values of the parameter $\frac{x}{c}$ is given in Fig. 11.

One clearly sees that a velocity distribution according to Fig. 11 is not uniform.

The consequences of this finding are:

1 The uniform velocity assumption in elastohydrodynamic squeeze problems may give an erroneous picture of what is happening in the region of small H^* values.

2 The true velocity distribution and thus more accurate oil film characteristics, can be found through a computing scheme based on successive approximations. The first approximation is given in this paper.

Conclusion

The integral equation approach proves to be successful to arrive at a solution of the elastohydrodynamic squeeze problem for two parallel cylinders approaching each other. The results are obtained assuming a uniform velocity distribution and an isoviscous lubricant.

It is shown that the former assumption may give an erroneous picture at small values of parameter $\frac{h_0}{R}\left(\frac{E'R}{W}\right)$ due to the nonuniformity of the velocity distribution (Fig. 11). It is thought therefore that a more refined analysis of the problem, taking into account the pressure dependency of the viscosity, must also include the nonuniformity of the velocity distribution.

DISCUSSION

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It is now a number of years since I first started to look at the problem of elastohydrodynamics of normal approach and comparatively little development of the theory has taken place since that time. I am therefore very glad to see the work of the author and I wish to congratulate him with an elegant formulation and solution of the constant viscosity problem for cylinders.

In my earlier studies I also arrived at a formulation of the problem in terms of an integral equation which, however, differed from the one presented by the author. This formulation was not, however, used as a basis for my solution which utilized a straight iteration algorithm.

The problem I had principally in mind at the time was the pressure dependent viscosity problem, and since the representation I obtained is different from the author's I shall give a brief outline of the method here.

Start by writing Reynolds equation in the following form

$$h = (\partial p/\partial x)h^4/12\eta Vx \qquad (42)$$

Now eliminate h between equation 42 and the elasticity equation. After an integration by parts and substitution of the boundary conditions $\lim_{x\to\infty} \partial p/\partial x = 0$ we get the following equation for $\partial p/\partial x$

$$\Gamma(x)\, \partial p/\partial x = x(h_0 + x^2/2R) - \frac{4x}{\pi E}\int_{-\infty}^{\infty} \frac{\partial p}{\partial \xi} K(x, \xi)d\xi \qquad (43)$$

where

$$\Gamma(x) = h^4/12\eta V \qquad (44)$$

$$K(x,\xi) = (x-\xi)\ln|x-\xi| + \xi \ln|\xi| \qquad (45)$$

$$\eta = \eta_0 \exp(\alpha p), \text{ a function of pressure.} \qquad (46)$$

This equation is highly nonlinear on account of the unkown function $\Gamma(x)$. If, however, for purposes of obtaining a solution to the equation we regard $\Gamma(x)$ as a given, known function, equation (43) is essentially a linear integral equation for $\partial p/\partial x$ of the Fredholm type. This equation is solved by assuming a (numerical) function $\Gamma^{(0)}$. On this basis a solution $\partial p/\partial x^{(1)}$ is obtained and from this solution the function $\Gamma^{(1)}$ is evaluated. The iteration proceeds until convergence.

Adopting the normalization used by the author, writing

$$\bar{x} = x/\sqrt{h_0 R},\ \bar{\xi} = \xi/\sqrt{h_0 R};\ H = h(x)/h_0$$

equation (43) becomes

$$(\partial \bar{p}/\partial \bar{x})\, H^4/\exp(\bar{p}) = Q\bar{x}(1 + \bar{x}^2/2) - T\bar{x}\int_{\infty}^{\infty} \frac{\partial \bar{p}}{\partial \bar{\xi}} \bar{K}(\bar{x},\bar{\xi})d\bar{\xi} \qquad (47)$$

with

$$\bar{p} = \alpha p$$

$$T = \frac{48}{\eta}\frac{\eta_0 VR}{Eh_0^2}\sqrt{\frac{R}{h_0}}$$

$$Q = \frac{12\alpha\eta_0 VR}{h_0^2}$$

The solution thus depends upon the two parameters T and Q. With constant viscosity equation (47) reduces to

$$(\partial \bar{p}/\partial \bar{x})H^4 = \bar{x}(1 + \bar{x}^2/2) - T\bar{x}\int_{-\infty}^{\infty} \frac{\partial \bar{p}}{\partial \bar{\xi}} \bar{K}(\bar{x},\bar{\xi})d\bar{\xi} \qquad (48)$$

with $\bar{p} = ph_0^2/12\eta_0 VR$ in accordance with the results of the author.

