

## ANFIS-based approach for scour depth prediction at piers in non-uniform sediments

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### ABSTRACT

An estimation of scour depth is a prerequisite for the efficient foundation design of important hydraulic structures such as bridge piers and abutments. Most of the scour depth prediction formulae available in the literature have been developed based on the analysis of the laboratory/field data using statistical methods such as the regression method (RM). Conventional statistical analysis is generally replaced in many fields of engineering by the alternative approach of artificial neural networks (ANN) and adaptive network-based fuzzy inference systems (ANFIS). These recent techniques have been reported to provide better solutions in cases where the available data is incomplete or ambiguous by nature. An attempt has been made to compare the performance of ANFIS over RM and ANN in modeling the depth of bridge pier scour in non-uniform sediments. It has been found that the ANFIS performed best amongst all these methods.

**Key words** | ANN, bridge pier, neuro-fuzzy, regression method, scour depth

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### NOTATION

$a_i, c_i$	parameters of premise	$\bar{w}_k$	firing strength of $k$ th inference rule
$b$	width of pier normal to the approaching flow	$x, y$	inputs to the ANFIS
$A_i^k$	fuzzy set for input variable $x$	$z$	output of the ANFIS
$B_j^k$	fuzzy set for input variable $y$	$z_k$	output of $k$ th inference rule
$d_s$	depth of scour	$\sigma_s$	sediment gradation
$d_{84}, d_{50}$	sediment diameters for which 84 and 50% of sediment material is finer by weight, respectively	$\beta$	mean absolute percentage error
$f_k$	consequent function	$\gamma$	root mean square error
$F$	Froude number	$\mu_{A_i}(x)$	membership value of $x$ in $A_i$
$g$	acceleration due to gravity		
$h$	depth of the approaching flow		
$k$	rule number		
$K$	coefficient of regression equation		
$n_1, n_2, n_3$	exponents of regression equation		
$p_k, q_k, r_k$	parameters of consequent part		
$r$	correlation coefficient		
$R_k$	$k$ th rule		
$U$	mean velocity of approach flow		
$w_k$	weight of $k$ th inference rule		

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### INTRODUCTION

Bridge pier scouring is an important issue in the safety evaluation of bridges (Huber 1991). It has been reported that the majority of bridge failures were related to the scour of foundation material. To avoid such failures of bridges, the pier foundation is normally taken deeper than the maximum possible scour depth in its lifetime. Hence, a reliable estimate of the maximum possible scour depth around

a bridge pier is of paramount importance for the safe and economic design of the foundations of bridge piers.

In the past, many investigators have attempted to develop conservative, analytical, semi-empirical or empirical equations based on an understanding of the mechanics of scour, dimensional analysis and data correlation of laboratory experiments and/or field observations (Breusers *et al.* 1977; Melville & Sutherland 1988; Richardson & Davis 1995; Melville 1997; Coleman & Melville 2001; Muzzammil & Gangadhariah 2003). In general, the scour phenomenon is extremely complex in nature and consequently experimental investigations of the scouring has been limited to the consideration of only certain aspects of the problem where other parameters are assumed to be constant. Consequently a simplified laboratory experiment may misinterpret the prototypical conditions. The calibration of the scour prediction model with field data is restricted mainly due to the lack of relevant size and precision of the field data (Yanmaz 2003). The existing scour equations would have, therefore, a considerable uncertainty due to the involvement of a number of uncertain flow, sediment and structural parameters.

A great deal of research effort has been devoted to exploring and refining the methods for improving traditional physical-based analysis in such situations. Recently artificial neural networks (ANN) and adaptive neuro-fuzzy inference systems (ANFIS) are commonly employed as alternative approaches to the traditional regression analysis (ASCE Task Committee 2000a,b; Azmathullah *et al.* 2005; Bateni & Jeng 2007; Bateni *et al.* 2007).

The problem of inherent uncertainties in the physical process and its modeling may be easily addressed by a fuzzy logic approach. The fuzzy approach is based on linguistic expressions that contain ambiguity rather than numerical probabilistic, statistical or perturbation approaches (Uyumaz *et al.* 2006). The fuzzy inference system (FIS) has been employed in the prediction of uncertain systems because its application does not require a knowledge of the underlying physical process as a precondition (Ahmad & Ayyub 2006; Ayyub 2006; Tiwari & Ayyub 2006, 2007; Bateni & Jeng 2007). Şen & Altunkaynak (2004) used fuzzy logic on hydrology for rainfall–runoff modeling. Fuzzy regression has been employed to investigate the modeling uncertainty in the prediction of bridge pier scour by

