ABSTRACT

Lorimer et al. have recently reported that the spin-down age ($\sim 7 \times 10^9$ yr) of the low-mass binary pulsar PSR J1012+5307 is much higher than the cooling age ($3 \times 10^8$ yr) of its white dwarf companion. The proposed solutions for this discrepancy are outlined and discussed. In particular, the revised cooling age estimate proposed by Alberts et al. agrees with data from other low-mass binary pulsar systems if a transition to the ‘classical’ cooling regime occurs between $\sim 0.14$ and $\sim 0.28$ $M_\odot$. If this transition is excluded, PSR J1012+5307 seems to have finished its accretion phase far from the spin-up line.

Key words: binaries: close – stars: magnetic fields – stars: neutron – pulsars: individual: PSR J1012+5307 – X-rays: stars.

1 INTRODUCTION: THE FORMATION SCENARIO

The currently accepted idea for the formation of low-mass binary pulsars is that these systems are the end-points of close binaries, in which an old pulsar is spun up by accretion from an evolved companion (see Bhattacharaya 1995 for a review). Once the companion star overflows its Roche lobe, it starts transferring mass (and angular momentum) to the neutron star, thereby spinning it up (recycling). The accretion occurs via a disc, which is truncated at the radius $r_{\text{in}}$ where the ram pressure of the accreting matter balances the magnetic pressure of the pulsar magnetic field. At this radius the matter is loaded on to the field lines and is funnelled on to the neutron star magnetic poles. Accretion is possible, provided that the centrifugal force experienced by the matter forced to corotate with the magnetic field lines is less than the gravitational force of the neutron star: this implies that the neutron star spin $\omega_{\text{spin}}$ cannot exceed the Keplerian angular frequency $\omega_K$. If enough angular momentum is accreted, $\omega_{\text{spin}}$ reaches a value very close to $\omega_K$ ($\geq 90$ per cent) and further accretion proceeds with the neutron star spin ‘locked’ at this equilibrium value. The equilibrium period can be written as (see e.g. Burderi, King & Wynn 1996):

$$P_{\text{eq}} = 5.3 \times 10^{-4} \Omega_i^{-1} \phi^{3/2} M^{-5/3} \mu_{26}^{6/7} R_6^{7/3} m^{-3/7} \text{s}.$$  \hspace{1cm} (1)

Here the ‘fastness parameter’ $\Omega_i$ (the ratio between the equilibrium angular frequency of the neutron star and $\omega_K$) is close to 1 ($\approx 0.9$; Ghosh & Lamb 1991), $\phi \sim 1$ depends on the details of the accretion process, $M$ is the neutron star mass in solar masses, $\mu_{26}$ is the magnetic moment in units of $10^{26}$ G cm$^3$, $R_6$ is the neutron star radius in units of $10^6$ cm and $m$ is the accretion rate in Eddington units, i.e. $1.5 \times 10^{-8} R_6 \eta^{-1} M_\odot$ yr$^{-1}$ (see e.g. Bhattacharaya & van den Heuvel 1991), where $\eta^{-1}$ is the ratio between the specific energy released and the specific binding energy at the neutron star surface. After the whole envelope has overflowed, the helium core of the companion is left as a very low-mass white dwarf orbiting a radio pulsar (low-mass binary pulsar system). At this point the pulsar spin period ($P_{\text{eq}}$) starts to decay under dipole radiation.

The time elapsed from this epoch can be estimated from the spin-down age,

$$t_{\text{spindown}} = \tau \times \left[2/(n - 1)\right] \left[1 - (P_0/P)^{(n-1)}\right],$$  \hspace{1cm} (2)

where $\tau = P/(2\dot{P})$ is the spin-down time-scale ($P$ and $\dot{P}$ are the spin period and its derivative), $P_0$ is the spin at the end of the accretion phase and $n$ ($\sim 3$ for dipole radiation) is the braking index. The model discussed above assumes that $P_0 = P_{\text{eq}}$.

