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Spatial Variability of Physical Parameters and Processes in Two Field Soils

Part II: Water Flow at Field Scale

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The horizontal variability of soil hydraulic parameters, water content and suction measured within two research fields in Denmark is analyzed and discussed.

A numerical analysis of the flow in the two spatially heterogeneous fields is carried out by assuming that the fields are composed of ensembles of one-dimensional non-interacting soil columns, each column representing a possible soil profile in statistical terms. Flow predictions based on the classical Richards' equation are carried out for all columns, and the mean and standard deviation of water content and suction in planes perpendicular to the flow direction is derived and compared to measurements. The analysis shows that the model to a reasonable degree is able to describe most of the variation in flow variables within the two fields.

The concept of equivalent soil properties is also investigated and applied to the two fields. Based on the average retention properties it is possible to predict the horizontal averages of flow variables within the fields.

Introduction

Part I of this series of papers (Jensen and Refsgaard, this issue) discusses and evaluates flow and transport simulations at the local scale, *i.e.* in individual soil profiles within the fields. It was demonstrated that flow and transport predictions based on the classical one-dimensional equations, Richards' equation, and the convection-dispersion equation (CDE), respectively, provide good descriptions of the observed variations in water content, suction and concentration. However, due to limitations in the model formulations where hysteresis and macropore flow are not considered, empirical parameter adjustments are needed in order to obtain reasonable comparisons.

In this paper the observed variability of soil and flow parameters within the two research fields is discussed, and the consequences of this variability in relation to modelling of flow integrated over the fields are analyzed. The interpretation of model and field results is performed from a statistical point of view assuming that soil properties and flow variables are stochastic variables. We analyze the results in terms of the statistical moments (mean and standard deviation) based on an assumption that the two fields can be considered as realizations of the ensemble of fields which all have the same properties at the sampling points as the two given fields.

Statistical Analysis of Soil Properties

The two basic parameters which determine unsaturated flow are the retention and the hydraulic conductivity functions. As discussed in Part I (Jensen and Refsgaard, this issue) the retention properties were measured in the laboratory on undisturbed soil samples, (5 cm diameter and 5 cm length) using a combination of hanging water column and pressure plate techniques. The hydraulic conductivity properties were only measured consistently in the two fields at full saturation on undisturbed soil samples of the same size as above.

The statistics of the spatial variation of retention properties in terms of frequency distributions of the water content at each of the suctions applied on the soil samples are listed in Table 1 and illustrated graphically in Fig. 1. In this paper we use the concept pF, which is defined as the logarithmic value of suction in cm.

For the Jyndevad site the distributions for three depths are shown. Although the field is located in an alluvial outwash plain which is considered rather homogeneous from a geological point of view, a significant variability is observed. The figure shows that the retention properties are more homogeneous in the upper soil horizon than in the lower, presumably due to agricultural activities. For the deepest horizon a considerable variability is recognized, particularly for pF-values between 1.5 and 2.0. Since the water content at this suction level represents field capacity, a similar large variability is expected in the field water conditions.

For the Taastrup site the retention characteristics of the top layer (10 cm) differ from the other layers, and we have therefore grouped the data into two intervals, Fig. 1. Also at this site we find that the soil properties are more homogeneous close to the soil surface than deeper in the profile. Note that due to entrapped air the retention curves have been modified in the wetter regime before they are used as input to the flow model.

Table 1 – Mean (μ) and standard deviation (σ) of retention characteristics (vol. %) Hansen *et.al.*, 1986

	Porosity				θ at pF $\equiv$ 1.0			
Depth	Jyndevad		Taastrup		Jyndevad		Taastrup	
	μ	σ	μ	σ	μ	σ	μ	σ
10			39.9	2.2			38.1	2.0
30			37.9	2.2			35.7	1.8
50			38.3	1.4			35.9	1.3
70			36.2	2.1			33.7	1.5
90			35.3	3.3			33.4	2.9
	θ at pF $\equiv$ 1.3				θ at pF $\equiv$ 1.5			
Depth	Jyndevad		Taastrup		Jyndevad		Taastrup	
	μ	σ	μ	σ	μ	σ	μ	σ
10	37.0	1.6			34.4	2.4	35.1	1.7
30	35.7	2.2			31.6	3.8	32.4	1.8
50	34.6	2.1			23.9	6.5	32.3	1.5
70							31.3	1.4
90							31.2	2.2
	θ at pF $\equiv$ 1.7				θ at pF $\equiv$ 2.0			
Depth	Jyndevad		Taastrup		Jyndevad		Taastrup	
	μ	σ	μ	σ	μ	σ	μ	σ
10	21.6	2.6			17.8	1.3	32.7	2.2
30	22.2	3.9			17.1	3.1	28.3	2.8
50	13.6	3.9			8.6	2.1	27.9	2.5
70							27.4	1.8
90							27.5	3.1
	θ at pF $\equiv$ 2.5				θ at pF $\equiv$ 3.0			
Depth	Jyndevad		Taastrup		Jyndevad		Taastrup	
	μ	σ	μ	σ	μ	σ	μ	σ
10	14.9	1.0	30.3	1.9	10.8	0.7	26.8	1.8
30	13.7	2.6	25.0	3.3	10.2	1.8	21.2	3.5
50	6.8	1.8	24.5	3.4	5.4	1.3	20.9	3.9
70			24.1	3.0			20.4	3.6
90			24.0	4.0			20.3	4.3
	θ at pF $\equiv$ 4.2							
Depth	Jyndevad		Taastrup					
	μ	σ	μ	σ				
10	3.8	0.7	11.4	1.1				
30	4.5	1.2	10.9	3.3				
50	3.1	0.7	11.5	3.8				
70			11.3	3.6				
90			10.5	3.3				

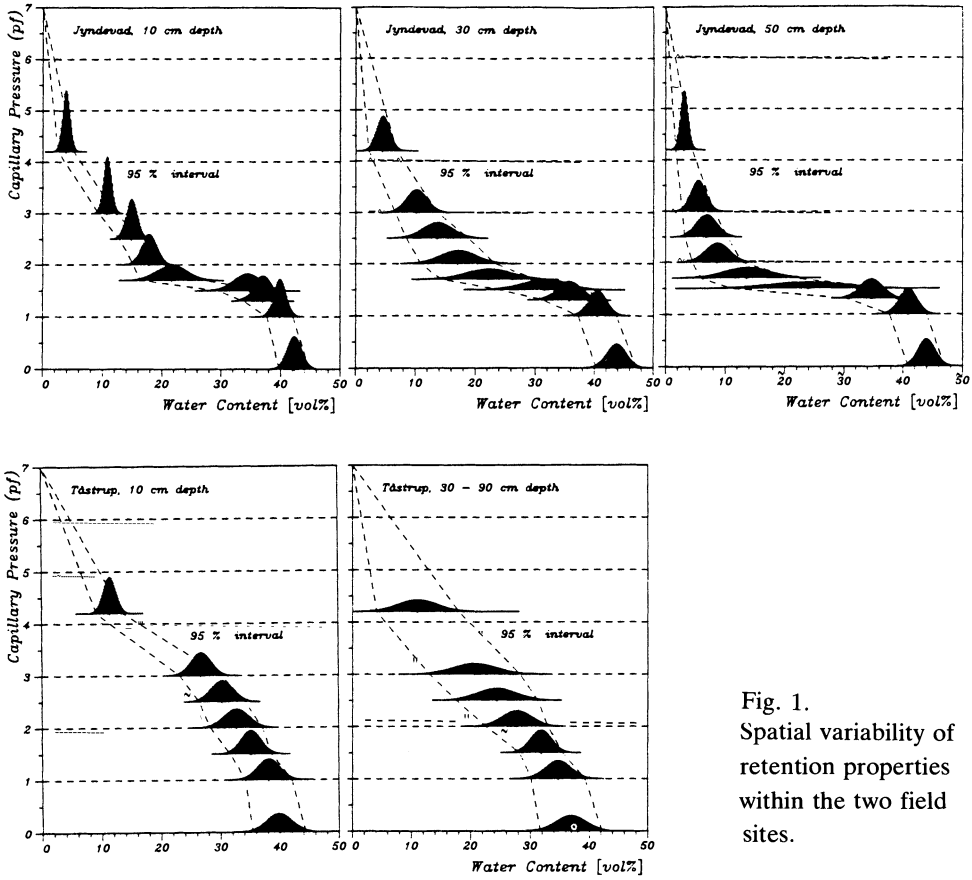

Fig. 1.
Spatial variability of retention properties within the two field sites.

