Galactic disc dark matter, terrestrial impact cratering and the law of large numbers

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ABSTRACT
A new approach is used to study the old question of missing mass in the Galactic disc. Invoking the law of large numbers, a simple average is formed of the many independent published values of the volume density of solar neighbourhood matter deduced from observational analyses of the gravitational force perpendicular to the Galactic plane. This average value is $0.15 \pm 0.01 M_\odot \text{pc}^{-3}$, which should be compared with $0.10 \pm 0.01 M_\odot \text{pc}^{-3}$ in known visible matter. The estimated 30 per cent dark matter significantly increases the Galactic z-force, thereby reducing the average half-period of vertical oscillation of the Solar system about the Galactic plane to $37 \pm 4$ Myr. A number of previously proposed Galactic mechanisms can both trigger and modulate the rate of gravitational perturbations of the Oort halo of comets. These mechanisms include encounters with massive interstellar clouds and the vertical Galactic tides. Predicted impacts of Oort halo comets on the Earth are broadly compatible with the terrestrial record of large impact craters. A specifically Galactic signature may appear in the $36 \pm 1$ Myr mean periodicity of cratering detected in Grieve’s updated list of the ages of large impact craters.


1 INTRODUCTION
Three long-standing hypotheses about terrestrial catastrophism, originally unrelated to each other, have recently been forged into a new tool for investigating the geologic record. These include (1) the old idea of episodic geological upheavals and biological mass extinctions, (2) their subsequent attribution to either comet or asteroid impacts on the Earth, and (3) their more recent association with the movement of the Solar system through the Galaxy.

The modern phase of investigation started with the discovery by Alvarez et al. (1980) of an iridium excess at the Cretaceous/Tertiary (K/T) boundary, 65 Myr ago, which pointed to a huge meteoroid impact at that time. The K/T mass extinctions were attributed to the giant impact. Raup & Sepkoski (1984) later found that the known record of marine mass extinctions might be nearly periodic, each cycle lasting 26 Myr. They inquired whether an astronomical cause was responsible. After Rampino demonstrated that an alternative period of 30 Myr might actually fit the data better, the present author proposed an astronomical theory explaining the mass extinctions as the result of Galaxy-induced periodic comet showers, which he found to be possibly reflected in an apparent periodicity of 31 Myr in dated terrestrial impact craters (Rampino & Stothers 1984a). Soon thereafter, Alvarez & Muller (1984) detected a comparable period, 28 Myr, in the subset of very large impact craters. Within a few months, a variety of astronomical theories to explain periodic mass extinctions had been proposed, involving different Galactic, stellar and planetary perturbers (see e.g. the review papers in Smoluchowski, Bahcall & Matthews 1986).

Of these astronomical theories, only the Galactic ones survive now with any reasonable credibility, and they are still being modified. In them, the basic clock is the space motion of the Solar system perpendicular to the Galactic plane, the time between successive plane crossings being somewhat uncertain but probably lying somewhere in the range 30–44 Myr (Matese et al. 1995). The principal mechanism causing large-body impacts on the Earth is gravitational perturbations of the Solar system’s halo of comets, known as the Oort (1950) cloud. Perturbations are either more frequent or stronger near the Galactic mid-plane, where matter is more highly concentrated.

Differences among these theories lie mainly in their identifications of what the most important quasi-periodic perturbers might be. Suggestions have included the following: intermediate-sized and large-sized molecular clouds (Rampino & Stothers 1984a); giant molecular clouds (Clube & Napier 1984); dark matter (Stothers 1984; Bailey, Wilkinson & Wolfendale 1987); the vertical Galactic tidal shock (Scalo & Smoluchowski 1984); and the adiabatically varying vertical Galactic tide (Napier 1987; Matese et al. 1995). All versions of the Galactic model predict infrequent,
2 LAW OF LARGE NUMBERS

The law of large numbers is a simple, although sometimes misunderstood, result from probability theory, most familiar to physicists from its application to practical problems in statistics. The law states that if the expectation, or the true mean \( \mu \), exists for a sequence of \( N \) independent random variables \( (x_1, \ldots, x_N) \) having a common distribution, the probability that the average of the random variables, or the sample mean \( \bar{x} \), will differ from \( \mu \) by less than an arbitrarily prescribed \( \varepsilon > 0 \) tends to 1 as \( N \to \infty \) (Feller 1957).