Another estimator of the age is the cooling age of the white dwarf, $t_{\text{cool}}$, because the cooling starts once the accretion has stopped. The white dwarf parameters can be determined from optical photometry and spectroscopy. Adopting a cooling model, $t_{\text{cool}}$ can be computed. The whole formation picture can therefore be checked from the comparison of $t_{\text{spindown}}$ and $t_{\text{cool}}$.

2 THE CASE OF PSR J1012+5307

This binary system contains the millisecond pulsar PSR J1012+5307 and a low-mass companion in a circular orbit (the mass function gives a companion mass of $0.11\sin i$ $M_\odot$, for a 1.4-$M_\odot$ neutron star, where $i$ is the inclination of the system; Nicastro et al. 1995). Measurements of optical colours and astrometry show that the low-mass companion is a white dwarf. Taking the derived luminosity and temperature for the white dwarf, and adopting the cooling model of Iben & Tutukov (1986), the cooling age has been estimated as $t_{\text{cool}} \sim 3 \times 10^8$ yr (Lorimer et al. 1995). In the same paper, the true period derivative ($\dot{P} = 1.19 \times 10^{-20}$) is determined after correction for the transverse motion of the system (inferred from astrometric measures of the white dwarf proper motion and interstellar scintillation). The spin-down time-scale is $\tau \sim 7 \times 10^9$ yr. For J1012+5307, $P_{\text{eq}} \ll P$, so $t_{\text{spindown}} \sim \tau$. The significant
discrepancy between the spin-down age and the cooling age poses a serious difficulty for the recycling model. There are two possible solutions to this age problem.

(i) Dropping the assumption that the pulsar spins in equilibrium at the end of the accretion phase. In this case $P_0 \gg P_{eq}$ and $t_{\text{spin-down}} \ll \tau$. Because the equilibrium period depends on the accretion rate, if the accretion rate is low enough it is possible that $P_{eq} \sim P$, and again $t_{\text{spin-down}} \ll \tau$. For J1012+5307 we have $P_{eq} \sim P$ for $m \sim 2 \times 10^{-5}$. Hence for accretion rates not too far from the Eddington value (typical of most low-mass X-ray binaries, which are believed to be the progenitors of binary millisecond pulsars), $P_{eq} \ll P$. J1012+5307 is then a young system of age $t_{\text{cool}} \sim 3 \times 10^6$ yr.

(ii) Revising the cooling age. If the adopted cooling model underestimates the age of the white dwarf by a factor ~20, J1012+5307 is actually an old system, and the pulsar was spinning in equilibrium at the end of the accretion phase.

A solution of type (i) is the scenario in which the neutron star is formed via accretion-induced collapse of a white dwarf driven over the Chandrasekhar mass limit by accretion from the companion. In this case $P_0$ is no longer constrained to lie close to the equilibrium value, provided that the mass (and hence the angular momentum) accreted after the formation of the neutron star is too low (a few $10^{-5} M_\odot$) to be able to spin up the neutron star to its equilibrium value. This scenario requires an extremely well-timed collapse.

A different solution of type (i) has been recently proposed by Burderi et al. (1996). It is shown that a non-equilibrium spin rate at the end of the accretion phase is very likely if the low value of the neutron star magnetic moment results from accretion-induced decay of the magnetic moment, because most of the transferred mass is then used to reduce the field rather than spin up the pulsar.

To yield a solution of type (ii), Alberts et al. (1996) have recently suggested that most of the earlier calculations underestimated the cooling age of the white dwarf. They revised a cooling model originally proposed by Webbink (1975) in which the cooling timescales of low-mass ($\lesssim 0.3 M_\odot$) helium white dwarfs are substantially longer than those predicted by the model of Iben & Tutukov (1986). The latter predict cooling ages very close to the classical result of Mestel (1952) at a few $10^6$ yr after the end of the accretion phase.

In the following section, we discuss these cooling models and compare their predictions with the results obtained from optical observations of the white dwarf companions of low-mass binary pulsars.