The global statistical parameters of saturated hydraulic conductivity K_s for the two fields are listed in Table 2. K_s follows a log-normal distribution (Hansen *et.al.* 1986) and mean and standard deviation are therefore listed in terms of the logarithmically transformed values ($\ln K_s$).

These parameters confirm that the soil at Jyndevad (coarse sand) is more permeable than the soil at Taastrup (sandy loam) and that the sandy soil is more homogeneous than the sandy loam.

Table 2 - Mean and standard deviation of logarithmically transformed saturated hydraulic conductivity K_s (m/s)

Site	Mean of $\ln K_s$	Standard deviation of $\ln K_s$
Jyndevad	- 9.42	1.31
Taastrup	-13.37	1.99

Saturated hydraulic conductivity was measured on undisturbed 100 cm^3 soil cores taken in triplicates. The measurements showed a very large variability between the triplicates, a variability which cannot be explained when compared to measurements of other parameters such as soil texture and retention characteristics. It appears that the sample size may be too small for this particular parameter because disturbances in the cores in the form of larger stones or cracks may give rise to abnormal flow conditions which are not representative on a larger scale. Due to the sensitivity of the measurements to such variations in pore geometry the two extreme values of the triplicate measurements have been excluded when deriving the statistical parameters given in Table 1. Still, the standard deviations of $\ln K_s$ at the two fields are larger than reported elsewhere (Nielsen *et al.* 1973, Bresler and Dagan 1983, Byers and Stephens 1983).

The spatial correlation of the hydraulic properties has been analyzed by Hansen *et al.* (1986). Using traditional geostatistical methods (Matheron 1963 and 1971) semivariograms have been developed for various parameters including retention characteristics and saturated hydraulic conductivity.

For Jyndevad field site the spatial correlation length expressed as the range of the semivariogram is typically about 30-50 m for soil water retention although it varies significantly for the individual depths and suction levels. The range of the logarithmically transformed data for saturated hydraulic conductivity is of the same order, 25-40 m.

The results from Taastrup field site are less consistent with respect to the spatial structure. For the retention characteristics close to the soil surface (10 cm) and those deepest in the profile (90 cm) the range is typically of the same order as above, 25-40 m. On the other hand, almost no correlation can be identified for the depth intervals in between. As opposed to the Jyndevad data for saturated hydraulic conductivity no spatial correlation at all can be deduced from the Taastrup data.

Statistical Analysis of Water Content

Water content was measured regularly in the two fields during approximately three years in order to analyze the variability in time and space. Figs. 2 and 3 illustrate the variability over the season at various depths for selected parts of the investigation period. The shaded area represents the range of variation between the mean added and substracted two times the standard deviation, respectively. The range of variation in water content is derived from measurements in 24 soil profiles, and it represents the 95% confidence interval if a normal distribution can be assumed of the horizontal variation at a particular measuring day.

For the Jyndevad site a marked seasonal variation is observed in the range of variation near the soil surface, where the horizontal variability tends to decrease

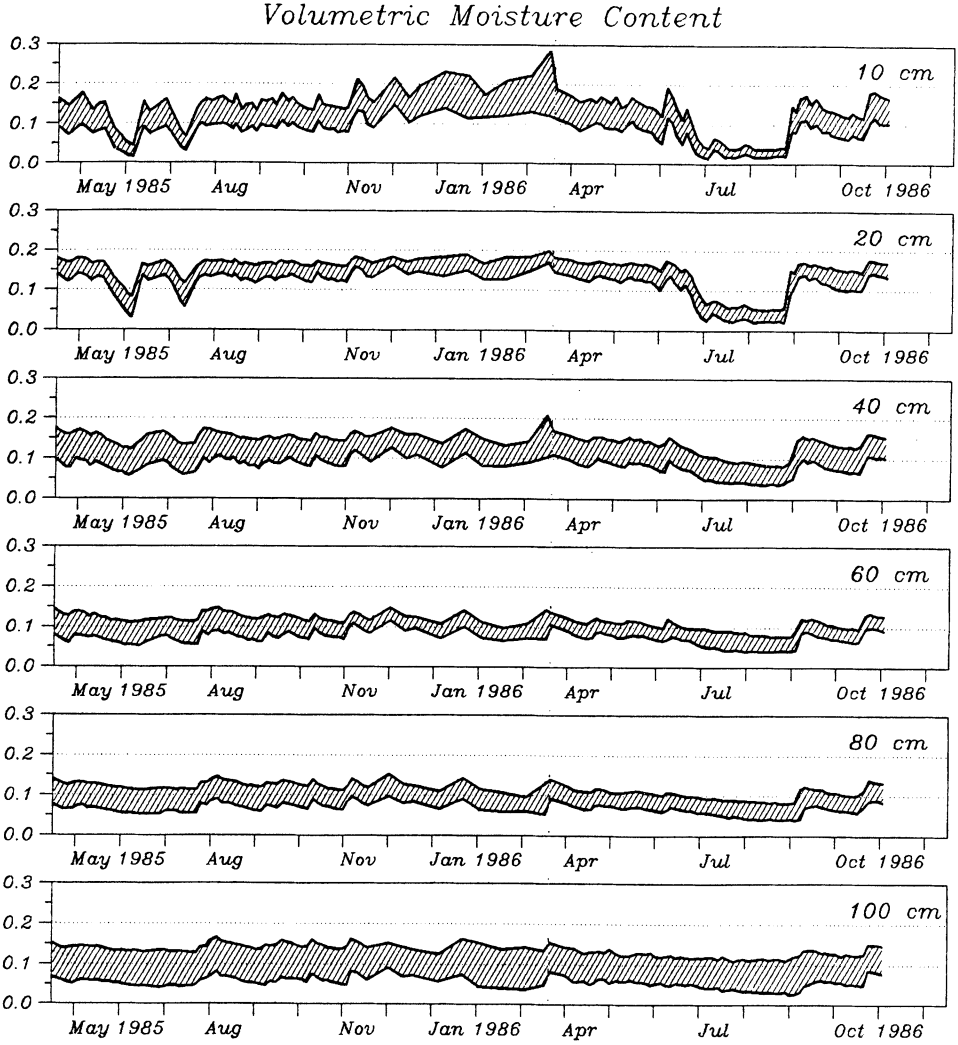

Fig. 2. Range of variation in water content of the Jyndevad site (95% confidence intervals).

during periods with high water stress. Furthermore, for periods outside the growing season, the variability tends to be larger near the surface than deeper in the profile.

For the Taastrup site the range of variation of water content across the field seems almost constant over the season and with depth. This difference between the two sites may be explained by the fact that the variability of the retention curve is more or less constant at all suctions in Taastrup whereas in Jyndevad the variability is significantly larger at pF 1.5-2.0 than at pF 4.2.

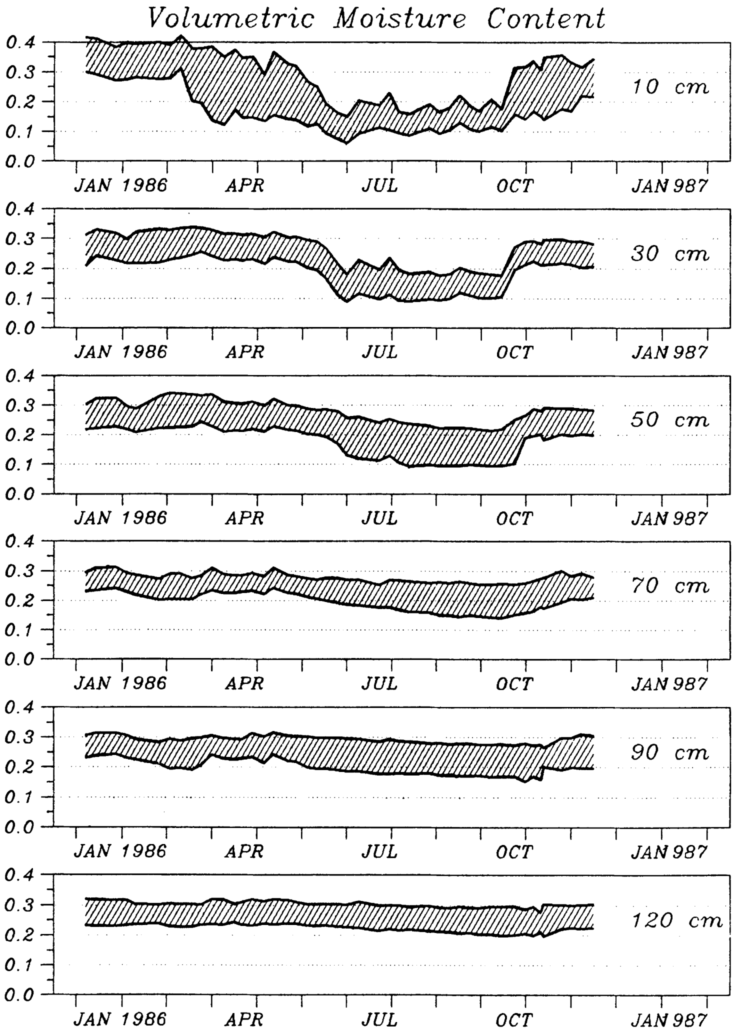

Fig. 3. Range of variation in water content at the Taastrup site (95% confidence intervals).