Physicists usually assume the law to mean that the average of \( N \) independent measurements is bound to be near \( \mu \). Although, theoretically, \( \bar{x} \) is not guaranteed by the law of large numbers to be close to \( \mu \) for all values of \( N \), statistical applications of the law do not usually lead to unpleasant surprises if \( N \) is sufficiently large. An important generalization of the law demonstrates that it remains valid in the case where the random variables do not have the same distribution, provided that the means and variances of the different distributions exist. Closely related to the law of large numbers is the central limit theorem, which states that, if both the mean \( \mu \) and the variance \( \sigma^2 \) exist, then the distribution of \( \bar{x} \) approaches the normal, or Gaussian, distribution with mean \( \mu \) and variance \( \sigma^2/N \) as \( N \to \infty \).

Physicists often interpret this to imply that \( N \) independent measurements have an approximately Gaussian distribution, on the supposition that every deviation from the mean is the result of a large number of independent causes, each contributing a small portion of the total deviation (Fraser 1958).

Nevertheless, physicists are sometimes reluctant to trust the law of large numbers in real applications in which the data do not possess high quality (the measurement errors are large). Even when they feel comfortable in averaging their own measurements, they often balk at including other physicists’ measurements, especially if the latter were obtained by different analytical methods. Alternatively, they may regard some or all of the measurements as not sufficiently independent if large systematic errors are believed to be present. In the latter case, if independent methods of measurement, having different systematic errors, are available, an average can be formed for each method separately, and then the averages based on the different methods can themselves be averaged, so that the systematic errors become accidental errors. In general, the average of enough measurements will reduce the effect of even large accidental errors. Feller has given some striking examples of the power of the various laws related to large numbers to reach correct conclusions, even when these results are counterintuitive to most scientists (and even to many statisticians).

2.1 Use of the law in an example from astronomy

To illustrate the remarkable power of the law of large numbers in an important case where astronomers have generally taken a strong stance against using it, the classical Cepheid period–luminosity relation will be examined. Most astronomers, instead of using the large number of past determinations of this relation to derive a reliable statistical result, have sought instead a ‘better’ method or ‘better’ data. This is not a wrong approach, but may be unnecessary. Such an example will prove relevant when we later discuss the local Galactic mass density.

The period–luminosity relation for Galactic Cepheids, based on an average of many published determinations of it, is given by (Stothers 1983; Carson & Stothers 1988)

\[
M_V = -2.85 \log P - 1.34, \\
\pm 0.03 \quad \pm 0.03
\]  

(1)

where \( M_V \) is the time-averaged visual absolute magnitude, \( P \) is the period in days, and errors quoted are standard errors of the mean, including both accidental and systematic errors. For the mean slope, five independent methods and a total of 20 determinations were used, while, for the mean zero-point, four independent methods and a total of 26 determinations were used. Although none of the methods depended on trigonometric parallaxes, or otherwise produced individually accurate results, the total number of determinations is large, which greatly brings down the standard errors of the mean slope and mean zero-point.

Recently, a highly accurate result has been derived from Hipparcos Catalogue trigonometric parallaxes for 20 Galactic Cepheids (Feast & Catchpole 1997; Oudmaijer, Groenewegen & Schrijver 1998):

\[
M_V = -2.81 \log P - 1.36, \\
\pm 0.05 \quad \pm 0.07
\]  

(2)

Although the slope of \(-2.81\) is an adopted one from the Large Magellanic Cloud, the zero-point is fully independent and clearly fundamental. Over the whole range of observed periods, equations (1) and (2) give values of \( M_V \) that differ from each other by only 0.02 mag. The smallness of the difference both confirms and illustrates the power of the law of large numbers in an astronomical application where the true mean is now quite accurately known.