It is evident, that for constant viscosity the representation used by the author, avoiding an iterative procedure, is more elegant and also more convenient from a computational point of view. However, with pressure dependent viscosity the representation given in equation (43) may, perhaps, possess certain computational advantages.

To bring this more clearly out the normalization is changed to

$$\bar{x} = x(\pi\alpha E)/4R;\ \bar{\xi} = \xi(\pi\alpha E)/4R;\ H = h(\pi\alpha E)^2/8R$$

With these variables equation (43) becomes

$$\Gamma\, \partial \bar{p}/\partial \bar{x} = \bar{x}(H_0 + \bar{x}^2) - 2\bar{x}\int_{-\infty}^{\infty} \frac{\partial \bar{p}}{\partial \bar{\xi}} \bar{K}(\bar{x}, \bar{\xi})d\bar{\xi} \qquad (49)$$

$$\Gamma = \frac{8R}{3\eta_0\alpha(\pi\alpha E)^4}\frac{H}{\exp(-\bar{p})} = f(\bar{p}_0)$$

$$\bar{p} = \alpha p$$

The solution is a function of the two parameters

$$H_0 = h_0(\pi\alpha E)^2/8R \text{ and the central pressure } \bar{p}_0$$

The constant viscosity equation can be found by replacing α by $\gamma = 1/p_0$ in the foregoing equation. The solution to this problem becomes a function of the single parameter $H_0 = h_0(\pi\gamma E)^2/8R$.

Let us now consider the author's discovery that his integral equation (equation (11) in the paper) in general yields two solutions to each value of the parameter T, or else no real solution. T consists essentially of the ratio of the relative velocity and the central film thickness separating the approaching surfaces. But V and h_0 are not completely independent quantities. The author has demonstrated this mathematically in his equation (38); intuitively it can be seen by performing the following "mental experiment."

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We wish to measure the relative approach velocity of the surface at a fixed value of h_0. The velocity is varied by changing the load on the cylinders. At a small load the velocity of the mass center of the cylinder will be small but so will the deformation velocity. In a repeat of the experiment with a larger load the velocity of mass-center will be larger but so will the deformation velocity. The relative approach velocity being the difference between the absolute velocity and the deformation velocity may still be small. It may in fact have the same value as in the first experiment. This type of argument therefore leads to the expectation that the same relative velocity can be obtained by the application of two different loads. Or in other words, that the same value of T should give rise to two separate solutions. From this it appears that the bifurcation phenomenon that the author has observed is directly caused by his mode of representation and need not have any particularly interesting fundamental interpretation. If the alternative representation (49) is used a unique solution is found and the difficulties associated with bifurcation do not arise. It is therefore perhaps a little optimistic to believe that the convergence troubles experienced in the variable viscosity case are principally caused by the bifurcation phenomenon.

The author points out that the maximum pressure in the film cannot exceed the Hertzian pressure. This is quite true with constant viscosity. With pressure dependent viscosity, however, the maximum film pressure can be very much larger than the corresponding Hertzian pressure. The author also points to the fact that the relative approach velocity will vary along the film, and suggest that this may have serious consequences for the solution at small separations. This fact becomes, if anything even more pronounced with pressure dependent viscosity, and appears to a larger extent to dominate the solution at small film thicknesses. It seems likely that to obtain good solutions under these circumstances it becomes quite necessary to account for the effect of this velocity variation.