3 COOLING MODELS AND OBSERVED LOW-MASS HELIUM WHITE DWARFS

The major difference between the cooling models for a low-mass ($\lesssim 0.3 M_\odot$) helium white dwarf in a close binary system, proposed by Webbink (1975) and Alberts et al. (1996) on the one hand and by Iben & Tutukov (1986) on the other, is the fate of the tenuous envelope (mass $=1.6 \times 10^{-3} M_\odot$), almost entirely composed of hydrogen, remaining at the end of the accretion phase. Two scenarios are possible.

(1) Stable hydrogen shell-burning continues for some time, lengthening $t_{\text{cool}}$ (Webbink 1975 and Alberts et al. 1996).

(2) Very short thermal flashes (~100 yr) occur at the beginning of the hydrogen-burning shell phase. The sudden expansion of the shell causes the loss of most of the hydrogen via Roche-lobe overflow; the remaining envelope mass is too low to sustain stable shell-burning. One recovers the classical result of Mestel (1952) $t_{\text{cool}} \propto A^{-1}(M_{\text{WD}}/L)^{3/2}$, where $M_{\text{WD}}$, $L$ are the white dwarf mass and luminosity and $A$ is the mean atomic number of the interior for a carbon ($A = 12$) white dwarf (model of Iben & Tutukov 1986).

The stable hydrogen shell-burning mentioned in scenario (1) is possible provided that the mass of the white dwarf does not exceed $0.25 M_\odot$ (Alberts et al. 1996). To compare the predictions of scenarios (1) and (2) with observations, therefore, the determination of the white dwarf mass is of crucial importance. Of the different methods for finding white dwarf masses in low-mass binary pulsars (see Bergeron, Saffer & Liebert 1992 for a review), four are relevant for the following discussion.

(i) Computation of the mass function from timing analysis of the pulse arrival times (e.g. Taylor & Weisberg 1989). Adopting a neutron star mass of $1.4 M_\odot$, a lower limit for the mass is obtained ($M_{\text{WD}}/\sin i$), where $i$ is the inclination of the system. As the probability of observing a binary system at an inclination $i$ less than a given value $\tilde{i}$ is $P(\tilde{i} < i) = 1 - \cos \tilde{i}$, an upper limit on the mass can be derived.

(ii) Using the magnitudes of the white dwarf from optical photometry (typically $V$, $B$, $R$ and $I$ bands) and the white dwarf effective temperature from spectroscopy, and taking the distance from the dispersion measure in the radio data, the timing parallax (Camilo, Foster & Wolszczan 1994) or the apparent change in the orbital period caused by the Doppler effect from proper motion (Bell & Bailes 1996), one obtains the white dwarf radius. The radius in turn implies a mass via the zero-temperature mass–radius relation of Hamada & Salpeter (1961) [some approximate estimates are available for the bloating of the white dwarf induced by the temperature, e.g. the Wood (1995) models for white dwarfs with pure carbon cores and thick hydrogen atmospheres]. The major uncertainty in this method is the distance determination.

(iii) Computation of the surface gravity of the white dwarf from fitting optical spectroscopy to model atmospheres, and direct determination of the mass from the appropriate mass–radius relation (van Kerkwijk, Bergeron & Kulkarni 1996). This method avoids the uncertainty introduced by the distance measure.

(iv) Shapiro delays: this relativistic effect, detectable with timing analysis, gives a very accurate and reliable estimate of both the masses of the system. However this method works well only for nearly edge-on orbits.

Besides PSR J1012+5307, PSRs B0820+02, J1640+2224, J0034+0534, J1713+0747 and J0437−47 have detected white dwarf companions with probable masses $< 0.3 M_\odot$. We have also considered PSR 1855+09 (for which only upper limits exist for the companion in the $R$ and $I$ band; Kulkarni, Djorgovski & Klemola 1991), because this is the only system for which the masses have been determined with great accuracy from the Shapiro delays (Kaspi, Taylor & Ryba 1994). The relevant parameters of the white dwarf companions in these systems are listed in Table 1.

