.

4) T. Takabayasi, Phys. Letters, in press.