Effects of the Magnetic Field on the Ionization Front

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The effects of interstellar magnetic field on the expanding HII region are investigated based on the simplified plane similarity solution. The conditions for the occurrence of $M$-type fronts are modified by the influence of the magnetic field. It is conjectured that the apparent form of the expanding HII region becomes not entirely circular but elliptic, elongated along the magnetic lines of force. From the observed ellipticity of the HII region, the intensity of interstellar magnetic field is estimated as $10^{-5} \sim 10^{-6}$ gauss. The efficiency of acceleration of interstellar clouds by the Oort-Spitzer mechanism is also discussed briefly.

§ 1. Introduction

The problems relating to ionization fronts* have been discussed by many authors, since the studies of Strömgren$^1$ and Oort and Spitzer.$^2$ Especially, the classification of I.F. and the condition for appearance of S.F. in HI region were investigated in detail by Kahn.$^3$ By Savedoff and Greene$^4$ and Kaplan,$^5$ dynamical effects have also been analysed using similarity solutions. Recently, Goldsworthy$^6$ and Axford$^7$ have investigated the states of flow taking account of recombinations, and obtained similarity solutions for cylindrical and spherical cases. Vandervoort$^8$ has given a more elaborate solution for spherical flow in the HII region neglecting recombinations.

Now, these authors have neglected the effect of magnetic fields existing in the interstellar medium, except for Kaplan$^9$ who derived formulae including the effect. However, it is natural that the magnetic field would play an important role in the interstellar gas motion.

In this paper, we take, explicitly, the magnetic field into account, and consider its various effects on the problem relating to the expansion of the HII region. In the present analysis, we consider similarity solutions of plane flow only, and the orientation of the magnetic field is assumed to be perpendicular to the flow. We expect that the results of this simple problem can be used for an approximate estimation of the effect of the magnetic field.

In § 2, we consider the condition for the co-existence of I.F. and S.F. for the non-magnetic case. This is essentially an application of Kahn's consideration for the isothermal flow.$^3$ Next, in § 3, we introduce the magnetic field

* Hereafter, the ionization front and the shock front are denoted by I. F. and S. F. respectively.
Effects of the Magnetic Field on the Ionization Front

817

and examine how the condition of appearance of M-type fronts and physical quantities are changed by the field. As a preparation for the application, the similarity relation is given in § 4. In § 5, the results obtained in the preceding section are applied to analyse the apparent elliptic form of the HII region and the acceleration of the HI clouds is briefly discussed.

§ 2. Region of co-existence of I.F. and S.F. (I): Non-magnetic case

<table>
<thead>
<tr>
<th>I.F.</th>
<th>S.F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>region 1</td>
<td>region 2</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>( T_2 )</td>
</tr>
<tr>
<td>( \rho_1 )</td>
<td>( \rho_2 )</td>
</tr>
<tr>
<td>( u_1 )</td>
<td>( u_2 )</td>
</tr>
<tr>
<td>( \mu_1 = 1/2 )</td>
<td>( \mu_2 = 1 )</td>
</tr>
<tr>
<td>( \mu_1 = 1 )</td>
<td>( \mu_2 = 0 )</td>
</tr>
<tr>
<td>( U_1 )</td>
<td>( U_2 )</td>
</tr>
</tbody>
</table>

Fig. 1. The régime of co-existence of I.F. and S.F. in plane case. Photon flux \( J \) comes from a bright star on the left. Here, I.F. and S.F. denote ionization front and shock front respectively.

Let us consider the system having both I.F. and S.F. as schematically shown in Fig. 1. Here usual notations of physical quantities are used. The temperatures \( T_1, T_2 \) and \( T_1 \) are taken to be given constants, which are determined by the cooling and the heating processes.\(^{10}\) \( U_s \) and \( U_i \) are the propagation velocities of S.F. and I.F. respectively.

As is well known, certain relations between these quantities are imposed by the conservation laws. Before writing down these relations, we introduce, for convenience, following non-dimensional quantities:

\[
V = u/a, \quad \omega = \rho/\rho_1
\]

and

\[
\theta = a^2/a_1^2.
\]

Here

\[
a = (RT/\mu)^{1/2}
\]

is the isothermal sound velocity. The quantities have their own suffixes corresponding to each region. For example,

\[
\theta_2 = (a_2/a_1)^2 = T_2/T_1 \quad \text{and} \quad \theta_1 = (a_1/a_1)^2 = 2T_1/T_1,
\]

since

\[
\mu_1/\mu_1 = 1 \quad \text{and} \quad \mu_2/\mu_1 = 2.
\]

Note that

\[
\theta_1 = \omega_1 = 1 \quad \text{and} \quad V_1 = 0,
\]
Also the non-dimensional propagation velocities of I.F. and S.F. are given by
\[ \xi_i = \frac{U_i}{a_i} \quad \text{and} \quad \xi_s = \frac{U_s}{a_s}. \]

Finally, the ionizing photon flux \( J \) arriving at I.F. is represented in non-dimensional form as
\[ j = \frac{m_H J}{\rho_i a_i}, \]
where \( m_H \) is the mass of hydrogen atom.