For each system we adopted the best available estimate for the companion mass. The methods by which these estimates were obtained are given in Table 1, along with the associated uncertainties. As $\tau \propto t_{\text{spin-down}}$ (because $P \propto P_0$), one expects $\tau \propto t_{\text{cool}}$. We therefore plot $\tau$ versus $M_{\text{WD}}$ for the selected systems as an upper limit (for J1640+2224 we adopted a spin-up time-scale of 10 Gyr, the age of the Galaxy). On the same plane we plot the theoretical curves from the models of Webbink (1975), Alberts et al. (1996) and Iben & Tutukov (1986).
and Iben & Tutukov (1986). To compare different systems we 'renormalized' the data and the theoretical curves to a standard luminosity of $10^{12} \, L_\odot$ (for the system containing PSR 1855+09 we adopted the upper limit for the luminosity reported by Kulkarni et al. 1991). The results are shown in Fig. 1.

4 DISCUSSION AND CONCLUSION

The cooling model of Alberts et al. (1996) (and only barely that of Webbink 1975) is able to explain PSR J1012+5307 without invoking any departure from the equilibrium spin period at the end of the accretion phase; however, it upsets the good agreement between $t_{\text{cool}}$ and $t_{\text{in}}$ in all of the other systems, unless it can be demonstrated that the white dwarf in J1012+5307 lies close to the boundary between the two regimes (1) and (2) characterizing the cooling of the tenuous white dwarf envelope. As can be seen from Fig. 1, to maintain equality between cooling and spin-down times in all four cases we are forced to assume a transition from regime (1) to

### Table 1. Parameters for the white dwarf companions: the mass estimate methods refer to those outlined in Section 3.

<table>
<thead>
<tr>
<th>PSR</th>
<th>$M/M_\odot$</th>
<th>Mass estimate method</th>
<th>Mass limits $M/M_\odot$</th>
<th>$L/L_\odot$</th>
<th>Ref.</th>
<th>$\tau , \text{Gyr}$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>J1012+5307</td>
<td>0.16</td>
<td>(iii)</td>
<td>0.14–0.18</td>
<td>3.70 $\pm$ 0.52 $\times 10^{-3}$</td>
<td>(1)</td>
<td>7.0 $\pm$ 1.4</td>
<td>(6)</td>
</tr>
<tr>
<td>B0820+02</td>
<td>0.25–0.35</td>
<td>(iii)</td>
<td>0.20–0.64</td>
<td>1.95 $\pm$ 0.65 $\times 10^{-2}$</td>
<td>(2)</td>
<td>0.11 $\pm$ 0.02</td>
<td>(7)</td>
</tr>
<tr>
<td></td>
<td>$\geq$ 0.5</td>
<td>(ii)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J1640+2224</td>
<td>0.26</td>
<td>(ii)</td>
<td>0.26–0.83</td>
<td>1.58 $\pm$ 0.32 $\times 10^{-4}$</td>
<td>(3)</td>
<td>$\geq$ 20</td>
<td>(3)</td>
</tr>
<tr>
<td>B1855+09</td>
<td>0.258</td>
<td>(iv)</td>
<td>0.242–0.286</td>
<td>$&lt; 1.4 \times 10^{-4}$</td>
<td>(4)</td>
<td>5.0 $\pm$ 1.0</td>
<td>(4)</td>
</tr>
<tr>
<td>J0437–4715</td>
<td>0.22–0.32</td>
<td>(ii)</td>
<td>0.22–0.32</td>
<td>1.3 $\pm$ 0.2 $\times 10^{-4}$</td>
<td>(5)</td>
<td>$\leq$ 0.6</td>
<td>(5)</td>
</tr>
<tr>
<td>J0034–0534</td>
<td>0.23</td>
<td>(ii)</td>
<td>0.15–0.48</td>
<td>6.3 $\pm$ 1.3 $\times 10^{-5}$</td>
<td>(3)</td>
<td>5.9 $\pm$ 1.2</td>
<td>(3)</td>
</tr>
<tr>
<td>J1713+0747</td>
<td>$&lt; 0.32$</td>
<td>(ii)</td>
<td>0.29–0.32</td>
<td>1.58 $\pm$ 0.32 $\times 10^{-4}$</td>
<td>(3)</td>
<td>9.6 $\pm$ 1.9</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Notes. For the systems PSR J1640+2224, PSR J0034–0534 and PSR B0820+02 the limits quoted for the mass are the 95 per cent confidence level from the mass function for the upper limit and the minimum of the mass function for the lower limit (adopting a neutron star mass of 1.4 $M_\odot$). For PSR J0437–4715 the lower limit on the mass is derived from a timing analysis, while the upper limit is derived from a cooling model. In the case of PSR J1713+0747 an upper limit is available from Lundgren, Foster & Camillo (1996b), while the lower limit is the minimum of the mass function. The uncertainties quoted for the system PSR 1012+5307 combine in quadrature an uncertainty in the surface gravity and the relative uncertainty of 0.05 assumed as an estimate of the uncertainty in the mass–radius relation by van Kerkwijk et al. (1996). For PSR B1855+09 the uncertainties are those derived from the timing of the Shapiro delays (Kaspi et al. 1994). The uncertainties in the luminosity (2$\sigma$ confidence level) are dominated by errors in the distance estimate. References: (1) van Kerkwijk et al. (1996); (2) van Kerkwijk & Kulkarni (1995); (3) Lundgren et al. (1996b); (4) Kaspi et al. (1994); (5) Shandu et al. (1997); (6) Lorimer et al. (1995); (7) Kulkarni (1986); (8) Koester, Chanmugam & Reimers (1992).

