The terminal bulk Lorentz factor of relativistic electron–positron jets

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ABSTRACT

We present a numerical simulation of the bulk Lorentz factor of a relativistic electron–positron jet driven by the Compton rocket effect from accretion disc radiation. The plasma is assumed to have a power-law distribution $n_e(\gamma) \propto \gamma^{-s}$ with $1 < \gamma < \gamma_{\text{max}}$ and is continuously reheated to compensate for radiation losses. We include the full Klein–Nishina (hereafter KN) cross-section, and study the role of the energy upper cut-off $\gamma_{\text{max}}$, spectral index $s$ and source compactness. We determine the terminal bulk Lorentz factor in the cases of supermassive black holes, relevant to AGN, and stellar black holes, relevant to galactic microquasars. In the latter case, Klein–Nishina cross-section effects are more important and induce a terminal bulk Lorentz factor smaller than in the former case. Our result are in good agreement with bulk Lorentz factors observed in Galactic (GRS 1915+105, GRO J1655–40) and extragalactic sources. Differences in scattered radiation and acceleration mechanism efficiency in the AGN environment can be responsible for the variety of relativistic motion in those objects. We also take into account the influence of the size of the accretion disc; if the external radius is small enough, the bulk Lorentz factor can be as high as 60.

Key words: radiation mechanisms: non-thermal – stars: individual: GRS 1915+105 – stars: individual: GRO J1655–40 – galaxies: active – galaxies: jets.

1 INTRODUCTION

Superluminal motion observed in active galactic nuclei (AGN), especially in the blazar class, seems to be closely linked with high-energy emission. Such motion was recently observed in the Galaxy (Mirabel & Rodriguez 1994; Hjellming & Rupen 1995; Tingay et al. 1995) in the so-called microquasars. Nevertheless, differences are noticeable in those two cases. The latter systems were observed to have a small value of bulk Lorentz factor (around 2.5), while in the former ones values of about 10–20 are frequent. It is well known that the radiation pressure acting on an electron–positron plasma in the vicinity of a near-Eddington accreting object is very efficient in accelerating the plasma outwards, since the gravitational force is around 1000 times weaker than for an electron–proton plasma. However, Phinney (1987) has shown that for a realistic accretion environment can be responsible for the variety of relativistic motion in those objects. We also take into account the influence of the size of the accretion disc; if the external radius is small enough, the bulk Lorentz factor can be as high as 60.

Key words: radiation mechanisms: non-thermal – stars: individual: GRS 1915+105 – stars: individual: GRO J1655–40 – galaxies: active – galaxies: jets.

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source, spectral index and upper cut-off $\gamma_{\text{max}}$, and then make comparisons between AGN and galactic microquasars. We also consider the influence of the scattered radiation of a broad-line region (BLR) and dusty torus around the central black hole. Finally we discuss the influence of the size of the accretion disc, which could be relevant to the high value of bulk Lorentz factors.

2 THE COMPTON ROCKET EFFECT WITH KLEIN–NISHINA CORRECTIONS

2.1 Notation

All energies are measured in units of $m_e c^2$. We refer to all quantities expressed in the blob rest frame by a prime $\prime$, all quantities in the particle rest frame by a star $^*$, and quantities in the disc frame are not labelled. Photon energies will be labelled by $e$, and the unit direction vector by $k$. We use the KN differential cross-section (Rybicki & Lightman 1979) given by

$$\frac{d\sigma}{d\Omega d\Omega'} = \frac{3\pi\sigma}{16} \left( \frac{e_1}{e'} \right)^2 \left( \frac{e_1 + e' - \sin^2 \theta}{1 + e_1 (1 - \cos \theta)} - 1 \right),$$

$$\sigma = \frac{8\pi^3}{3} \frac{r_e^4}{2}$$ is the Thomson cross-section, where $n_e = e^2 / 2\pi^2 \rho m_e c^2$ is the electron classical radius. This expression applies to the scattering of a photon with energy $e'$ and direction $k'$ by a photon with energy $e_1$ and direction $k_1$, and $\cos \phi = k \cdot k'$. $\theta$ is the scattering angle.

2.2 The general picture

Fig. 1 shows the general configuration of the model. The pair plasma is assumed to be described in the bulk rest frame by an energy distribution $n_\gamma(z, \gamma') \propto \gamma^{\gamma' - 3}$ for $\gamma_{\text{min}} < \gamma' < \gamma_{\text{max}}$, with $s$, $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$ independent of $z$ (see subsection 3.2.1 for a further discussion of this assumption). The radiation force is caused by soft photons coming from a standard accretion disc (Shakura & Sunyaev 1973) around a Schwarzschild black hole. The inner radius of the accretion disc is $r_i = 3r_g$ (where $r_g$ is the Schwarzschild radius of the black hole). The outer radius $r_o$ is a free parameter. We use the blackbody approximation for the disc emission so that the specific intensity is

$$I_r(r) = B_r(T_{\text{eff}}(r)),$$

where $B_r$ is the Planck function and the effective temperature $T_{\text{eff}}$ is given by

$$T_{\text{eff}}(r) = \left( \frac{3G M}{8\pi \sigma T^4} \right)^{1/4} \left[ 1 - b(r_0/r)^{1/2} \right]^{1/4}.$$

$M$ is the accretion rate, $M$ is the mass of the central black hole and $b$ is a parameter describing the inner torque on the disc ($0 \leq b \leq 1$).