Now, using these non-dimensional quantities, the conservation laws may be written as follows:
\[
\begin{align*}
\dot{\xi}_s &= \omega_s (\xi_s - V_s), \\
1 + \dot{\xi}_s^2 &= \theta_4 \omega_s + \omega_s (\xi_s - V_s)^2
\end{align*}
\]
for S.F. and
\[
\begin{align*}
\omega_i (\xi_i - V_i) &= \omega_s (\xi_s - V_s) = j, \\
\theta_i \omega_i + \omega_s (\xi_s - V_s)^2 &= \theta_4 \omega_s + \omega_s (\xi_s - V_s)^2
\end{align*}
\]
for I.F.

In addition, as was emphasized by Kaplan, the Chapman-Jouguet condition is imposed on I.F., i.e.
\[
\xi_i - V_i = \sqrt{\theta_i} \tag{2.3}
\]
in our notation.\(^*\)

From the above six equations, the unknown six quantities, \( \omega_s, V_s, \xi_s, \omega_i, V_i \) and \( \xi_s \), can be determined, provided that the parameters \( \theta_s, \theta_i \) and \( j \) are fixed. Moreover, the following two conditions must be satisfied in order that a régime such as shown in Fig. 1 should exist:

i) \( \xi_i > \xi_s \), which is imposed by the requirement that S.F. should not be overtaken by I.F.,

ii) \( \omega_i > 1 \), which requires that the shock is really compressive.\(^**\)

Owing to these conditions, allowed values of parameters \( \theta_i, \theta_s \) and \( j \) are limited for this régime, i.e. co-existence of I.F. and S.F.\(^***\) to exist.

Possible values of \( j \) are then limited by
\[ j_D (\theta_i, \theta_s) < j < j_R (\theta_i, \theta_s). \]

\( * \) This means that I.F. is of \( D \)-critical type.

\( ** \) In general case, when \( \theta_2 > 1 \), it is physically required that \( \theta_{ad} > \theta_2 \), where \( \theta_{ad} \) is the temperature ratio in the case of adiabatic shocks. Therefore, the condition ii) is really applied only to the isothermal case (\( \theta_2 = 1 \)) for all shock strengths.

\( *** \) This is nothing but \( M \)-type régime of Kahn.\(^5\)
Effects of the Magnetic Field on the Ionization Front

Here \( j_R \) and \( j_D \) correspond to the values for \( \xi = 0 \), and \( \omega = 1 \), respectively, \( \theta_i \) and \( \theta_1 \) being fixed. Incidentally, they represent the values for the critical I.F., i.e., \( R \)-critical and \( D \)-critical for the isothermal case \( \theta_i = 1 \). This may be easily seen by the replacement \( \omega \rightarrow 1 \), \( V_1 \rightarrow V_1 = 0 \), \( \theta_i \rightarrow 1 \). Then, they eventually take the following simple form:

\[
\begin{align*}
\frac{j_R}{j_\infty} &= V\sqrt{\theta_1} + V\sqrt{\theta_1 - 1}, \\
\frac{j_D}{j_\infty} &= V\sqrt{\theta_1} - V\sqrt{\theta_1 - 1}.
\end{align*}
\]

In Fig. 2, the allowed \( M \)-type region is represented for this case in \( \log \theta_i - \log j \) plane. Note that S.F. can exist only when

\[
\theta_1 > 1 \quad \text{or} \quad T_1 > \frac{T_1}{2}.
\]

When either \( j \geq j_R \) or \( j \leq j_D \), only I.F. can exist, and these cases correspond to \( R \)- and \( D \)-type respectively. In other words, near a bright star, where the photon flux is large, there exists \( R \)-type I.F. and it propagates rapidly outwards. Because of geometrical spreading and recombination, the photon flux becomes smaller and smaller. In \( \log \theta_i - \log j \) plane, this is expressed by the downward motion along a vertical line \( \theta_i = \) const. approximately. When \( j \) arrives at \( j_R \), S.F. is generated and propagates ahead.\(^{a)}\) When \( j \) further decreases, the strength of S.F. becomes weaker, and ultimately disappears when \( \omega = 1 \), i.e., \( j = j_D \). Thereafter, only a weak \( D \)-type I.F. propagates slowly.

