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FORCED, NONLINEAR, NONPLANAR OSCILLATIONS OF A CANTILEVERED BEAM DURING TRANSITION THROUGH RESONANCE

In-Ming K. Shyu, Dean T. Mook, and Raymond H. Plaut
Virginia Polytechnic Institute and State University
Blacksburg, Virginia

EXTENDED ABSTRACT

A slender, elastic, inextensible, cantilevered beam is considered. The principal moments of inertia of the beam cross section are equal (e.g., the cross section may be circular or square). The beam is subjected to a uniformly distributed vertical force which has a static component (e.g., just the weight of the beam) and a dynamic component with constant amplitude and varying frequency $\Omega(t)$. Planar and nonplanar motions are analyzed.

The governing equations of motion are an extension of those derived by Crespo da Silva and Glynn [1,2], taking into account the static deflection. Nonlinear curvature, nonlinear inertia, torsional displacements, and viscous damping are included in the formulation. A perturbation method is applied, and quadratic and cubic terms are retained in the equations. Torsional inertia is neglected, and with the use of the inextensibility condition, the longitudinal motion is written in terms of the horizontal and vertical motions, which are then represented in terms of three vibration modes in the application of Galerkin's method. In some cases, the time histories of the modal amplitudes are determined by numerical integration; in others, the method of multiple scales is used to obtain approximate solutions for the response.

When the forcing frequency Ω is fixed, the system exhibits primary resonance when Ω is close to a natural frequency of the beam, subharmonic resonance when Ω is close to twice or three times a natural frequency, and superharmonic resonance when Ω is close to one-half or one-third of a natural frequency. Under these resonance conditions, the planar (vertical) steady-state motion of the beam may be unstable and a nonplanar (whirling) motion may occur. These stationary conditions have been examined previously [3]. Here, nonstationary conditions are studied, in which the forcing frequency $\Omega(t)$ either increases or decreases slowly through a resonance.

The time histories of the vertical and horizontal motions are determined. For the various resonances, the magnitude and form of the response depends on many factors, including the rate of change of $\Omega(t)$, the static component of the force, the amplitude of the dynamic component of the force, the amount of damping, and the initial conditions (at the beginning of the acceleration or deceleration). Attention is focused on cases that lead to nonplanar motions, and some interesting types of dynamic behavior are exhibited by the beam.

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REFERENCES

[1] Crespo da Silva, M. R. M., and Glynn, C. C., "Nonlinear Flexural-Flexural-Torsional Dynamics of Inextensional Beams. I. Equations of Motion," Journal of Structural Mechanics, Vol. 6, 1978, pp. 437-448.

[2] Crespo da Silva, M. R. M., and Glynn, C. C., "Nonlinear Flexural-Flexural-Torsional Dynamics of Inextensional Beams. II. Forced Motions," Journal of Structural Mechanics, Vol. 6, 1978, pp. 449-461.

[3] Shyu, I.-M. K., Mook, D. T., and Plaut, R. H., "A Nonlinear Analysis of the Whirling Motions of Slender Beams under Various Resonant Excitations," presented at the 13th Biennial ASME Conference on Mechanical Vibration and Noise, September 22-25, 1991, Miami, Florida.