In Section 4, we also consider the case of a two-temperature disc model (Shapiro, Lightman & Eardley 1976). This model applies to the hot inner part of the disc ($r < r_s$). The specific intensity is then(Sunyaev & Titarchuk 1980)

$$I_r(r) \propto \frac{x^4}{e^{x}} \quad \text{for} \quad x > 1,$$

$$I_r(r) \propto x^4 \quad \text{for} \quad x < 1,$$

where $x = h\nu/kT_e$ and $T_e$ is the electronic temperature, given by

$$T_e(r) = 7 \times 10^8 \left( \frac{M}{3M_\odot} \right)^{1/4} \left( \frac{10^{17} g s^{-1}}{M} \right)^{1/6} \frac{(1 - (r/r_0)^{1/2})^{-1/6}}{(2r/r_0)^{1/4}}.$$

In AGN, a BLR surrounding the central black hole reprocesses a fraction of about 10 per cent of the disc radiation. In subsection 3.2.5, following Sikora et al. (1996), we choose two extreme cases to describe this emission. The first case is case ‘a’ in Sikora et al. (1996), where a fraction $\chi$ of the total disc luminosity $L_d$ is intercepted by the outer part of the disc and reprocessed in Lyα lines with a specific intensity given by

$$I_r(r) = \frac{\chi L_d}{4\pi v^2} \int_{r_i}^{r} \frac{\rho(v - \nu_{\text{Ly}})}{v^2} dr.$$
course by an energy loss that tends to cool the plasma. However, as mentioned above, we assume that there exists a continuous acceleration mechanism that compensates for these losses, so that the particle distribution remains constant. We assume that this acceleration acts isotropically in the plasma frame, and so it does not contribute to the expression given in equation (8). We also neglect the gravitational force, since for near-Eddington objects and cold pairs we have $F_{\text{rad}}/F_{\text{grav}} = m_e/m_c < 10^3$.

To calculate $F_{\text{\Lorentz}}$ more easily, one can use two different approximations. For low-energy particles, the KN cross-section becomes important but we can assume the head-on approximation (Blumenthal & Gould 1970) to be valid. In this approximation, the velocity of all photons in the particle frame is assumed to be antiparallel to the particle velocity in the observer frame. The momentum loss rate is therefore necessary in the direction of the particle motion. Let $x$ denote this direction; the electron energy and momentum loss rate in its rest frame are then, in units of $m_e c^2$ ($\ast$ refers to this frame),

$$
\frac{dE^*}{dt} = -c \int \frac{d\sigma}{d\Omega^*_i} (e^* - e^*_0 \cos \phi^*) de^*_i d\Omega^*_i,
$$

and

$$
\frac{dE}{dt} = -c \int \frac{d\sigma}{d\Omega_i} (e_0 - e^*_0) de^*_i d\Omega^*_i.
$$

$dE^*$ is the differential photon density in the electron rest frame. The momentum change rate in the blob rest frame is then given by

$$
\frac{dp^*_i}{dt} = \frac{dp^*_i}{dt} + \beta \frac{dE^*}{dt}.
$$

Note that, using the KN cross-section, the rate of momentum change in the electron rest frame is not zero, contrary to the cases of the Thomson limit or synchrotron emission. One must take this into account in computing the energy loss rate in the blob rest frame, which is not Lorentz-invariant any more (Blumenthal & Gould 1970). In the head-on approximation the integrals (11) and (12) over $e^*_i$ and $\Omega^*_i$ can be readily performed, leading to

$$
\frac{dp^*_i}{dt} = -\sigma_T \int d\sigma [f_0(e^*) + \beta f_0(e^*)].
$$

The two functions $f_0$ and $f_\ast$ are given by

$$
\begin{align*}
\frac{dE^*}{dt} &= \frac{dE^*}{dt} + \beta \frac{dE^*}{dt}, \\
\frac{dE}{dt} &= -c \int \frac{d\sigma}{d\Omega_i} (e_0 - e^*_0), \\
\frac{dp^*_i}{dt} &= -\sigma_T \int d\sigma [f_0(e^*) + \beta f_0(e^*)].
\end{align*}
$$

where the function $f(e^*)$ corresponds to the ultrarelativistic case ($\beta^* = 1$) and is given by

$$
\begin{align*}
\frac{dE^*}{dt} &= \frac{1}{8} \left[ 18e^* + 102e^{3*} + 186e^{4*} + 102e^{5*} - 20e^{6*} \\
&\quad - 1 \left( -9 - 141e^{2*} - 60e^{3*} - 12e^{4*} + 24e^{5*} \right) \right].
\end{align*}
$$

We then use these expressions in equation (8).