2.2 Misuse of the law in an example from geology

Many authors who have speculated that impact cratering gives rise to detectable secondary geologic phenomena have looked for corresponding periodicities in the geologic record. Unless the impact cratering periodicity is considered known a priori, however, a detected geologic period has not usually been found to be statistically significant. A prominent recent exception is the finding of a significant (at the 5 per cent level) a posteriori period in major geologic events by Rampino & Caldeira (1992, 1993) (see also Clube & Napier 1996). Their claimed value for the period, 27 Myr, however, is at variance with many earlier studies suggesting values closer to 33 \( \pm 3 \) Myr (e.g. Rampino & Stothers 1984b, 1986), and it also conflicts with the best-fitting impact cratering period of 36 \( \pm 1 \) Myr (see below). In fact, large impact craters correlate in age very well statistically with major geologic events, such as mass extinctions at stage boundaries (Hildebrand et al. 1991; Stothers 1993; Matsumoto & Kubotani 1996).

What Rampino & Caldeira (1992, 1993) have done is to identify and date 77 major geologic events of various kinds during the past 250 Myr. In an attempt to build up the statistics, they combined all
of the event data, and binned the 77 ages in small time intervals. They then smoothed the resulting histogram and subjected the final curve to Fourier spectral analysis. Different types of geologic events, however, are very likely to be physically related. Consequently, many of these ages are not independent, and so duplication, as well as close bunching, of the ages occurs in the composite time series. The approach adopted by Rampino & Caldeira, therefore, artificially exaggerates by a large factor the number of degrees of freedom that were needed for their Monte Carlo simulations based on random time series (d.f. = 77). The null hypothesis of uniformly distributed random times can then hardly fail to be rejected. Instead, one should adopt as independent observations just the 11 highly conspicuous time peaks in Rampino & Caldeira’s histogram. When this is done, their periodicity of 27 Myr loses all statistical significance (the significance level becomes ~40 per cent). The 11 time peaks are really the product of only two wholly independent time series: the Palaeozoic stage boundary ages (selected by the major mass extinctions, orogenies, etc.) and the volcanic flood basalt ages. Both series, however, are so severely affected by dating uncertainties that one may derive either no period at all or, at best, a weakly possible period of 24–33 Myr for major mass extinctions (Stothers 1989) and 31–33 Myr for volcanic flood basalt (Rampino & Stothers 1988) if the estimated errors of the ages are included in the time series analysis. A more serious problem arises if one needs specifically to select events that might be due to comet or asteroid impacts. The standard 11 events could conceivably shrink to one or a few. The approach adopted by Rampino & Caldeira, therefore, especially since the Oort cloud (Oort 1950) was discovered, has led to a series of geologic studies which have conjectured that major mass extinctions, orogenies, etc.) and the volcanic flood basalt ages, etc., but only in a ‘period’ derived from such proxy geologic data could range anywhere from ~5 to ~200 Myr. Although a number of large impact craters, major mass extinctions and flood basalts do correlate in age, it seems preferable to deal only with the impact crater ages themselves (Rampino & Stothers 1998; Matase et al. 1998).

### 3 LOCAL MASS DENSITY IN THE GALAXY

As the Solar system moves up and down through the Galactic plane on its orbit through the Galaxy, its maximum attain height above or below the plane is probably always sufficient to moderate that the Galactic restoring force on it remains effectively linear in $z$. In that case, the time between plane crossings (the half-period of vertical oscillation) is

$$P_{1/2} = (\pi^2/4G\rho_0)^{1/2},$$

(3)

where $\rho_0$ represents the mean volume density of matter near the plane (Chandrasekhar 1942). Numerically, if $\rho_0$ is in units of M$_\odot$ pc$^{-3}$, $P_{1/2} = 13.2\rho_0^{1/2}$ Myr. The Solar system today lies very close to the Galactic plane.

Determination of the volume density in the solar neighbourhood can be done either by counting the visible matter, consisting of stars, gas and dust, or by analysing the positions and velocities of tracer stars in the direction perpendicular to the Galactic plane. The known matter, according to the most recent studies, amounts to no more than $0.10 \pm 0.01$ M$_\odot$ pc$^{-3}$ (Bahcall 1984a; Gould, Bahcall & Flynn 1996; Crezé et al. 1998). Analysis of the observed response of tracer stars to the Galactic $z$-force, using the Boltzmann and Poisson equations combined in various forms, has given widely varying results, both for the same class of stars and among different classes of stars, ever since Oort’s (1932) classic study. For the older stars, the assumption that the stars are dynamically well mixed in the $z$-direction must be made (Oort 1932), while for the younger ones, like the O and B stars, their ages can be used explicitly (Jöeveer 1972) or else their distribution in phase space can be examined for approximate uniformity on the assumption that the gas and dust out of which they recently formed was already roughly relaxed (Stothers & Tech 1964; von Hoerner 1966; Flynn 1987).