\section*{§ 3. Region of co-existence of I.F. and S.F. (II): Magnetic case}

We treat a simple case of homogeneous magnetic field, whose orientation is assumed parallel to the plane of discontinuities. From the condition of "frozen in" fields, the relation

\[ j_R \]

\[ j_D \]

\[ j_R \]

\[ j_D \]

\[ j_i \]

\[ j_\infty \]

\[ j_R \]

\[ j_D \]

\[ j_i \]

\[ j_\infty \]

\[ j_R \]

\[ j_D \]

\[ j_i \]

\[ j_\infty \]

\[ j_R \]

\[ j_D \]

\[ j_i \]

\[ j_\infty \]

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\[ j_\infty \]
follows. Using this relation, it is easily shown that the momentum equations (the second equations) of (2·1) and (2·2) are replaced by
\[ 1 + \beta + \xi \omega_2 + \beta \omega_2 + \omega_1 (\xi - V_1)^2 = 0 \] (3·2)
and
\[ \theta_1 \omega_2 + \beta \omega_2 + \omega_2 (\xi - V_1)^2 = \omega_i + \beta \omega_2 + \omega_i (\xi_i - V_i)^2 \] (3·3)
respectively, where \( \beta \) is the ratio of magnetic and gas pressure at region 1:
\[ \beta = \frac{P_{ml}}{P_{g1}}. \] (3·4)

The Chapman-Jouguet condition (2·3) is also replaced by
\[ \xi_i - V_i = [\theta_i + 2\beta \omega_2]^{1/2}. \] (3·5)

The right-hand term here is nothing but the propagation velocity of infinitesimal disturbances for this case.

Thus, in this case, there is one more parameter \( \beta \), the relative strength of magnetic field. Because of the quadratic terms in \( \omega \)'s, \( j_R \) and \( j_D \) cannot be expressed as simple functions of \( \theta_i \) and \( \beta \), even for \( \theta_i = 1 \) which is treated here. They are obtained numerically by solving the equations
\[ 2\beta \omega_2 + (\theta_i - 3\beta) \omega_2 + 2\theta_i \omega_i + (1 + \beta) = 0 \]
and
\[ j = \omega_i [\theta_i + 2\beta \omega_i]^{1/2}, \]
which are easily obtained by putting \( \xi = \xi_i \) and \( \omega_i = 1 \) in the above equations, (3·2), (3·3), (3·5) and the first equations of (2·1) and (2·2).

In Fig. 3, curves of \( j_R (\theta_i) \) and \( j_D (\theta_i) \) are shown for different values of the \( \beta \). The points of intersection of \( j_R (\theta_i) \) and \( j_D (\theta_i) \) curves are given by \( \omega_i = \omega_i = 1, \theta_i = 1 \) and \( j = \sqrt{1 + 2\beta} \). It is seen from this figure that, for the same value of \( \theta_i \), \( j_R \) and \( j_D \) are larger for stronger magnetic field. This may be understood by the following consideration: With the increase in the intensity of magnetic field, the propagation velocity of the infinitesimal disturbances becomes higher. Therefore, the disturbances of the expanding HII region can propagate more rapidly for the same star, i.e. for the same ability of ionizing radiation. This makes the overtaking of the isothermal shock (see the foot note on p. 819 earlier, and thus the critical \( j_R \) becomes larger. The larger values of \( j_D \) may be understood similarly, by the higher propagation velocity of disturbances in the HI region owing to the magnetic field.

The behaviours of various physical quantities due to the magnetic field are shown in Fig. 4. It is seen from this figure that, as the magnetic field becomes stronger,

a) the compression of the HII region (\( \omega_2 \)) becomes smaller,
**Effects of the Magnetic Field on the Ionization Front**

Fig. 3. The critical curves $j_R(\theta_j)$ and $j_D(\theta_j)$ for different values of parameter $\beta$, i.e. the relative intensity of the magnetic field defined by (3.4).