We can estimate the errors in the two extreme regimes described above. In the head-on approximation the first corrections are roughly of order $1/\gamma$, while in the Thomson limit these corrections are $\sim \gamma(e)$. Here $\gamma(e) = M^{14}/M^2$ is the average photon energy emitted from an accretion disc. We connect the two regimes by defining a critical Lorentz factor $\gamma_{\text{crit}}$ for which errors in the head-on approximation ($\gamma > \gamma_{\text{crit}}$) and in the Thomson limit ($\gamma < \gamma_{\text{crit}}$) are of the same order. This gives $\gamma_{\text{crit}} \sim (e)^{-1/3}$. We may therefore have errors of the order of $1/\gamma_{\text{crit}} \sim \gamma_{\text{crit}}(e) - (e)^{2/3}$. For AGN, $(e) \sim 10^{-4}$ and we find $\gamma_{\text{crit}} \sim 20-30$ with a maximum error $\sim 0.2$ per cent while for a microquasar $(e) \sim 10^{-2}$ and $\gamma_{\text{crit}} \sim 5$ with a maximum error $\sim 5$ per cent.

### 2.4 Equation of motion

Following Phinney (1982) we determine the acceleration of a pair plasma by considering the conservation of the stress–energy tensor, leading to Phinney’s equations (7) and (8) in the bulk rest frame:

$$
\frac{\partial}{\partial t} [(\rho^* + \rho^*) \gamma_b \rho_b] - \rho_b F_{\ast} \gamma_b = F^0 + \beta_b F_{\ast},
$$

and

$$
\frac{\partial}{\partial t} [(\rho^* + \rho^*) \gamma_b \rho_b \delta_b] = F_{\ast} + \beta_b F^0.
$$

Combining these two equations and for a reheated relativistic plasma with $\rho^* = \rho/s$ one finds the equation of motion (with $\gamma_b = 1/3\gamma_b^2$ and $\gamma^* = \gamma^* - 1$)

$$
\frac{\sigma_b}{\partial z} = \frac{1}{\rho^*} (\rho^* + \rho^* \gamma_b \rho_b) \gamma_b
$$

To compute the radiative force using equation (14) we need the differential photon distribution in each electron rest frame. For this we use the Lorentz invariant $d\sigma/e$ (see Blumenthal & Gould 1970) and the relation between energy in the accretion disc frame and the electron rest frame

$$
\gamma^* = \gamma_b (1 - \beta_b \mu_e) \gamma (1 - \beta \mu_e).
$$

In the last equation $\mu_e$ is the cosine of the angle between the photon direction and the jet axis in the accretion disc frame (see Fig. 1) and $\mu = \mu_b \mu_e$ (1 - $\mu_e^2$) cos $\phi_e$ is the cosine between the electron and photon directions in the bulk rest frame. We use the averaged scattered photon energy over the azimuthal angle $\phi_e$, and equation (18) with $\mu_b = \mu_b \mu_e$. Finally we integrate equation (17) between $z = 10\gamma_b$ and $z = 10\gamma_b f_g$ (Galactic case) or $z = 10^2 f_g$ (extragalactic case) for different configurations to determine the final bulk Lorentz factor.

### 3 RESULTS

#### 3.1 Equilibrium Lorentz factor

In any axisymmetric radiation field different from a plane wave, there exists an equilibrium Lorentz factor $\gamma_{\text{eq}}$, for which the radiation force vanishes $[F_{\ast}(\gamma_{\text{eq}}) = 0]$; see Fig. 2. For $\gamma_b > \gamma_{\text{eq}}$ the pair plasma sees many more photons coming forward and then decelerates, while if $\gamma_b < \gamma_{\text{eq}}$ the plasma is pushed by radiation.
The qualitative behaviour of a general solution of equation (17) is therefore the following: $\gamma_b$ is first set to the value $\gamma_{beq}$. This value increases gradually until an altitude $z = z_{crit}$ the radiation force becomes too weak to accelerate the plasma. Ballistic motion at a constant Lorentz factor $\gamma_{beq}$ therefore follows. This mechanism ensures that in most cases the initial value of $\gamma_b$ does not influence the terminal value, if the blob is injected below $z_{crit}$.

The Thomson regime has been well studied by various authors (Phinney 1982, 1987; Marconi et al. 1996). In this case the equilibrium Lorentz factor is determined by the simple condition $H' = 0$, that is the radiation flux vanishes in the blob rest frame. Analytical calculations in the Thomson regime give the behaviour of the equilibrium Lorentz factor and the solutions of the equation of motion for a standard accretion disc (Shakura & Sunyaev 1973). This yields $\gamma_{beq} \propto z^{1/4}$ for $z < r_e$ and $\gamma_{beq} \propto z$ for $z > r_e$ (see Appendix A for this case). Fig. 3 shows the function $\gamma_{beq}$ obtained in the Thomson limit for different values of $r_e$ in comparison with these two asymptotic regimes. Note that for decreasing size of the accretion disc, the radiation field is more and more anisotropic and the function $\gamma_{beq}$ increases more rapidly with $z$. For $z_{crit} < r_e$ one finally obtains for a hot plasma $\gamma_{beq} \propto (\gamma z^{-5/2})^{0/3} H(\gamma')^{1/3}$, while for $z_{crit} > r_e$, $\gamma_{beq} \propto (r_e/r_e)^{3/4} (\gamma z^{-5/2})^{0/3} H(\gamma')^{1/3}$ (see Appendix A).

$X_{\text{crit}} \propto |\gamma|^{-1/2} \cdot \lambda \cdot \mu \cdot \nu$ is the compactness of the source, and $\zeta = \int \frac{n_0(\gamma')d\gamma'}{\gamma'}$.