Figure 1. Time-scales versus white dwarf mass. The dashed, dash-dotted and solid lines represent $t_{\text{cool}}$ for a fixed white dwarf luminosity $L_0 = 10^{-2} \, L_\odot$ in regime 1 [dashed, adopting the cooling model of Alberts et al. (1996), and dash-dotted, adopting the cooling model of Webbink (1975)] and 2 [solid, adopting the cooling model of Iben & Tutukov (1986)]. The boxes indicate the errors for the individual systems.
regime (2) occurring between 0.14 and 0.28 $M_\odot$. Actually, Alberts et al. (1996) adopted this solution in their paper. On the other hand, if such a transition does not occur the age discrepancy $t_{\text{cool}} < \tau$ for J1012+5307 is removed only at the cost of requiring the other systems to be much older than their spin-down ages $\tau$, which would be very hard to explain.

Adopting the Iben & Tutukov (1986) model, the agreement between the two estimates of the age is good except for PSR J1012+5307. However, for this system the difficulty could be resolved simply dropping the assumption that the pulsar spins in equilibrium at the end of the accretion phase. In this case, any scenario adopting the solution (i) outlined above is able to explain the peculiarity of this system.

In conclusion, we have shown that combined optical and radio data from the low-mass binary pulsar systems seem to indicate the following.

(i) Above $0.18 M_\odot$ the white dwarf companion cools in agreement with the classical result:

$$t_{\text{cool}} \propto A^{-1}(M_{\text{WD}}/L)^{5/7}. \quad (3)$$

(ii) If the Alberts et al. (1996) model is adopted, a transition from a cooling white dwarf showing a temporary stable hydrogen-burning shell to a cooling white dwarf in which almost all the residual hydrogen is lost in a few short ($\sim 100$-yr) thermal flashes via Roche-lobe overflow should occur between 0.14 and 0.28 $M_\odot$.

(iii) The cooling model of Iben & Tutukov (1986) describes six of the seven systems well. Moreover, it is not in conflict with the system J1012+5307. For very low accretion rates ($\sim 10^{-2}$ times the Eddington value), the equilibrium spin period is high enough to keep the spin-down age close to the cooling age of Iben & Tutukov (1986). Alternatively, adopting more standard accretion rates, the discrepancy is resolved if the neutron star in this system finished its accretion phase with a spin far from the equilibrium predicted by the standard spin-up model.

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