Because the variability of the latest determinations of the Oort limit $\rho_0$ remains as high as in the earlier work (Table 1), the most recent values will be given merely the same weight as the others. Systematic and accidental errors are known to be large in all of these determinations, and often cannot even be estimated. Assumptions and methodologies themselves are imperfect, even if the individual data have high accuracy. The problem at hand, therefore, is an ideal task for the law of large numbers. All 28 determinations listed in Table 1 show an approximately Gaussian distribution with a mean of $0.141 \pm 0.010$ M$_\odot$ pc$^{-3}$; the median is also 0.14 M$_\odot$ pc$^{-3}$. Fig. 1 shows the distribution. We find that Krischunas’s (1977) earlier list of only 14 determinations likewise gives a mean of $0.149 \pm 0.014$ M$_\odot$ pc$^{-3}$. It can be argued that many of these determinations do not depend on fully independent data sets. To remove such a bias, means are computed for each spectral class of stars separately, in those cases in which spectral classes were not grouped together in the original dynamical analyses. Six independent spectral classes can be used: O and B; A; F; classical Cepheids; K III; and M III. The mean for each spectral class being accorded equal weight, the mean of the mean volume densities becomes $0.148 \pm 0.016$ M$_\odot$ pc$^{-3}$. Clearly, we are statistically justified in believing that the true mean must lie close to $0.15 \pm 0.01$ M$_\odot$ pc$^{-3}$.

Two consequences of this follow. First, the half-period of vertical oscillation for a low-velocity object like the Solar system in our local neighbourhood must be $34 \pm 1$ Myr. Under plausible assumptions about the structure of our Galaxy, the time average of $P_{1/2}$

<table>
<thead>
<tr>
<th>$\rho_0$ (M$_\odot$ pc$^{-3}$)</th>
<th>Stars</th>
<th>Reference</th>
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<tr>
<td>0.09</td>
<td>A–G V; G I–III; K–M III</td>
<td>Oort 1932</td>
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<td>0.23</td>
<td>B–G V; G–K III</td>
<td>Nahon 1957</td>
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<td>0.18</td>
<td>A</td>
<td>Woolley 1957</td>
</tr>
<tr>
<td>0.13</td>
<td>K III</td>
<td>Hill 1960</td>
</tr>
<tr>
<td>0.15</td>
<td>K III</td>
<td>Oort 1960</td>
</tr>
<tr>
<td>0.15</td>
<td>K III</td>
<td>Yasuda 1961</td>
</tr>
<tr>
<td>0.075</td>
<td>K III; F V</td>
<td>Eelsalu 1961</td>
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<td>0.14</td>
<td>A0</td>
<td>Jones 1962</td>
</tr>
<tr>
<td>0.11</td>
<td>O and B</td>
<td>Stothers &amp; Tech 1964</td>
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<td>A</td>
<td>Woolley &amp; Stewart 1967</td>
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<td>Perry 1969; Eegn 1969</td>
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<td>K III</td>
<td>Turon Lacarrieu 1971</td>
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<td>0.085</td>
<td>B8–B9</td>
<td>Jöeveer 1972</td>
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<td>0.235</td>
<td>A</td>
<td>Gould &amp; Vandervoort 1972</td>
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<td>0.21</td>
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<td>Jones 1972</td>
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<td>G–K III, IV</td>
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<td>A and F</td>
<td>Hill, Hilditch &amp; Barnes 1979</td>
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<td>0.185</td>
<td>F V</td>
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<tr>
<td>0.076</td>
<td>A and F</td>
<td>Crézé et al. 1998</td>
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during the orbit of the Solar system around the Galactic Centre, which takes 250 Myr, would be 0—20 per cent larger than this (Stothers 1985; Matee et al. 1995). Accordingly, the expected mean value of \( P_{1/2} \) over the past 250 Myr would be 37 ± 4 Myr.