Johnson & Ayyub (1996). Shrestha *et al.* (1996) carried out the fuzzy-rule-based control systems for reservoir operation. Kindler (1992) applied fuzzy logic for optimal water allocation. Bardossy & Disse (1993) employed it to model the infiltration and water movement in the unsaturated zone. Pongracz *et al.* (1999) reported that fuzzy-rule-based methodology on regional drought provided an excellent tool. Altunkaynak *et al.* (2004a,b) applied the fuzzy logic approach in the modeling of time series and reported its superiority over classical approaches. Uyumaz *et al.* (2006) developed a fuzzy logic model for equilibrium scour downstream of a dam's vertical gate and indicated that a fuzzy logic model has superiority over the regression model. Bateni & Jeng (2007) adopted the ANFIS-based approach for the prediction of pile group scour and found that the errors of the ANFIS model were much less than those of the conventional technique of statistical curve fitting.

The main objective of the present study is to develop an ANFIS model (based on fuzzy logic) for scour depth prediction in non-uniform sediments. The performance assessment of the ANFIS model is compared with that of the regression model and ANN model.

## CONVENTIONAL APPROACH

Most of the scour depth prediction formulae available in the literature have been developed based on the analysis of laboratory/field data using statistical methods such as the regression method. Johnson (1992) had developed a modified scour depth prediction model similar to the Colorado State University (CSU) formulae for non-uniform sediments in the following form:

$$d_s = K_s K_\alpha K h \left(\frac{b}{h}\right)^{n_1} F^{n_2} \sigma_s^{n_3} \quad (1)$$

where  $K_s$  and  $K_\alpha$  are the factors accounting for the pier shape and the flow alignment, respectively, and are well documented (Melville 1997).  $K$  is the coefficient and  $n_1$ ,  $n_2$  and  $n_3$  are the exponents of the equation,  $d_s$  is the maximum depth of local scour at bridge pier,  $b$  is the width of the bridge pier perpendicular to the flow direction,  $h$  is depth of the approach flow,  $F = U/(gh)^{1/2}$  = approach Froude number, where  $U$  is the mean velocity of approach flow,

**Table 1** | Influence of sediment non-uniformity on the scour depth

Sediment gradation ( $\sigma_s$ )	1.0	1.50	2.00	2.40	2.75	3.30	4.00	7.80
Factor for sediment gradation ( $K_\sigma$ )	1.0	0.90	0.75	0.50	0.38	0.25	0.16	0.08

$K_\sigma = d_{sn}/d_{su}$ ;  $d_{su}$  = scour depth for uniform sediment;  $d_{sn}$  = scour depth for non-uniform sediment.

$g$  is acceleration due to gravity and  $\sigma_s$  = sediment gradation =  $d_{84}/d_{50}$ , where  $d_{84}$  and  $d_{50}$  are the sediment diameters for which 84 and 50% of sediment material is finer by weight, respectively. In Equation (1),  $\sigma_s$  is assumed to be 1, representing uniform sediment. The sediment gradation ( $\sigma_s$ ) is an index for the sediment non-uniformity and it has a strong influence on the scour depth. When standard deviation of the sediment is large, the sediment contains some non-moving sizes for a given discharge; the coarser material would tend to accumulate in the scour hole and inhibit the development of scour. As such, for the same median size, scour depth will be smaller for material with a larger standard deviation. The influence of sediment gradation on the scour depth may clearly be observed in Table 1 (Kothyari 2003). The coefficient ( $K$ ) and exponents ( $n_1$ ,  $n_2$  and  $n_3$ ) may be easily obtained from regression analysis of the observed data of scour.

Laboratory scour data from the University of Auckland is used in the present study, which was used by Johnson (1992). The regression model was developed for scour depth prediction at a bridge pier in non-uniform sediments. The data were collected in an 11.8 m long, 440 mm wide and 380 mm deep glass-sided flume. The sample size is 130. Table 2 indicates the mean and coefficients of variation (COV) of the dataset. The COV is the largest for  $b/h$  and it is smallest for  $h$ . The COV for the dataset considered indicates a moderate range of data for each of the parameters. The correlation matrix of the data is given in Table 3.

**Table 2** | Mean and coefficients of variation of scour parameters

x	Mean	COV
$h$ (mm)	152.0	0.2294
$b/h$	0.382	0.6897
$F$	0.537	0.3789
$\sigma_s$	1.922	0.6497
$d_s$ (mm)	70.30	0.3539

The values obtained for mean, COV (for all the scour parameters) and correlation (among the parameters) is almost the same as that given by Johnson (1992). However, the correlation of sediment gradation  $\sigma_s$  is different from that given by him. The linear correlation of  $\sigma_s$  with scour depth  $d_s$  is found to be  $-0.44$  against  $0.32$  estimated by Johnson (1992).