Fig. 4 shows the equilibrium Lorentz factor calculated including KN corrections for different configurations. We compare the results with the corresponding Thomson solution. The effect of the KN corrections is to reduce the contribution of highly energetic collisions ($e^+ \approx 1$) to the net radiation force seen by the plasma. As the more energetic photons come from the inner part of the accretion disc, in the blob rest frame they contribute to accelerating the plasma. On the other hand, the outer and colder parts contribute to decelerating it. We then expect that the equilibrium Lorentz factor is reduced when including KN corrections, as shown in Fig. 4. The importance of this difference depends on the particle and photon distributions. It is predominant for microquasars, in which the accretion disc radiates more energetic photons than in the extragalactic case.

### 3.2 Influence of the different parameters

#### 3.2.1 General considerations

To determine the terminal Lorentz factor for different configurations, we solve numerically the equation of motion for various radiation fields and particle distributions. We show in Fig. 5 typical solutions in the case of a stellar black hole. The terminal value of the bulk Lorentz factor does not depend on the initial value of $\gamma_{init}$ as discussed in Section 3.1. The general behaviour of the solution discussed in this section still holds, even including KN corrections.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The radiative force as a function of the bulk Lorentz factor $\gamma_b$ at different altitudes from the central black hole.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Equilibrium Lorentz factor $\gamma_{beq}$ as a function of $z$ in the Thomson limit. $\gamma_{beq}$ is given for $n_0 = 10^6$, $r_e = 10^2 r_g$, $r_b = 10^2 r_g$, and $r_e = 10^3 r_g$. We also represent the two asymptotic regimes $\gamma_{beq} \propto z^{1/4}$ for $z < r_e$ and $\gamma_{beq} \propto z$ for $z > r_e$, with $r_e = 3 \times 10^3 r_g$.

![Figure 4](https://example.com/figure4.png)

**Figure 4.** The equilibrium Lorentz factor calculated including KN corrections compared with the Thomson solution. The figure corresponds to a stellar black hole $M = 5 M_\odot$ with $\gamma_{max} = 10^7$ for spectral index $s = 1.5$ and $s = 5$. The external radius is $r_e = 3 \times 10^3 r_g$ and $L = L_{\text{Edd}}$.

![Figure 5](https://example.com/figure5.png)

**Figure 5.** Solutions of the equation of motion in a case of a stellar black hole ($M = 5 M_\odot$). We chose two different initial conditions $\gamma_{init} = 2$ and $\gamma_{init} = 5$. $r_e = 3 \times 10^3 r_g$ and $L = L_{\text{Edd}}$.
The critical point $z_{\text{crit}}$ is reached rather close to the central engine (before $10^4$ Schwarzschild radii). For $z > z_{\text{crit}}$ the motion is nearly ballistic and therefore independent of the radiation force, which has become too weak. It is also independent of any variation of the pair distribution, unless the variation strengthens the radiation force. This scenario would require a more efficient acceleration mechanism when moving away from the central source, which is very unlikely. Therefore our assumption of a stationary pair energy distribution over a large range of $z$ does not strongly influence the terminal value of $\gamma_b$, or in other words this value is essentially determined by the local parameters at the critical distance.

3.2.2 Influence of the energy upper cut-off and spectral index

Fig. 6 illustrates the influence of the spectral index (for $1.5 \leq s \leq 5$) and the energy cut-off (for $10^3 \leq \gamma_{\text{max}} \leq 10^7$) on the terminal Lorentz factor. We chose $M = 5 M_\odot$ to be representative for stellar black holes and $M = 10^9 M_\odot$ for supermassive black holes. The calculations were carried out for $L = L_{\text{Edd}}$ and $L = 0.1L_{\text{Edd}}$, where $L$ is the luminosity of the accretion disc.

Results are very sensitive to the spectral index value. There are three different types of behaviour according to the value of $s$.

(i) for $s < 2$ there exists a maximum terminal Lorentz factor, which is a function of $\gamma_{\text{max}}$.

(ii) for $2 < s < 3$ there still exists a maximum, but less pronounced. The variation as a function of $\gamma_{\text{max}}$ is smoother.

(iii) for $s > 3$ there is no variation $\gamma_{\text{max}}$.

We find that for low values of $\gamma_{\text{max}}$, our solutions agree with the Thomson regime solutions. Nevertheless, KN corrections reduce the efficiency of the Compton rocket effect. As a matter of fact, in the Thomson regime, an increase of $\gamma_{\text{max}}$ leads to an increase of $(\gamma^2)$ (for $s < 3$) and therefore of $\gamma_b$. This mechanism is valid until KN corrections begin to dominate, roughly when $\gamma_{\text{max}}(\varepsilon) \sim 1$. When $\gamma_{\text{max}}$ is greater than $(\varepsilon)^{-1}$, therefore, the radiation force does not increase any more whereas the plasma inertia $\rho'$ is much more important. This leads to a less efficient acceleration mechanism. This effect is larger for small indexes, explaining the inversion of the curve for high $\gamma_{\text{max}}$. Indeed we find that acceleration is much more efficient for $s = 2$ and $\gamma_{\text{max}} = 10^7$ than for $s = 1.5$ and the same value of $\gamma_{\text{max}}$ in the case of a stellar black hole. When steepening the pair distribution, the radiation force is dominated by the low-energy part of the distribution. This fact explains why no variation is apparent with $\gamma_{\text{max}}$ for $s > 3$. The plasma behaves...
dynamically like a cold plasma, and we find a small value of the terminal Lorentz factor. Finally, including KN corrections in the calculations gives rise to an absolute upper limit to the maximal Lorentz factor for a given luminosity.