Statistical Analysis of Suction

Suction was measured at five depths in 12 sampling profiles using tensiometers. Before discussing these observations several matters should be pointed out. Compared to the measurements of water content by the neutron probe technique, tensiometer recordings represent a smaller measuring scale, and hence they are more sensitive to small-scale variations. Furthermore, a proper function of the device requires that the water in the soil and in the instrument is in contact, and this

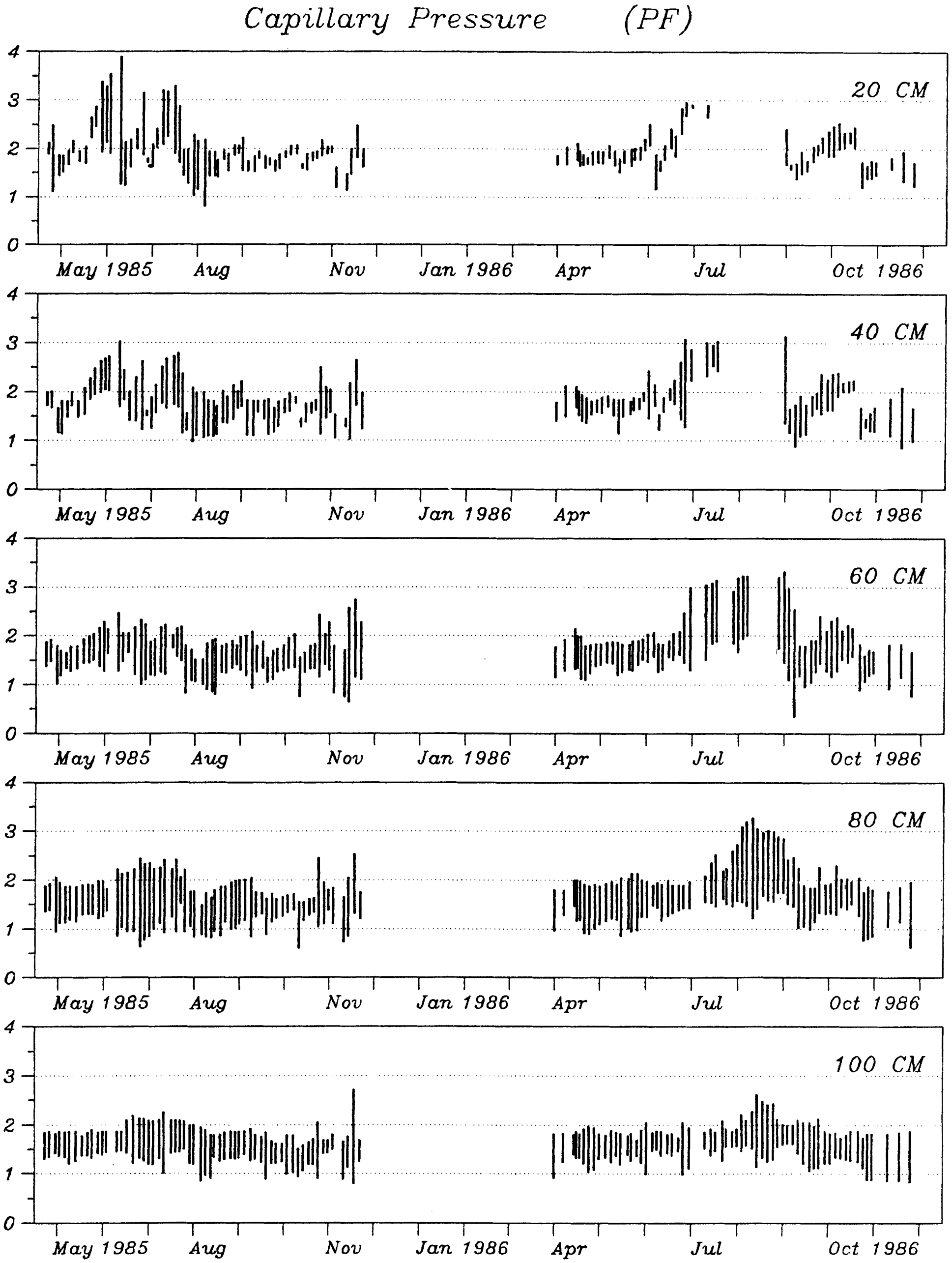

Fig. 4. Range of variation in suction (log values) at the Jyndevad site (95% confidence intervals).

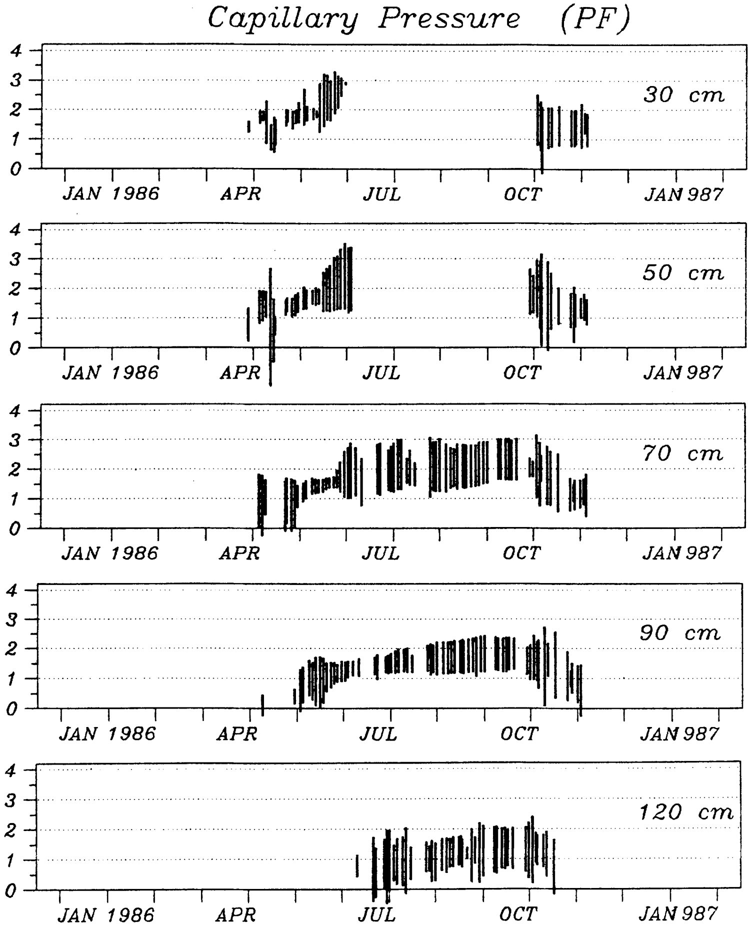

Fig. 5. Range of variation in suction (log values) at the Taastrup site (95% confidence intervals).

is often difficult to establish in a coarse sandy soil. In addition, the instrument fails for suctions larger than 10 m implying that measurements are difficult to obtain in the root zone during the growing season.

These circumstances in combination with a small number of measurements (often less than 12) imply that the variability patterns shown in Figs. 4 and 5 may be rather uncertain. The results are depicted on a logarithmic scale (pF), and the shaded area reflects the range of variation between upper and lower limits represented by the mean of the logarithmically transformed measurements added and substracted two times the standard deviation, respectively. Assuming a log-normal distribution this interval represents the 95% confidence interval.

For the two field sites the spatial variability of suction, as a function of both depth and time, has a somewhat erratic behaviour which makes it difficult to extract general conclusions. No general trend with depth or season of the horizontal variation is present.

Stochastic Modelling of Flow in Field Soils

In practice, measurements of soil hydraulic properties are constrained to a limited number of point values. The actual distribution of these hydraulic properties across a field or catchment is difficult to predict from these point values because of the complex heterogeneity of natural soils. A useful approach is to treat soil properties as stochastic variables so that their distribution in space can be described by a few statistical parameters. By assuming statistical stationarity and ergodicity, ensemble averages and space averages can be interchanged. Hence, the expectation of a variable represents the average over the field, and similarly the standard deviation reflects the variation. In many applications it is of little interest to give a detailed description of the flow within the field, and the useful information is rather contained in the main statistical moments (mean, variance).