Fig. 4. Effect of the magnetic field on various physical quantities for $\theta_1=200$ and $j=10$. $\omega_\beta/\omega_0(0)$ is the density in region 2 (see Fig. 1) relative to the non-magnetic value. $V^2(\beta)/V^2(0)$ is the relative kinetic energy per unit mass. $[\xi_\beta(\beta) - \xi(0)]/[\xi_\beta(0) - \xi(0)]$ denotes the relative value of the difference between the propagation velocity of I.F. and S.F.
b) the kinetic energy per unit mass \( (V^2/2) \) is decreased,
c) the difference of the propagation velocities \( (\xi_1 - \xi_s) \) becomes larger.

These results are easily understood by a consideration similar to the above. The results for typical values are shown in Fig. 4, i.e. \( \theta_s = 200 \) and \( j = 10 \).

§ 4. Similarity relation

In order to apply the results to practical problems, we have to know the positions of I.F. and S.F., or at least, the relative distance between them. To do so, the solution of the equation of motion is required. Moreover, in the preceding sections, we assumed implicitly that the region 2 is homogeneous, i.e \( \rho_1 \), \( u_1 \) and \( H_1 \) are constant quantities in this region. However, whether such a solution can exist should be examined also by use of the equation of motion.

Here, for simplicity, we adopt the similarity solutions given by Kaplan, although its justification must be checked by physical considerations. The non-dimensional quantities

\[
\begin{align*}
\omega &= \rho/\rho_1, \\
V &= u/a_1
\end{align*}
\]

(4·1)

are the functions of a similarity variable \( \xi \) only, which is defined by

\[
\xi = \frac{r}{a_1 t}.
\]

(4·2)

Here, \( r \) is the distance from a certain origin, and \( t \) is the time. Note that \( \xi_1 \) and \( \xi_s \), which are represented as the propagation velocities in the preceding sections, are now also interpreted as the non-dimensional positions of the fronts.

The solutions of the equations are the following two, which were suggested by Kaplan for the non-magnetic case:

i) Homogeneous solutions:

\[
V = \text{const.} \quad \text{and} \quad \omega = \text{const.}
\]

(4·3)

ii) Inhomogeneous solutions:

\[
\begin{align*}
\xi - V &= V \theta + 2 \beta \omega, \\
\frac{dV}{d\xi} &= \frac{\xi - V}{\omega} \cdot \frac{d\omega}{d\xi}
\end{align*}
\]

(4·4)

Now, we show that the inhomogeneous solution (4·4) cannot be applied to the region 2. In fact, at the point just behind S.F., the first equation of (4·4) becomes

\[
\xi_1 - V_1(\xi_s) = [\theta_s + 2 \beta \omega_1(\xi_s)]^{1/\alpha},
\]

which suggests that the fluid velocity behind the shock front is just the (gen-e
Effects of the Magnetic Field on the Ionization Front

ralised) sound velocity. This can occur only when the strength of shock becomes infinitesimal, i.e. the shock effectively disappears. Thus, the presence of S.F. itself requires homogeneous solutions (4·3).

The régime depicted in Fig. 1 is therefore justified, if the similarity is admitted. As for the HII (i) region, both solutions are possible. Note that at \( \xi = \xi_i \), the first equation of (4·4) represents the Chapman-Jouguet condition, and so the boundary condition is automatically satisfied on I.F. The constants \( \omega_i \) and \( V_i \) for the homogeneous solution also need to satisfy this relation at \( \xi = \xi_i \).

So far as the homogeneous solution in the region 2 is allowed, the distance between the fronts can be given by the relation

\[
rs - ri = (\xi_s - \xi_i) a_i t,
\]

if only the time \( t \) is given. Here, \( \xi_s \) and \( \xi_i \) are determined by the equations of § 2 and § 3 for the given parameters \( j \), \( \theta \), and \( \beta \).

Although we adopted similarity solutions for simplicity, we do not regard them as valid in whole regions, especially around the bright stars, as did Goldsworthy \(^6\) and Axford. \(^7\) Nevertheless, for relative estimations in the next section, it might be permitted to take the similarity solutions, and connect the time \( t \) and the non-dimensional photon flux \( j \) as follows.

Owing to the geometrical spreading and the absorption, the photon flux at I.F. (for example) is given by

\[
J(r_i) = \frac{\alpha L^*}{4\pi r_i^2},
\]

where \( L^* \) is the luminosity of the star and \( \alpha \) is the factor due to the absorption (recombination). Then, from the above reasoning, the relation

\[
j = \frac{A^3}{\xi_i^2 t^2},
\]

or

\[
t = \frac{A}{\xi_i V_j} \quad (4·5)
\]

holds, where the constant \( A \) is given by

\[
A = \left( \frac{\alpha m e L^*}{4\pi \rho_0 a_i^2} \right)^{1/3}.
\]

Since \( \xi_i \) is the function of \( j \), \( t \) is determined by \( j \) only and thus \( t_R \) or \( t_D \) corresponds to \( j_R \) or \( j_D \) respectively.