Secondly, some kind of disc dark matter must apparently exist, because the most probable value of the Oort limit exceeds the visible mass density by 0.05 ± 0.01 \( M_\odot \) pc\(^{-3}\). A conclusion similar to this has already been reached in most, although not all, of the individual Galactic z-force analyses published in the past (Table 1), notably in the work of Bahcall. Since the numbers of faint disc stars are unlikely to have been underestimated by this much (Trimble 1987; Gould et al. 1996; Gould, Bahcall & Flynn 1997; Kirkpatrick et al. 1998), the best candidate for dark matter is probably more or bigger cold molecular clouds (Stothers 1984; Lequeux, Allen & Guilloteau 1993). Being inevitably relatively few in number, such clouds would not violate the upper mass limit of 2 \( M_\odot \) for any unseen individual disc stars that is implied by the existence of undisrupted wide binary star systems (Bahcall, Hut & Tremaine 1985).

The local Galactic surface density, \( \Sigma_0 \), can also be derived from dynamical studies of tracer stars and compared with counts of visible matter. However, \( \Sigma_0 \) is a more difficult quantity to evaluate accurately than \( \rho_0 \), in part because of its sensitivity to the uncertain vertical scaleheights of the various objects studied. Results scatter widely, \( \Sigma_0 \) in \( M_\odot \) pc\(^{-2}\) being estimated as \( \sim \)47 (Kuijken & Gilmore 1989a, 1991), \( \sim \)53 (Gould 1990; Flynn & Fuchs 1994), \( \sim \)67 (Bahcall 1984a,b) and \( \sim \)84 (Bahcall, Flynn & Gould 1992). In view of the fact that the visible matter shows only \( \sim \)40 \( M_\odot \) pc\(^{-2}\) (Gould et al. 1997), some dark matter seems, again, to be indicated.

### 4 METHOD OF ANALYSING POINT SERIES FOR THE CRATER AGES

A commonly used method of analysing point series is the one sometimes referred to as Broadbent’s method. Strictly speaking, Broadbent (1955, 1956) considered only point series with a known starting position, and his analytical criterion for statistical significance is valid only for infinite \( N \). In his method, the deviation \( d_i \) of each observed time \( t_i \) from the nearest predicted time in a regular time series of period \( P \) is calculated, and the lumped variance is then computed as a goodness-of-fit statistic.

In several independent versions of this method, other authors have made generalizations to include an unknown starting position (the phase). Although often taken for granted, this improvement is non-trivial, as the chosen statistic used to determine the phase for a specified period has widely varied: a root-mean-square (rms) deviation (Stothers 1979), a mean deviation that depends on the signs of the individual deviations (Raup & Sepkoski 1984), a median absolute deviation (Stothers 1994), and a mean absolute deviation (Stothers 1996). Although only the choice of a rms deviation yields a fully consistent, maximum-likelihood (least-squares) solution, the other choices are expected to give equivalently good solutions, except for Raup & Sepkoski’s choice, which can sometimes lead to multiple solutions (Stothers 1991). To assign the starting position to the first observed time, as in Broadbent’s approach (Grieve et al. 1985), biases the solution.

The smallest value of the chosen statistic at a given period \( P \) determines the best-fitting phase for that period. In so-called linear spectral analysis (Stothers 1979, 1991), the rms deviation divided by \( P \) is subtracted from a constant to yield the ‘residuals index’,

\[
RI = \frac{1}{N} (N^2 - 1) (12N^2)^{1/2} - \left[ \frac{1}{N^2} \sum_{i=1}^{N} (d_i/P)^2 \right]^{1/2},
\]

which is a sort of analogue of Fourier spectral power. In this version of the method, a plot of \( RI \) as a function of \( P \) represents the spectrum.

Statistical significance of any selected period can be tested for by using Broadbent’s (1955, 1956) criterion or, better, a semi-analytic modification of his criterion for arbitrary \( N \) (Stothers 1991). The most realistic approach, however, is to perform Monte Carlo simulations in exactly the same way as was used to analyse the observed time series.