Also climatic variables such as rainfall and radiation exhibit a spatial variation. However, the correlation scale for these variables is much larger than for soil properties, and consequently it is assumed in the study that the climatic observations at the two fields apply uniformly over the investigated areas.

The characteristics of the grass cover such as density, green active material, albedo and root growth will also vary spatially but the variability of these factors is not considered either, and only the heterogeneity of soil properties is treated.

The present numerical analysis of the flow in spatially heterogeneous fields assumes that a field is composed of an ensemble of vertical non-interacting soil columns, each column representing a possible soil profile in statistical terms. This is a common approach for modelling field-scale flow and transport, see *e.g.* Dagan and Bresler (1979), Destouni and Cvetkovic (1991), and Bresler and Dagan (1981).

By distributing the hydraulic properties amongst the ensemble of soil profiles according to the statistical distributions found in the actual field, the flow in each column can be described deterministically using the model developed for the local scale. A subsequent statistical analysis of the model-predicted flow variables in all columns can then provide an estimate of the statistical moments over the field at given depths as a function of time.

In the present analysis it is assumed that the statistical variation of hydraulic soil properties within the fields, both vertically within soil columns and horizontally between soil columns, is represented by the data from the 24 sampling profiles established in the fields. The statistical outcome of the model predictions can subsequently be analyzed against the statistical properties for moisture content and suction.

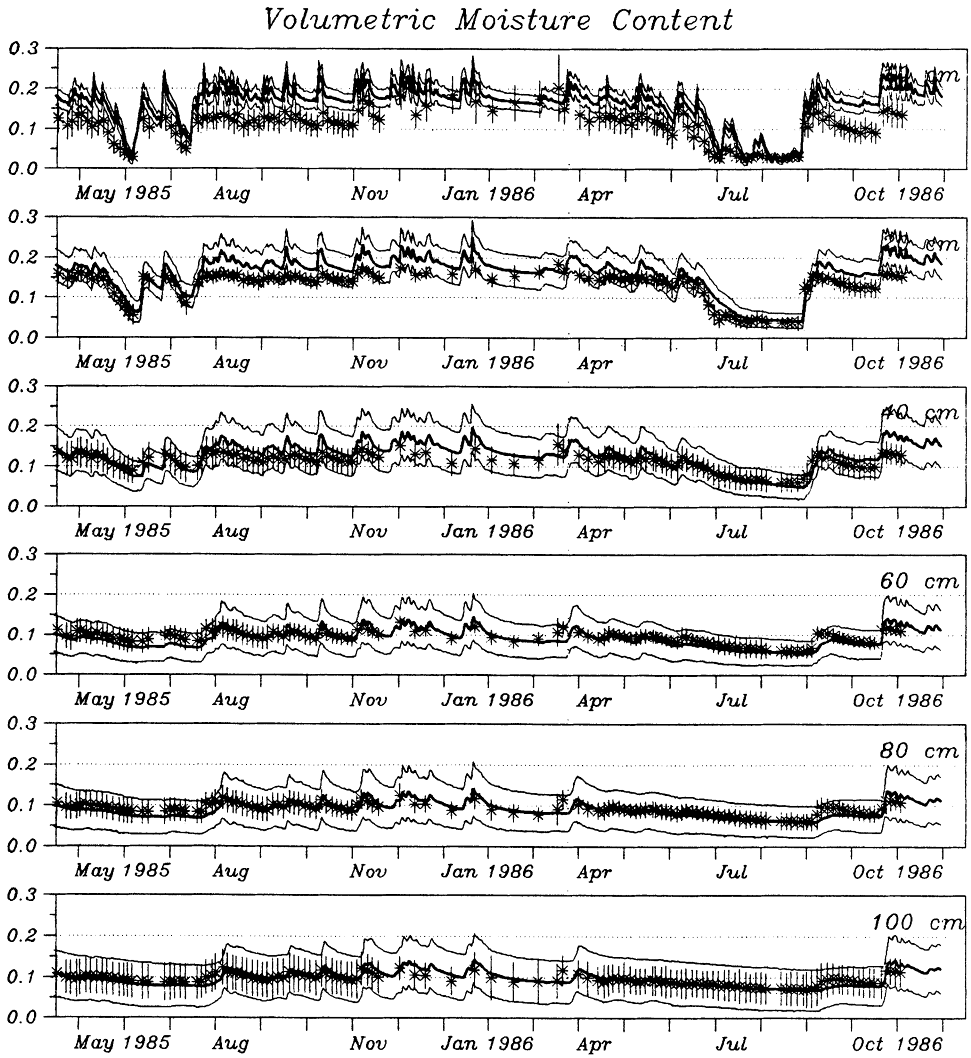

Fig. 6. Measured (*) and simulated (–) 95% confidence intervals of water content for Jyndevad.

Jyndevad Field Site

The stochastic simulation results for the Jyndevad site of water content and suction are presented in Figs. 6 and 7 together with the observations previously shown in Figs. 2 and 4. The simulation results of the average water content show the same discrepancies near the soil surface as discussed in Part I (Jensen and Refsgaard, this issue) for a selected profile, *i.e.* values higher than measured in periods outside the

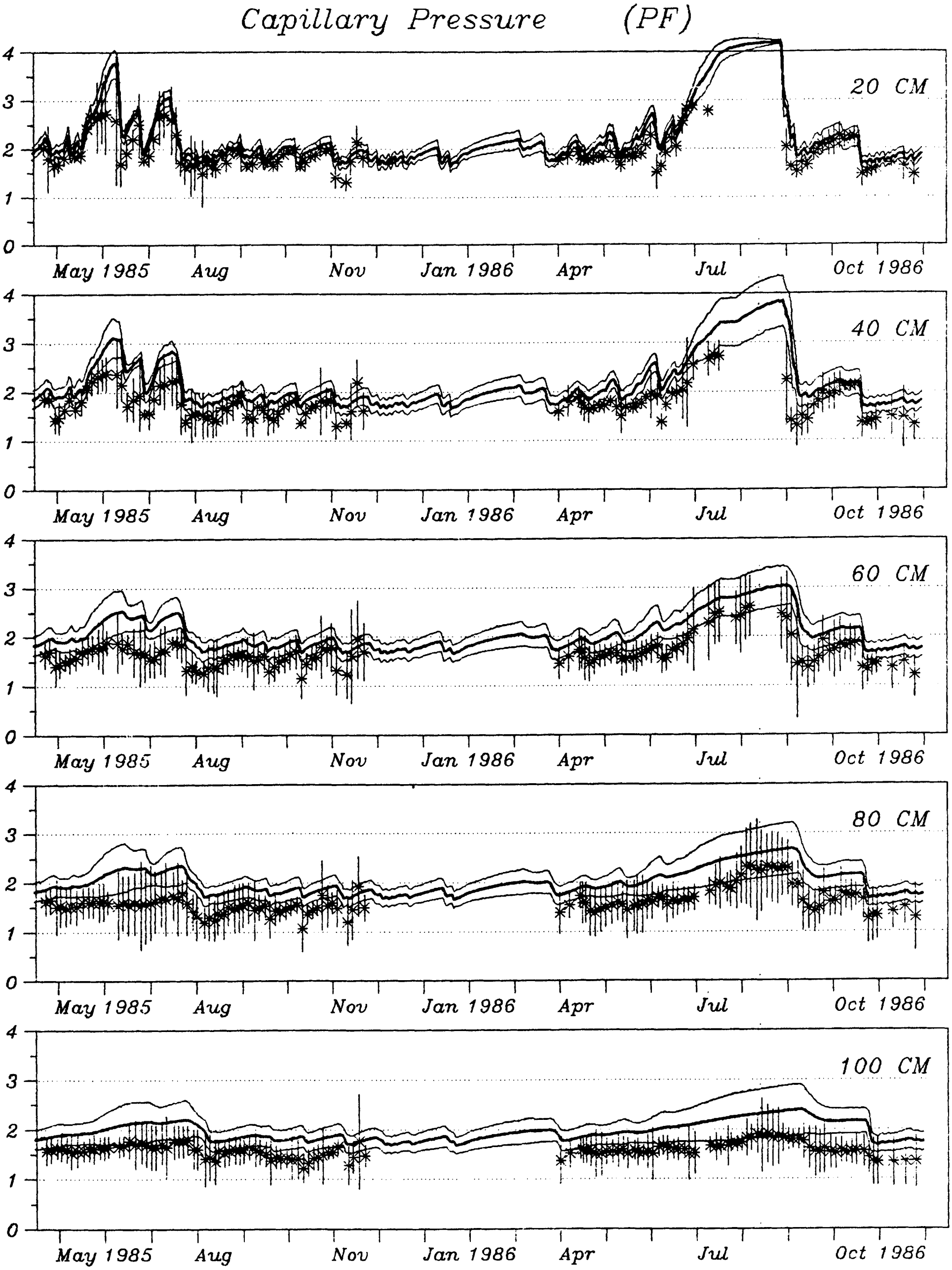

Fig. 7. Measured (*) and simulated (–) 95% confidence intervals of suction for Jyndevad.

growing season. However, deeper in the profile the general level of the simulations as determined by the spatial averages compares very well to the measurements.