3.2.3 Influence of the black hole mass

As discussed above, the influence of the mass of the central black hole is predominant. Stellar black holes with soft X-ray emission ($\langle e \rangle \sim 10^{-5}$) are less efficient in accelerating a blob of pair plasma than supermassive black holes with softer radiated emission ($\langle e \rangle \sim 10^{-3}$), because KN saturation effects occur at a much lower energy. A more realistic description of the accretion disc around stellar black holes reinforces this role. As shown in Fig. 7, the radiation emitted from a two-temperature disc (Shapiro et al. 1976) leads to a smaller $\gamma_{\text{beq}}$ than in the case of standard accretion discs around stellar black holes should radiate up to a few keV (as in a two-temperature disc), KN corrections play an important role in this case.

3.2.4 Influence of the luminosity

As $\langle e \rangle \propto M^{1/4} / M^{-1/2}$, the luminosity of the disc also directly influences the maximum of the function $\gamma_{\text{beq}}$ as a function of $\gamma_{\text{max}}$. As shown in Fig. 6, the maximum occurs at increasing $\gamma_{\text{max}}$, when the luminosity decreases. Also, less luminous systems contribute a lower radiation force and therefore less efficient acceleration. In the Thomson regime one has a dependence $\gamma_{\text{beq}} \propto L^{1/7}$.

3.2.5 Effect of scattered radiation

All the results described above are obtained when studying the disc radiation alone. We also include in our calculation BLR radiation fields corresponding to two cases:

(i) re-emission from a ring located between $r_1$ and $r_2$ and with an emissivity given by equation (5);

(ii) re-emission from spherically distributed matter at a distance $r_0$ from the central black hole with an emissivity given by equation (6).

Fig. 8 displays the equilibrium Lorentz factor in the presence of a BLR located between $r_1 = 10^3 r_g$ and $r_2 = 10^5 r_g$ (case i) in the Thomson regime. We also plotted the equilibrium Lorentz factor including KN corrections, as well as the solution of the equation of motion for a plasma with $s = 2$ and $\gamma_{\text{max}} = 10^5$. As one can see, the effect of BLR on $\gamma_{\text{beq}}$ is considerably weakened by KN corrections. This can be understood because the photons coming from the BLR are blueshifted by the relativistic motion in the blob rest frame, whereas the photons coming from the disc are redshifted. The dragging force from the BLR is therefore much more reduced by KN corrections than the accelerating force caused by disc photons. Owing to the weakness of the radiation field, the dynamical solution $\gamma_{\text{beq}}(z)$ is still less affected than the equilibrium value. The case of a spherical shell located at $r_0 = 10^3 r_g$ (case ii) is illustrated in Fig. 9.

One can see that the diffused radiation field strongly affects the motion, which is almost stopped at the crossing of the shell, whereas the radiation density is dominated by the isotropically scattered photons. However, the plasma is quickly reaccelerated after the crossing, and a high Lorentz factor can be reached again.

Sikora et al. (1996) argued that scattered radiation should lead to efficient radiation drag. However, the influence of scattered radiation is strongly governed by the position of the scattering clouds with respect to the critical distance $z_{\text{crit}}$: if they lie below this critical distance, the plasma will be temporarily braked during the crossing of the scattering region, but will be quickly reaccelerated after it. If the clouds lie above $z_{\text{crit}}$, the terminal Lorentz factor can indeed be strongly affected. Figs 10 and 11 display the terminal Lorentz factor as a function of $\gamma_{\text{max}}$ for different distances of the BLR. It can be seen that if it is close enough, the BLR can even give a higher Lorentz factor than for the disc alone for the highest value of $\gamma_{\text{max}}$. This is because the solid angle subtended by it at $z_{\text{crit}}$ is so small that its radiation field has an accelerating rather than decelerating effect. Even for a BLR between $10^4$ and $10^5 r_g$, however, terminal Lorentz factors around 10 are clearly reachable. We conclude that the presence of broad lines can affect $\gamma_{\text{beq}}$, but does not prevent highly relativistic motions in general.

We also discuss the effect of possible emission from a hot dusty torus surrounding the central black hole outside the BLR region. The effect of such a scattered radiation field is much more important, because KN corrections are almost negligible here. Fig. 12 shows the terminal Lorentz factor in the presence of an IR-emitting ring heated to $T = 1500$ K, located between $r_3$ and $r_4$. It is obvious that the terminal Lorentz factor is strongly reduced to
values around 3 for the less favourable case ($r_3 = 10^3 r_g$, $r_4 = 10^2 r_g$). The situation is a little less dramatic for more distant sources ($r_3 = 10^5 r_g$, $r_4 = 10^4 r_g$) because the isotropic radiation density is lowered, and a $\gamma_{\text{bm}}$ of 5 can be reached.