### 5 IMPACT CRATERS

The many published time series analyses of dated impact craters have made use of the age lists compiled (and continually updated) by R. A. F. Grieve or, sometimes, the semi-independent age lists that are published by others. The derived ages of the impact craters, determined at different times, vary greatly in quality, depending on the dating method used, namely radiometric, fission-track or stratigraphic. In addition, selection effects have led to more young craters than old being discovered (simply as a result of preservation bias), as well as there being a strong concentration of craters located on stable continental cratons and a prevalence of long-lasting large craters. Despite these obvious problems with the data, times series analyses have been performed by using some version of either linear spectral analysis or Fourier spectral analysis. The best-fitting period over the past 250 Myr has almost always turned out to lie in the range 27–32 Myr (Rampino & Stothers 1984a,b, 1986; Alvarez & Muller 1984; Muller 1986; Shoemaker & Wolfe 1986; Sepkoski & Raup 1986; Shuter & Klatt 1986; Chen, Liu & Zheng 1986; Trefil & Raup 1987; Napier 1987; Stothers 1988; Clube & Napier 1996). Widening the time window to 600 Myr ago produces essentially the same result (Rampino & Stothers 1984b; Yabushita 1991, 1992a,b, 1994, 1996a). In three analyses, however, the best-fitting period turned out to be 34–36 Myr (Durrheim & Reimold...
1986, 1987; Stothers & Rampino 1990). This dichotomy between possible periods of $30$ and $35$ Myr has evidently stemmed from the different selections of craters as well as of crater ages in the various studies (see fig. 2 of Stothers 1988). In all of these studies except for two (Alvarez & Muller 1984; Clube & Napier 1996), the most recent epoch (or phase) has always fallen near $563$ Myr ago.

Some authors deny the reality of any periodicity, mainly on the grounds of the known crater age errors, data gaps, shortness of the record, and unavoidable presence of non-periodic components in the data (e.g. random asteroid impacts) (Grieve et al. 1985, 1988; Weissman 1985; Tremaine & Tremaine 1986; Heisler & Tremaine 1989; Fogg 1989; Baksi 1990; Grieve 1991; Grieve & Shoemaker 1994; Grieve & Pesonen 1996; Yabushita 1996b). Nevertheless, actual Monte Carlo simulations that have included the presence of random components and large age errors (Stothers 1985, 1988; Treffil & Raup 1987) show a roughly even chance of detecting a hidden crater periodicity, which is a result not seriously in disagreement with more recent simulations (Heisler & Tremaine 1989; Fogg 1989; Yabushita 1992b; Valtonen et al. 1995). Jetsu (1997), on the other hand, has argued that the apparent periodicity arises from the rounding to the nearest integer of many of the crater ages; this possibility, however, has been previously tested and ruled out by detecting the rounding signal directly (Rampino & Stothers 1984a) and also by slightly randomizing the published crater ages within their estimated analytical errors (Stothers 1988). All authors agree that a cratering periodicity, if it exists at all, is likely to be statistically significant only as an a priori period, i.e. as one that is already known from some other source of information.

To re-examine the periodicity question with the most recent data, the list of well-dated impact craters published by Grieve & Pesonen (1996) will be used. This compilation contains craters with diameters $D \geq 5$ km that formed within the last 250 Myr and have analytical age errors of $\approx 20$ Myr. To it can be added the Chesapeake crater (90 km, $35 \pm 3$ Myr), the Mjølnir crater (35 km, $142 \pm 2.6$ Myr), the Morokweng crater ($100$ km, $145 \pm 3$ Myr), and also corrections for the ages of the Rochechouart (214 $\pm 8$ Myr) and Popigai (35.7 $\pm 0.8$ Myr) craters (Grieve 1996, 1997). Rejecting craters with analytical age errors exceeding 9 Myr, we have 31 craters in all. 11 of them show $D \geq 35$ km, and five have $D \approx 90$ km; these large craters follow a nearly stationary distribution in time. The smaller craters populate well only the time interval from the present back to 150 Myr ago, as only one of them is older than this value. Fig. 2 shows the two distributions in time.