With respect to the simulated variation in water content, the interval is somewhat underestimated at 10 cm depth, while the simulated range of variation is exaggerated for the lower depth levels. Several reasons are possible for these discrepancies. Obviously, the proposed modelling approach is a simplification of the flow processes taking place in the field, however, we also tend to believe that the inaccuracies of the neutron probe calibration may be a significant factor.

For the suctions, Fig. 7, the simulated spatial variation is generally underestimated, yet the general tendency of smaller variations at the upper level and larger variations deeper in the profile is also reflected by the simulations. For the upper two levels the simulated average suctions are in reasonable accordance with the measurements, while at the three lowest depths the simulations exceed the measurements.

Apart from the fact that the field data for both water content and particularly for suction are rather uncertain, an apparent inconsistency between the simulations of the two flow variables appears. When applying the main drainage curve as retention characteristics in the model simulations, it is not possible to describe the mean field conditions of both water content and suction with the same accuracy. Furthermore, the simulated ranges of variation for the two variables are conflicting in the sense that the range is overestimated for moisture content and underestimated for suction. This discrepancy cannot be eliminated when using a monotonic relation for the retention characteristics.

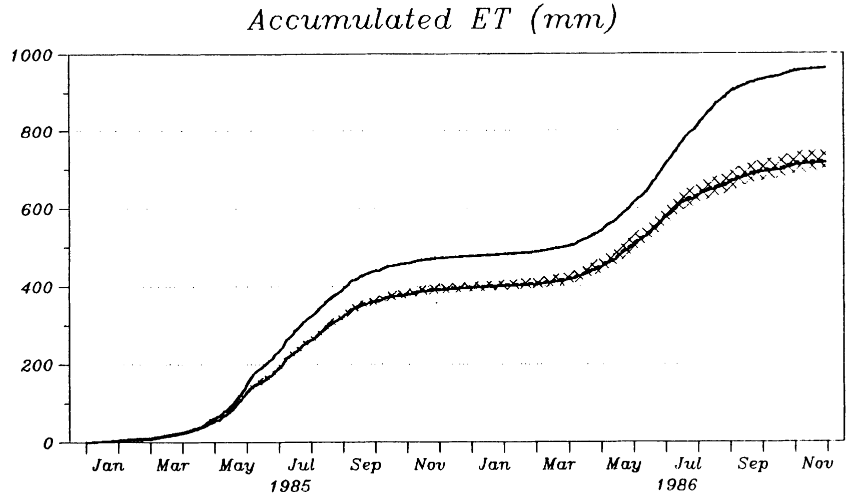

Fig. 8. Predicted accumulated evapotranspiration for Jyndevad. Potential values (upper –), simulated mean (lower –) and variation (cross-hatched area), together with values based on effective hydraulic parameters (--).

When viewing the simulations of the two variables together, Figs. 6 and 7, the results suggest that the water retention in the field is affected by hysteresis. Introducing hysteresis in the retention function instead of just using the main drying curve in the simulations we first of all expect a decrease in the general level of the simulated suction, and at the same time we will expect an increase in the range of variation without necessarily enlarging the interval of the simulated moisture content.

In Fig. 8 the results of predicted accumulated evapotranspiration are shown. The cross-hatched area represents the range bounded by the current maximum and minimum values, respectively. The figure indicates that a rather small variation over the field is predicted. This is in consequence of the combination of the sandy soil type at the site and a shallow root zone which leads to a small water-holding capacity. Thus, during dry periods in the growing season the available water is consumed rather quickly, and the evapotranspiration is subsequently very low, which implies that the spatial variation in accumulated evapotranspiration will be relatively small.

Taastrup Field Site

The stochastic simulation results for the Taastrup site of water content and suction are presented in Figs. 9 and 10 together with the observations previously shown in Figs. 3 and 5.

The simulation results show a general disagreement with the measured water content near the soil surface (10 cm depth) both with respect to the mean, which is generally simulated too high, and the range of variation, which is generally simulated too narrow. The discrepancies experienced for the mean values were also present at the selected profile shown and discussed in Part I (Jensen and Refsgaard, this issue). With respect to the range of variations the results indicate that the model description does not include all the important processes taking place in the ploughing layer, such as hysteresis and macropore flow phenomena. At larger depths the simulations of water content correspond reasonably well with the observations.

In Fig. 10 the measured and simulated suctions are compared. Since the suction is not log-normally distributed near pF = 0, the lower line af the confidence interval has not been drawn when it crosses the zero line. Generally, the mean values correspond reasonably well to the measurements, whereas the ranges of variation are simulated too narrow. The fact that the variations in water content are simulated well, while the variations in simulated suction are underpredicted, indicates shortcomings in the process description, such as the exclusion of hysteresis. Both water content and suction values at 120 cm depth are seen to be influenced considerably by the movement of the shallow ground water table. This is most evident in January, April, and May, where the water table is above this level in all sampling profiles.

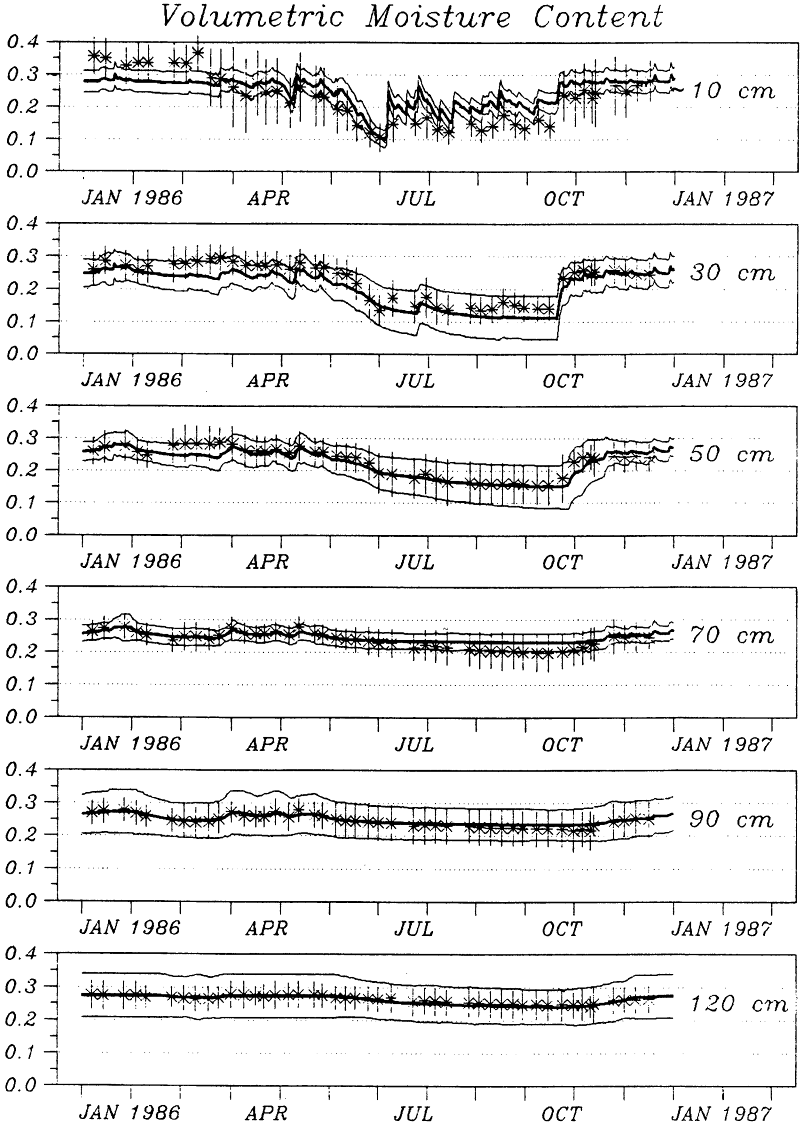

Fig. 9. Measured (*) and simulated (–) 95% confidence intervals of water content for Taastrup.