### 3.2.6 Influence of the accretion disc size

For small values of $r_e$ the radiation is more anisotropic and so more efficient at accelerating the pair plasma (see Fig. 13). In this figure $r_e = 10 r_g$ and we show the solutions for which we obtain the largest $\gamma_{\text{bm}}$ in extragalactic and Galactic cases. In this configuration the radiation force is more efficient and $\gamma_{\text{bm}}$ can be as high as 60 in the extragalactic case. The dependence on $s$ and $\gamma_{\text{max}}$ is shown in Fig. 14, where we extend the previous calculation to the case $r_e = 10 r_g$. We find the same global behaviour of $\gamma_{\text{bm}}$ with spectral index and $\gamma_{\text{max}}$.

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**Figure 10.** Terminal Lorentz factor $\gamma_{\text{bm}}$ as a function of $\gamma_{\text{max}}$ for the spectral index $s = 2$. The solid line shows the solution obtained including KN corrections for a standard accretion disc. The scattered radiation is described by equation (5) with $\chi = 0.1$, $\alpha = 2$ and different locations of the BLR ring.

**Figure 11.** Same as Fig. 10 for scattered radiation described by equation (6) with $\chi = 0.1$ and different locations of the BLR shell.

**Figure 12.** The influence of a dusty ring combined with a BLR on the terminal Lorentz factor $\gamma_{\text{bm}}$ as a function of $\gamma_{\text{max}}$ for the spectral index $s = 2$. The solid line shows the solution obtained including KN corrections for a standard accretion disc. The BLR ring emissivity is described by equation (5) with $\chi = 0.1$, $\alpha = 2$. The dusty ring emissivity is given by equation (7) with $\chi = 0.1$ or $\chi = 0.05$. We chose different locations for these two components.

**Figure 13.** Solutions of the equation of motion for a supermassive black hole ($M = 10^9 M_\odot$) and a stellar black hole ($M = 5 M_\odot$) with a compact accretion disc. $r_e = 10 r_g$ and $L = L_{\text{Edd}}$. $\gamma_{\text{bm}}$ is the initial condition of the solution. We also represent the equilibrium Lorentz factor in the Thomson regime.

**Figure 14.** The terminal Lorentz factor $\gamma_{\text{bm}}$ as a function of $\gamma_{\text{max}}$ for different value of spectral index $s$. The two panels correspond to $r_e = 10 r_g$, $L = L_{\text{Edd}}$, $M = 10^9 M_\odot$ (left) and $M = 5 M_\odot$ (right).

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Our model gives good agreement with observations of relativistic blob ejection in AGN and microquasars for the ‘disc alone’ solution. Fig. 6 shows that in the most favourable configuration ($s = 1.5$, $\gamma_{\text{max}} \sim 10^6$ and $L = L_{\text{Edd}}$), the Compton rocket effect is able to accelerate pair plasma up to a Lorentz factor $\gamma_{\text{max}} \geq 20$. This value is only gradually reached, the jet being much slower at small distances. One can obtain a higher value for super-Eddington systems. We notice that observations of the faster extragalactic superluminal motion correspond to such a value of the bulk Lorentz factor (Vermeulen & Cohen 1994).

This value is, however, strongly dependent on the spectral index $s$ and the high-energy particle distribution cut-off $\gamma_{\text{max}}$. The precise value of $s$ is not obvious to derive from observations. High-energy spectra show typical X-ray spectral indexes around 0.5, which correspond to $s = 2$. There is very often a spectral break in the MeV range: this could be attributed to a break in the particle distribution, which would correspond to $\gamma_{\text{max}} \sim 10^3$. However, Marcowith et al. (1995) have shown that this break could be very well reproduced by $\gamma-\gamma$ absorption, with an actual particle distribution giving a primary photon spectrum that can extend to much higher energy. High-energy spectra may therefore not be good indicators of the upper cut-off $\gamma_{\text{max}}$. Moreover, the high-energy emission probably takes place at relatively small distances (around $10^2 r_g$) from the centre, well below $\gamma_{\text{crit}}$: the final bulk Lorentz factor is not yet reached at this distance. On the other hand, the detection of photons with energies of at least 30 GeV, and even above TeV for some BL Lacs, implies an upper cut-off $\gamma_{\text{max}} \geq 10^5$.

Radio spectral indexes are also difficult to assess because of synchrotron self-absorption, especially for radio-flat quasars where the observed spectrum probably results from the superposition of many self-absorbed spectra. The optically thin index ranges mostly from 0.5 to 1, which corresponds to $2 \leq s \leq 3$. With reasonable parameters ($2 \leq s \leq 3$ and $\gamma_{\text{max}} \leq 10^5$), our model predicts typical values $4 \leq \gamma_{\text{obs}} \leq 10$, which are those most frequently observed (Vermeulen & Cohen 1994). Moreover, as shown in Section 3.2.5, KN effects avoid strong Compton drag induced by BLR radiation in the vicinity of the central black hole. The fastest superluminal motions may be attributed to those objects for which the BLR is weakest and/or closest to the central object.