A large number of linear spectral analyses of the crater ages have been performed for various sets of craters with $D \approx D_0$, starting with $D_0 = 5$ km (the full set) and then incrementing $D_0$ in steps of 5 km. Ignoring trial periods shorter than 22 Myr, which belong in the noise band (Alvarez & Muller 1984; Treffil & Raup 1987; Stothers 1988), we find high spectral peaks only at periods of $36 \pm 1$ and $29 \pm 1$ Myr. The residuals index, $RI$, for these spectral peaks is plotted as a function of $D_0$ in Fig. 3. Notice that, contrary to most earlier work, the dominant period is here the longer one, being especially prominent in the case of the largest craters. Although the many smaller craters tend to shift the best-fitting period to $\sim 30$ Myr (see also Yabushita 1996a,b), the largest craters constitute the ones that would form principally from comet impacts and so would be expected to reflect best the quasi-periodic comet showers (Shoemaker, Wolfe & Shoemaker 1990; Matese et al. 1998; Rampino & Stothers 1998).

**Figure 2.** Number of impact craters per interval of 8 Myr, for 31 well-dated craters with $D \geq 5$ km. Statistics for craters with $D \geq 35$ km are indicated in black. Arrows mark the maxima of the best-fitting mean cycle for craters with $D \geq 35$ km.

**Figure 3.** Residuals index, $RI$, of the highest and second-highest spectral peaks in time series analyses for impact craters younger than 250 Myr. The analyses are based on craters with diameters $D \approx D_0$, as indicated along the abscissa. Periods of the two highest spectral peaks are noted.
6 DISCUSSION AND CONCLUSIONS

Our revised estimate of the volume density of local Galactic matter, \( \rho_0 = 0.15 \pm 0.01 \, \text{M}_\odot \, \text{pc}^{-3} \), leads us to predict a quasi-period of 37 \pm 4 Myr for comet impacts on the Earth. This prediction is at least consistent with the mean periodicity of 36 \pm 1 Myr that emerges from Grieve’s list of the largest dated impact craters. The derived cratering periodicity, although it is not formally statistically significant, is definitely robust against data selection.

Two versions of the Galactic model for terrestrial impact cratering are currently viable, however. Which one is likely to be the more important? Although both molecular cloud encounters and the vertical Galactic tides must gravitationally perturb the Oort comet halo, only a few comets are expected actually to strike the Earth as a result in either case (Stothers 1988; Stagg & Bailey 1989; Valtonen et al. 1995). An important observable difference, however, is anticipated: the adiabatically varying Galactic tide should deliver comets individually, whereas a close molecular cloud encounter should trigger a comet shower. Clusters of similar-age impact craters have long been recognized in the cratering record (Seyfert & Sirk 1979; Rampino & Stothers 1984a; Hut et al. 1987), and Grieve’s new catalogue specifically shows two or more large craters at \( \sim 1, 35, 65, 74 \) and 90 Myr ago. Yet the record is nowhere near as periodic as is sometimes claimed (Shoemaker & Wolfe 1986), and this semirandom aspect seems compatible with either of the two Galactic theories, both of which predict considerable stochasticity in the cratering record, since the triggering perturbations do not take place exclusively at the Galactic mid-plane.

If, however, it should happen that the 30 per cent disc dark matter implied by our new value of \( \rho_0 \) consists of objects of small mass (e.g. brown dwarfs) distributed fairly far above and below the Galactic plane, the tidal model would not yield a noticeably large variation in the Oort halo comet flux, which is proportional to the total local Galactic mass density (Stothers 1984; Morris & Muller 1986; Heisler & Tremaine 1986; Torbett 1986). On the other hand, the disc dark matter more probably consists of additional, cold molecular clouds distributed in a flat subsystem like the known molecular clouds.

The main new datum in this paper, the revised value of the local Galactic mass density, will probably be regarded as controversial, although it should not be. Use of the law of large numbers is tantamount to a vote by popular referendum, nor is it unrelated to the way in which the real world works. It simply provides a best bet in a controversial situation. This argument, however, is not meant to discourage further observational and theoretical studies to determine a more precise value of \( \rho_0 \).

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Galactic disc dark matter

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