The simulated evapotranspiration is shown in Fig. 11 with the cross-hatched area representing the range of variation between the 24 columns. As compared to the corresponding Fig. 8 for Jyndevad, the variation is seen to be relatively higher for Taastrup. This is due to the relatively larger water-holding capacity and the more shallow ground water table at Taastrup.

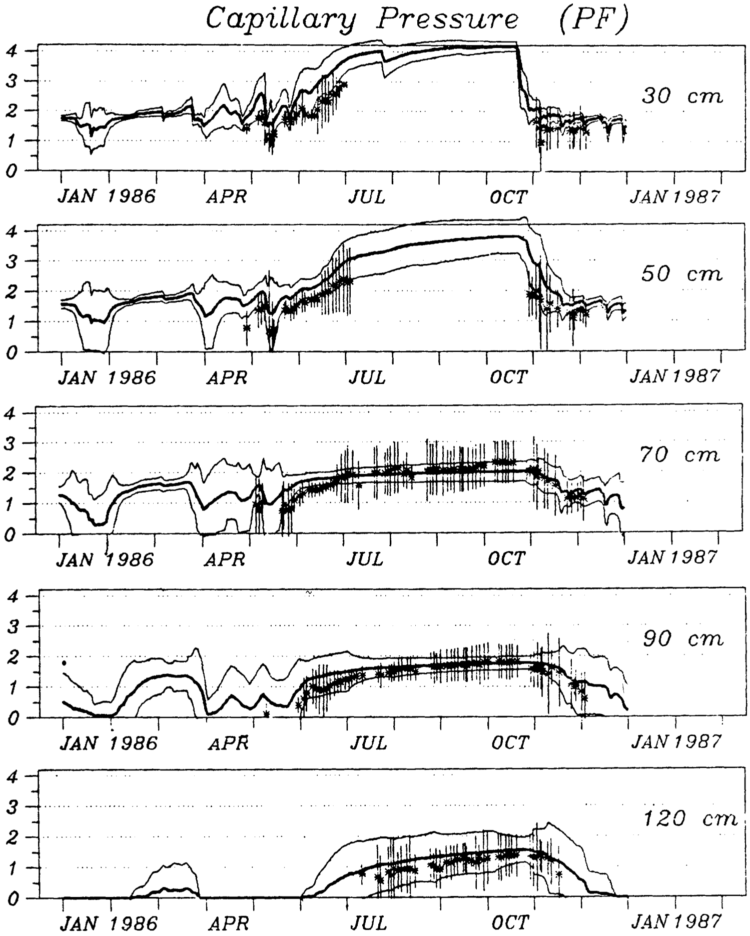

Fig. 10. Measured (*) and simulated (–) 95% confidence intervals of suction for Taastrup.

Equivalent Model – Effective Parameters

In many practical problems of water flow in heterogeneous fields, only the field-averaged one-dimensional flow pattern is of interest. This is *e.g.* the case for distributed hydrological models, where one numerical grid element may represent the size of a field. If a significant spatial variability of the hydraulic parameters is present within the field, the question arises whether it is possible to define a set of effective parameters such that the solution of the flow problem in an equivalent soil is identical to the expectation values over the field. Studies by *e.g.* Smith and Hebbert (1979) and Bresler and Dagan (1983) have questioned the feasibility of the concept of an equivalent soil due to the highly non-linear nature of unsaturated

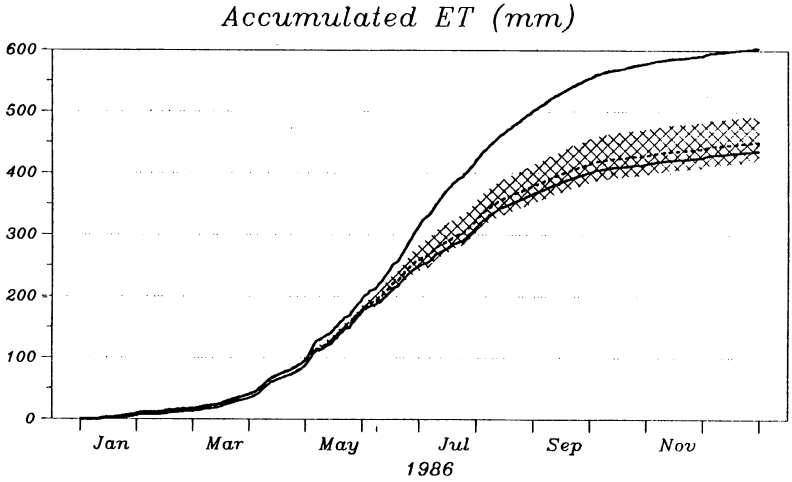

Fig. 11. Predicted accumulated evapotranspiration for Taastrup. Potential values (upper –), simulated mean (lower –) and variation (cross-hatched area), together with values based on effective hydraulic parameters (--).

flow processes. Yeh *et al.* (1985 a, b, c), Mantoglou and Gelhar (1987 a, b, c) and Mantoglou (1992) have shown theoretically that a large-scale model for unsaturated flow systems exists and that it is of the same form as the Richards' equation for local flow. Based on a stochastic representation of the spatial variability of the local hydraulic soil properties they have shown that the effective parameters of the large-scale model depend in a complicated manner on the statistics of local variability (*i.e.* mean, variances, and correlation lengths).

The approach taken in this study is to establish the effective parameters by introducing an averaging procedure of the available data on retention and hydraulic conductivity within the two fields.

The effective retention function is defined by taking the arithmetic averages of all retention data for the individual suctions applied in the laboratory, *cf.* Fig. 1, and subsequently connecting these points in order to establish the complete retention function. This procedure is applied to all the horizons from which data are available.

For the same horizons the effective saturated hydraulic conductivity is derived as the mean of the available measurements assuming the data follow a log-normal distribution. The complete effective hydraulic conductivity function is subsequently established using the same procedure as applied at local scale, *cf.* Part I (Jensen and Refsgaard, this issue). As discussed here this procedure is somewhat different for the two field sites.

Figs. 12-13 and 14-15 present the simulation results based on effective parameters for Jyndevad and Taastrup respectively. As shown in the figures, the simulated

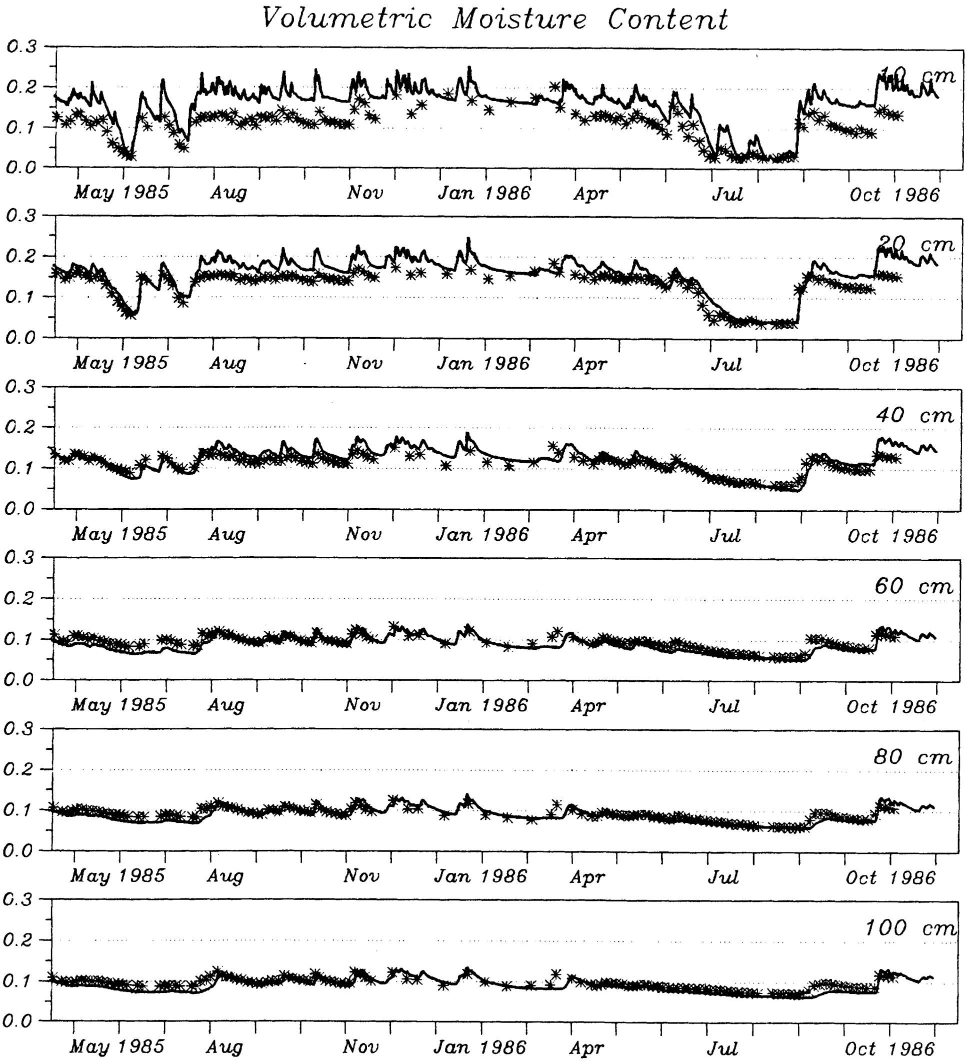

Fig. 12. Mean measured water content (*) against simulated values using effective soil parameters (–) at Jyndevad.

values of water content as well as suction correspond reasonably well to the spatially averaged values, thus supporting the validity of equivalent soil properties for these particular applications.