§ 5. Applications

i) Ellipticity of HII region

The apparent forms of the observed HII region around bright stars are not entirely circular but there are many of elliptic forms.\(^1\) According to earlier ob-
observational data, the eccentricity of the HII ring in the Orion region is about 0.5 and those of other nebulae are of the same order.\textsuperscript{10} We will show that this fact can be attributed to the effect of the magnetic field along the galactic arms as schematically shown in Fig. 5. In the first place, the Strömgren sphere can be regarded approximately as corresponding to the state in which the flux becomes \( j_\beta \). The critical value \( j_\beta(\beta) \) is larger for stronger magnetic field, provided that \( \theta_i \) is the same. The inverse relation holds for the radius \( R_\beta(\beta) \) owing to equation (4·2). Now, the direction parallel to the field corresponds to \( \beta = 0 \) and the radius \( R_\beta(0) \) is the largest, while on the
Effects of the Magnetic Field on the Ionization Front

The perpendicular direction the radius $R_\beta(\beta)$ is the smallest. Thus, in a homogeneous field the HII region is supposed to form a spheroid (see Fig. 5), having the eccentricity

$$e(\beta) = \left[ R_\beta^2(0) - R_\beta^2(\beta) \right]^{1/2}/R_\beta(0) = \left[ 1 - (j_\beta(0)/j_\beta(\beta)) \right]^{1/2}. \quad (5 \cdot 1)$$

Putting the observed value $e = 0.5$ in this equation, we obtain the value $\beta = 0.3 \sim 0.4^\circ$ from Fig. 6.

Taking the values $T_1 = 100^\circ$K, $n_1 = 10$ atoms/cc, which correspond to the HI clouds, we finally get

$$H_1 = 10^{-4} \sim 10^{-5} \text{ gauss.}$$

This value of the interstellar field intensity happens to be in agreement with that obtained from other estimations. It is remarkable that in this manner we can get information of the strength as well as of the orientation of the magnetic field from the apparent form of the HII region**).

ii) Acceleration of HI clouds.

This problem is important concerning the energetics of Galaxy and several investigations have been performed since the work of Oort and Spitzer. Here, we consider very roughly the effect of the magnetic fields on the acceleration.

The mass and the kinetic energy of the HI region driven in motion by the stellar irradiation are given respectively by

$$M = \rho_2 \mathcal{Q},$$

and

$$K = \frac{1}{2} \rho_2 u^2_2 \mathcal{Q},$$

at each time. $\mathcal{Q}$ is the volume of the region 2. For getting the total mass and the kinetic energy in our similarity consideration, we put

$$M = \int_{t_R}^{t_D} \frac{dM}{dt} dt = \left\langle \frac{dM}{dt} \right\rangle_{av} (t_D - t_R),$$

and

$$K = \int_{t_R}^{t_D} \frac{dK}{dt} dt = \left\langle \frac{dK}{dt} \right\rangle_{av} (t_D - t_R)$$

and make further approximation that the mean values are taken at

$$\langle j \rangle_{av} = \frac{[j_\beta + j_\gamma]}{2}.$$

Then, using the relation (4 \cdot 5) we can calculate $M$ and $K$. Figure. 7 shows

\footnotetext{\textsuperscript{*} The value $\theta_2$ or $T_1$ is not sensitive to the eccentricity.}

\footnotetext{\textsuperscript{**} We have to consider the angle between direction of the field and of the line of sight.}
the behaviors of $M(\beta)/M(0)$ and $K(\beta)/K(0)$ for a typical value of $\theta_i$, since they depend on $\theta_i$ not so strongly. It is seen how the efficiency of the acceleration is lowered owing to the magnetic field.

![Graph](https://academic.oup.com/ptp/article-abstract/30/6/816/1907864)

**Fig. 7.** Variation of the relative mass and the relative kinetic energy of the HI region driven in motion by the stellar irradiation versus the intensity of the magnetic field $\log (1+\beta)$. The results are for the typical values of $\theta_i$, i.e. $\theta_i=200$.

In order to obtain more reliable results, it is of course necessary to treat exactly the spherical case and, moreover, to take into account the tension of the magnetic fields.

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**References**