On the other hand, the slowest motions ($\gamma_{\text{obs}} \leq 4$) can be obtained in the presence of a rich and extended environment of scattering material, such as BLR clouds and dusty tori. The presence of dust is inferred from an enhancement in the near-infrared in the spectra of some quasars (Barvainis 1987). Nevertheless, a non-thermal infrared component, attributed to synchrotron radiation from the relativistic jet, is also usually observed in radio-loud AGN. This component is generally predominant in flat-spectrum radio quasars (Neugebauer et al. 1986) and is necessary to explain rapid variations observed in infrared flaring objects. We can speculate that those objects with the largest superluminal motion are those where the scattered thermal component is particularly weak. Such ‘non-thermal’ ones should have the lowest terminal Lorentz factors. In conclusion, the diversity among observed superluminal motions can be reproduced by our model by considering the influence of both the plasma acceleration mechanism and the AGN environment.

The observed value of the Lorentz factor of about 2.5 for the two microquasars (GRS 1915+105, Mirabel & Rodriguez 1994, and GRO J1655−40, Hjellming & Rupen 1995) with large spectral indexes (respectively $s \sim 4$, Finoguenov et al. 1994 and $s \sim 4.6$), $L_{\text{ER}}/L_{\text{Edd}}$ is not obvious to derive from observations. High-energy spectra show typical X-ray spectral indexes around 0.5, which would correspond to $\gamma_{\text{max}} \sim 10^3$. However, Harmon et al. (1995) is consistent with our results. We show in Fig. 15 the dependence of $\gamma_{\text{obs}}$ on compactness of the source for $s = 4$ and $s = 5$. We find that observations require $L \sim 0.2-0.3L_{\text{Edd}}$, which is very close to the result by Li & Liang (1996). This result is not strongly affected by the mass of the central black hole if the compactness of the source remains the same. The steep spectrums observed in these two objects assure us that the terminal Lorentz factor is only dependent on the low-energy cut-off of the electron distribution. In this case a more realistic description of the accretion disc emission does not change our results for such spectral indexes (Fig. 7). The maximal value of $\gamma_{\text{obs}}$ is model-dependent, however. The high-energy part of the emission contributes to decrease this value from the one obtained for a standard accretion disc. We find that value of about 6 can be reached in the most favourable case.

Finally, $\gamma_{\text{obs}}$ is much higher for a small-sized accretion disc. We find a value of about 60 in the case of an external radius of $10r_g$ in the case of a supermassive black hole.

5 CONCLUSION

We have considered the bulk acceleration of an electron–positron pair plasma in the vicinity of a central black hole. The acceleration is a result of the Compton rocket effect on the plasma and the radiation force originates from standard accretion disc emission. The pair plasma is continuously reheated by an efficient turbulent mechanism which takes place in the frame of the ‘two-flow’ model. We therefore assumed in our calculations a stationary power-law energy distribution for the pair. We included KN corrections in the computation of the radiation force, and solved the equation of motion numerically. We studied configurations relevant to relativistic motion in AGN and galactic microquasars. The main results of our calculations can be summarized as follows.

(i) For a given luminosity, the terminal Lorentz factor $\gamma_{\text{obs}}$ reaches an absolute maximum due to KN corrections. Values of about 20 can be reached in the extragalactic case, for a sufficiently flat spectrum ($s \sim 1.5$) and accretion at the Eddington rate, which may correspond to the highest observed relativistic motion. For more reasonable plasma parameters ($s \leq 2$ and $\gamma_{\text{max}} \leq 10^5$), the Compton rocket effect can account for the typical value of the terminal Lorentz factor inferred from observations ($\gamma_{\text{obs}}$).

(ii) Scattered radiation from an extended BLR or dusty torus can efficiently brake relativistic motion, even for a high-energy plasma. This Compton drag and weak plasma heating may be responsible for the lowest terminal Lorentz factor observed. The highest superluminal motion could be attributed to objects with a particularly weak diffuse component and very efficient heating.
(iii) For stellar black holes, KN corrections are important, leading to smaller values of the terminal Lorentz factor than for a supermassive black hole. Recent observations of relativistic ejection in Galactic microquasars are consistent with our results.

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APPENDIX A: EQUILIBRIUM LORENTZ FACTOR AND TERMINAL LORENTZ FACTOR IN THE THOMSON REGIME

The Eddington parameters are defined as follows:

\[ J = \frac{1}{4\pi} \int I_\alpha(\Omega) d\Omega d\nu, \]

\[ H = \frac{1}{4\pi} \int \mu I_\alpha(\Omega) d\Omega d\nu, \]

\[ K = \frac{1}{4\pi} \int \mu^2 I_\alpha(\Omega) d\Omega d\nu. \]

Marcowith et al. (1995) show that these parameters can be expressed, in the case of standard accretion disc (Shakura & Sunyaev 1973), as a function of the integral

\[ I(\alpha, \beta, \gamma) = \int_1^{r_c} [1/u^{(1 + e^2)/(e^2 - 1)} - (1/u^{1/2})] du, \]

with \( u = r/r_c, \) \( u_e = r_e/r_c \) and \( e = r/e_c. \) They obtain

\[ J = (L/8\pi^2 r_e^2)(1 - 2b/3)^{-1} e^2 I(2, 3/2, 1), \]

\[ H = (L/8\pi^2 r_e^2)(1 - 2b/3)^{-1} e^{-1} I(2, 2, 1), \]

\[ K = (L/8\pi^2 r_e^2)(1 - 2b/3)^{-1} e^{-1} I(2, 5/2, 1). \]