The predictions of accumulated evapotranspiration based on the effective parameters are compared to the average values in Figs. 8 and 11 for the two sites respectively. As indicated in the figures a very good accordance is obtained.

Note that the averaging procedure proposed in this study, which is based on

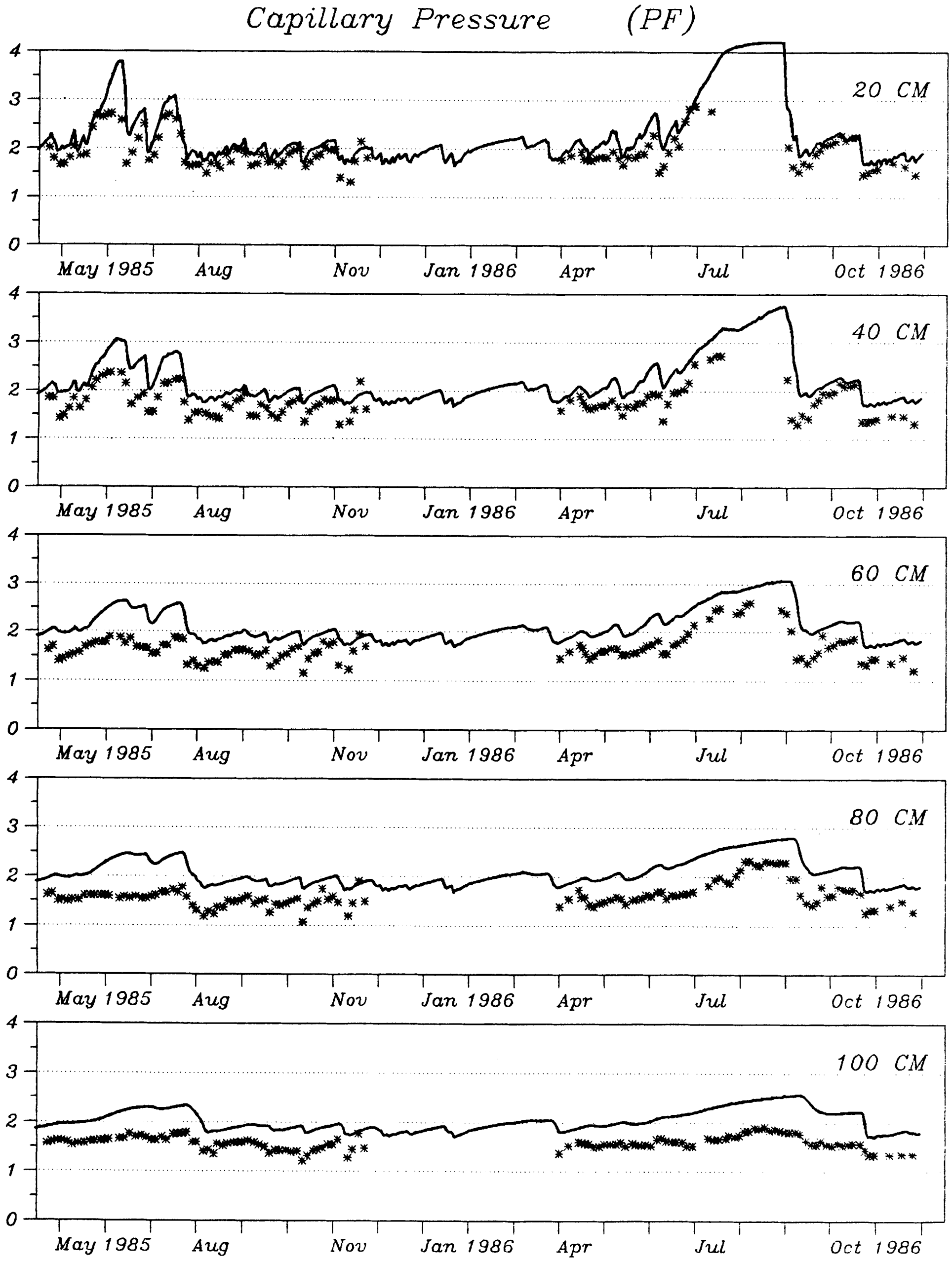

Fig. 13. Mean measured suction (*) against simulated values using effective soil parameters (–) at Jyndevad.

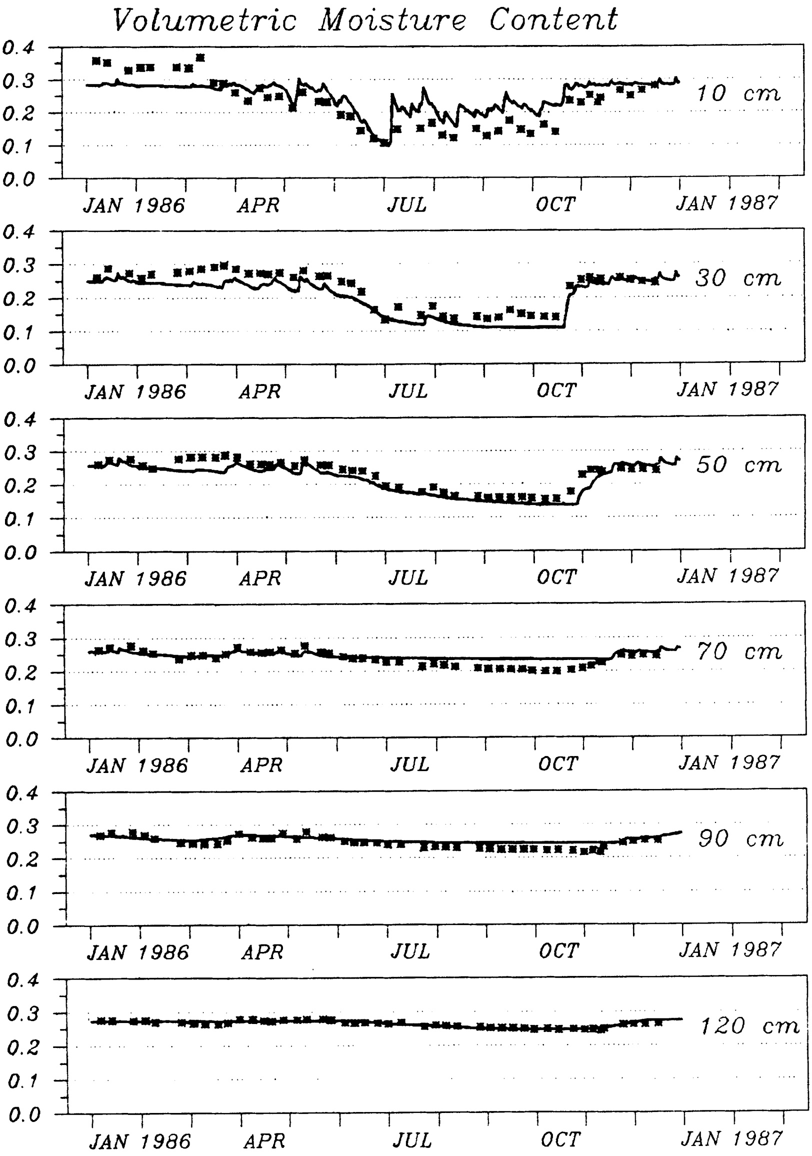

Fig. 14. Mean measured water content (*) against simulated values using effective soil parameters (–) at Taastrup.

simple arithmetric and geometric averages of the local hydraulic properties measured in the field, is not theoretically founded but may rather be considered as a practical approach to flow predictions in large-scale systems. Furthermore, since the large-scale flow model attempts to describe the average large-scale system