They study the case \( r_i < z < r_e \) for which \( e < 1 \) and \( u \approx 1. \) They find that the equilibrium Lorentz factor is given by

\[ \gamma_{eq} = e^{1/4} = \frac{\epsilon^{1/4}}{r^{1/4}}. \]  

In the case \( z > r_c \) we have \( e u = r/c < 1 \) for \( 1 < u < u_c. \) We can therefore expand the term \((1 + e^2 u^2)\) into a series:

\[ (1 + e^2 u^2)^{-\beta} = \sum_{k} \beta(\beta + 1) \cdots (\beta + k - 1) (e^2 u^2)^k. \]  

Hence in the case of \( \alpha = 2, \beta(1) = 1 \) and integrating over \( u, \) we obtain

\[ I(2, \beta, 1) = \sum_{k} e^{2k}(\beta(\beta + 1) \cdots (\beta + k - 1)) \]

\[ \times \left( \frac{u_c^{2k-1} - 1}{2k - 1} - \frac{u_c^{2k-2} - 1}{2k - 3/2} \right). \]

The first coefficients in \( \epsilon \) are then

\[ A_0 = \frac{1}{3} - \frac{u_c^{-1}}{3}, \]

\[ A_1 = u_c - 2u_c^{1/2} + 1 \sim u_c, \]

\[ A_2 = \frac{u_c^{3/2}}{3} - \frac{2u_c^{3/2}}{5} + \frac{u_c^{3/2}}{3}. \]

The equivalents are given for \( u_c \gg 1. \) This gives the Eddington parameters

\[ J = (L/8\pi^2 r_e^2)(1 - 2b/3)^{-1} e^2 A_0 - 2A_1 e^2 + 15/8A_2 e^4, \]

\[ H = (L/8\pi^2 r_e^2)(1 - 2b/3)^{-1} e^{-1} A_0 - 2A_1 e + 3A_2 e^2, \]

\[ K = (L/8\pi^2 r_e^2)(1 - 2b/3)^{-1} e^{-1} A_0 - 5/2A_1 e^2 + 35/8A_2 e^4. \]

The equilibrium Lorentz factor is \( \gamma_{eq} = (1 - \beta_{beq})^{-1/2}, \) where \( \beta_{beq} = x - (x^2 - 1)^{-1/2}. \)

\[ x = \frac{J + K}{2H}. \]

Using equations (A9) we find, with \( u_c \gg 1, \)

\[ x = 1 + \frac{1}{8} u_c^{3/2} e^4, \]

\[ \beta_{beq} = 1 - \frac{u_c^{3/2}}{e^2}, \]

and finally

\[ \gamma_{eq} = u_c^{3/2} e = \frac{z}{r_e^{1/4} u_c^{1/4}}. \]

The asymptotic solution of equation (17) is approximately

\[ \gamma_{eq} \approx \gamma_{eq0} / (\gamma_{eq0}^{z_{crit}}), \]

where \( z_{crit} \) is the location where the radiative force becomes too weak to maintain \( \gamma_e \approx \gamma_{eq0} \). This occurs when the evolution of \( \gamma_{eq0} \) i.e. \( \Delta z_0 = \gamma_{eq0} \) (\( \gamma_{eq0} \)) is larger than the evolution of \( \gamma_e \) towards \( \gamma_{eq} \), i.e. \( \Delta z_1 = (\gamma_{eq}/\gamma_{eq0}) \). Using equation (17), one finds

\[ z_{crit} \sim \left( \frac{dF_{\gamma_e}}{d\gamma_e} \right)^{-1}. \]
In the Thomson regime we can use equations (9) and (10), which give in the relativistic case
\[
\frac{dF}{d\gamma_b} = -\frac{\sigma_T}{c}\frac{8\pi}{3}\langle\gamma^2\rangle\frac{dH'}{d\gamma_b} = \frac{2H}{\gamma_b}(1-x/\beta_b),
\] (A16)

With (A9), (A12), (A13), (A14), and ignoring some terms of order unity, one finally obtains
\[
z_{\text{crit}} \sim \left(\frac{q_0}{r_i}\frac{L_0}{r_i} \frac{\langle\gamma^2\rangle}{\langle\gamma^2\rangle} \right)^{1/4},
\] (A17)

and,
\[
\gamma_{b,\text{eq}} \sim \left(\frac{r_i}{r_e}\right)^{3/4}\left(\frac{\langle\gamma^2\rangle}{\langle\gamma^2\rangle}\right)^{1/4}.
\] (A18)

In the case \( r_i < z < r_e \), using equation (A4) one finds
\[
z_{\text{crit}} \sim r_i \left(\frac{L_0}{m_e c^3} r_i \frac{\langle\gamma^2\rangle}{\langle\gamma^2\rangle}\right)^{4/7},
\] (A19)

and finally
\[
\gamma_{b,\text{eq}} \sim \left(\frac{\langle\gamma^2\rangle}{\langle\gamma^2\rangle}\right)^{1/7}.
\] (A20)