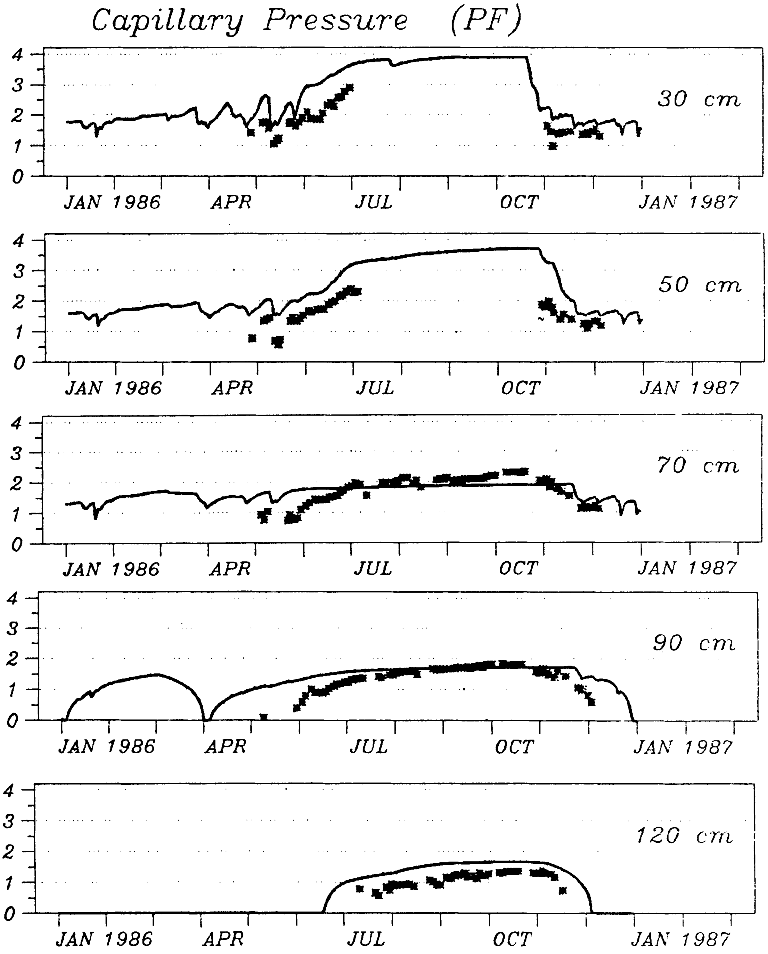

Fig. 15. Mean measured suction (*) against simulated values using effective soil parameters (–) at Taastrup.

behaviour rather than local details, an error is introduced, which cannot be quantified from the present approach. A more complete description of large-scale unsaturated flow problems is provided by the stochastic theory of Mantoglou and Gelhar (1987 a, b, c) which is also capable of estimating the model error. Application of the stochastic model to the Jyndevad data is presented by Jensen and Mantoglou (1992).

Apparently there is conflicting evidence on the suitability of the effective parameter concept between this study and the study by Bresler and Dagan (1983). A possible explanation for this inconsistency may be the different flow conditions considered. The present investigation analyses the flow under naturally occurring rainfall conditions in Denmark, which only give rise to ponding at a few occasions every year, whereas Bresler and Dagan (1983) only consider ponded infiltration events. Under such conditions the non-linearity may tend to be more pronounced due to the very large variability of saturated hydraulic conductivity, and these circumstances may invalidate the equivalence principle.

Conclusion

The water flow model, which was calibrated on a few soil columns within two research fields was applied to simulate the water flow in 24 vertical soil columns at each of the two fields, where retention curves and saturated conductivities were measured. The mean and variation over the field of simulated water content and suction were compared to measured values. Generally, the agreement between simulated and measured mean values followed the same pattern as for the single column simulations, *i.e.* some discrepancies at 10 cm depth and reasonable agreement deeper in the profile.

With respect to the horizontal variation over the fields the water content was generally simulated reasonably well, while the simulated suctions showed a too small variation. To some extent the disagreements are believed to be due to an insufficient process description, particularly with respect to hysteresis and macropore flow.

For both field sites the concept *equivalent soil* was applied by using arithmetic averages for retention and geometric averages for saturated conductivity as the effective parameters. This method seems to provide a practical approach to simulating the field-averaged values of water flow and evapotranspiration, however, it is expected only to be valid for water flow in hydrological regimes like the Danish where the rainfall intensities generally are moderate implying that ponding and overland flow are infrequently occurring phenomena.

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References

Bresler, E., and Dagan, G. (1981) Convective and pore scale dispersive solute transport in unsaturated heterogeneous fields, *Water Resour. Res., Vol. 17,* pp. 1683-1693.

Bresler, E., and Dagan, G. (1983) Unsaturated flow in spatially variable fields, 2. Application of water flow models to various fields, *Water Resour. Res., Vol. 19 (2),* pp. 421-428.

Byers, E., and Stephens, D. B. (1983) Statistical and stochastic analyses of hydraulic conductivity and particle-size in fluvial sand, *Soil Sci. Am. J., Vol. 47,* pp. 1072-1087.

Dagan, G., and Bresler, E. (1979) Solute dispersion in unsaturated heterogeneous soil at field scale, I. Theory, *Soil Sci. Soc. Am. J., Vol. 43,* pp. 461-467.

Destouni, G., and Cvetkovic, V. (1991) Field-scale mass arrival of sorptive solute into the groundwater, *Water Resour. Res., Vol. 27,* pp. 1315-1325.

Hansen, S., and Jensen, H. E. (1988) Spatial variability of soil physical properties. Theoretical and experimental analyses, II. Soil water variables – Data acquisition, processing and basic statistics. Royal Veterinary and Agricultural University, Denmark (internal report).

Jensen, K. Høgh, and Mantoglou, A. (1992) Application of stochastic unsaturated flow theory, numerical simulations and comparisons to field observations, *Water Resour. Res., Vol. 28(1),* pp. 269-284.

Jensen, K. Høgh, and Refsgaard, J. C. (1991) Spatial variability of physical parameters and processes in two field soils, Part I: Water flow and solute transport in soil profiles, *Nordic Hydrology, Vol. 22(5),* pp. 275.

Mantoglou, A. (1992) A Theoretical Approach for Modeling Unsaturated Flow in Spatially Variable Soils: Effective flow models in finite domains and nonstationarity, *Water Resour. Res., Vol. 28(1),* pp. 251-267.

Mantoglou, A., and Gelhar, L. W. (1987 a) Stochastic modelling of large-scale transient unsaturated flow systems, *Water Resour. Res., Vol. 23 (1),* pp. 37-46.

Mantoglou, A., and Gelhar, L. W. (1987 b) Capillary tension head variance, mean soil moisture content, and effective specific soil moisture capacity in transient unsaturated flow in stratified soils, *Water Resour. Res., Vol. 23 (1),* pp. 47-56.

Mantoglou, A., and Gelhar, L. W. (1987 c) Effective hydraulic conductivities of transient unsaturated flow in stratified soils, *Water Resour. Res., Vol. 23 (1),* pp. 57-67.

Matheron, G. (1963) Principles of geostatistics, *Econ. Geology, Vol. 58,* pp. 1246-1266.

Matheron, G. (1971) The theory of regionalized variables and its application, *Les Cahiers du Centre de Morphhologie Mathematique, Fas. 5,* C. G. Fontainebleau.

Nielsen, D. R., Biggar, J. W., and Erh, K. T. (1973) Spatial variability of field-measured soil-water properties, *Hilgardia,* Vol. 42 (7), pp. 215-260.

Smith, R. E., and Hebbert, R. H. B. (1979) A Monte Carlo analysis of the hydrological effects of spatial variability of infiltration, *Water Resour. Res., Vol. 15 (2),* pp. 419-429.

Yeh, T. C., Gelhar, L. W., and Gutjahr, A. L. (1985 a) Stochastic analysis of unsaturated flow in heterogeneous soils, 1, Statistically isotropic media, *Water Resour. Res., Vol. 21 (4),* pp. 447-456.

Yeh, T. C., Gelhar, L. W., and Gutjahr, A. L. (1985 b) Stochastic analysis of unsaturated flow in heterogeneous soils, 2. Statistically anisotropic media with variable α, *Water Resour. Res., Vol. 21 (4),* pp. 457-464.

Yeh, T. C., Gelhar, L. W., and Gutjahr, A. L. (1985 c) Stochastic analysis of unsaturated flow in heterogeneous soils, 3, Observations and applications, *Water Resour. Res., Vol. 21 (4),* pp. 465-471.

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