The nonlinear regression was employed to get the regression parameters of the scour prediction model using only 80% (104 dataset) of the available entire dataset selected randomly. It leads to the following equation for the estimation of scour depth at the bridge pier for the non-uniform sediments:

$$d_s = 1.38h \left(\frac{b}{h}\right)^{0.72} F^{0.21} \sigma_s^{-0.24} \quad (2)$$

A quantitative comparison between observed and predicted values of scour was made in terms of three error measures (performance indices) such as (i) correlation coefficient ( $r$ ), (ii) the mean absolute percentage error ( $\beta$ ) and (iii) root mean square error ( $\gamma$ ). The performance of Equation (2) was evaluated during the calibration as well as the validation process and is shown in Table 4. The remaining unseen 20% of data (26) were used for the validation. It may be observed that the performance of the proposed model is satisfactory. However, the calibration set of data performs relatively better than that of the validation set of data. Figure 5(a) shows the scatter diagram of observed versus predicted values of scour depth for regression analysis for the qualitative assessment of the regression model for scour depth prediction.

The scour depth prediction Equation (2) has been obtained using the regression method (RM). The following drawbacks in any RM application have been pointed out by Sen *et al.* (2003, 2004) and Uyumaz *et al.* (2006):

- (i) The deviations of scatter points from the fitted curve have zero average with assumed constant. However,

**Table 3** | Correlation matrix for scour parameters

$x$	$h$	$b/h$	$F$	$\sigma_s$	$d_s$
$h$	1.00	-0.88	-0.33	0.27	-0.60
$b/h$	-0.88	1.00	0.43	-0.26	0.77
$F$	-0.33	0.43	1.00	-0.16	0.54
$\sigma_s$	0.27	-0.26	-0.16	1.00	-0.44
$d_s$	-0.60	0.77	0.54	-0.44	1.00

**Table 4** | Performance of scour model for calibration and validation process for regression method

Performance index	Calibration	Validation
$r$	0.88	0.91
$\beta$	15.35	19.64
$\gamma$	0.0112	0.0114

in the actual scatter diagram most often the variance is not constant but changes depending on the independent variable value.

- (ii) The regression curve may pass close to a certain percentage of points in the scattered diagram, but this cannot account for the validity of the method.
- (iii) The prediction error is expected to abide with a Gaussian distribution function, which is not the case in many practical studies.
- (iv) The prediction errors are also expected to be independent from each other, i.e. completely random (noise).

Further, the use of regression analysis, where high inter-correlation among the independent variables exists, will result in standardized correlation coefficients that do not accurately reflect the importance of the variables (McCuen 1985; Johnson 1992). In order to avoid such problems in the application of the regression analysis, an alternative approach is generally advocated (Johnson 1992; Uyumaz *et al.* 2006).

## ARTIFICIAL NEURAL NETWORKS APPROACH

Artificial Neural Networks (ANN) are considered to be a flexible modeling tool capable of learning the mathematical mapping between input and output variables of nonlinear systems (Bateni & Jeng 2007). ANNs with commonly used

algorithms such as the Feed-Forward Back-Propagation (FFBP), Feed-Forward Cascade correlation (FFCC) and Radial Basis Function (RBF) algorithm were developed in the MATLAB environment for the scour depth prediction modeling in non-uniform sediments. Most of the engineering applications of the networks are usually based on the FFBP, while the FFCC algorithm ensures an optimum network configuration, whereas the RBF algorithm is considered to be an advanced training scheme (ASCE Task Committee 2000a; Azmathullah *et al.* 2005).

The training data was the same randomly selected 80% of the available entire data for the network that was used in the regression analysis above. The remaining 20% of data was used for validation.

The ANN models of various training algorithms were employed for scour depth prediction using flow depth ( $h$ ), relative pier width ( $b/y$ ), Froude number ( $F$ ) and sediment gradation ( $\sigma$ ) as input and scour depth ( $d_s$ ) as the output. The number of neurons in the input and the output layers is thus fixed. The next step is to select the number of hidden layers and the number of neurons in each layer. The number of hidden layers as well as the number of neurons in the hidden layer is crucial for the successful application of ANNs. The use of one hidden layer is generally recommended at least in preliminary studies. Kumar & Ray (1997) reported that the performance of a single hidden layer is better than that of a double hidden layer for the rainfall-runoff modeling. Furthermore, an increase in the hidden layer generally slows down the training process without substantially improving the efficiency of the network. A single hidden layer was therefore considered in the present study. The network employed herein consists of three layers with a tan-sigmoid transfer function in the hidden layer and a linear transfer function in the output layer. This is a useful structure for function approximation problems.

The determination of an appropriate number of neurons in the hidden layer is another important aspect for an efficient network since it generally enhances the performance of the neural network. Several authors have given guidance for the number of neurons to be used in a hidden layer. Hecht-Nielsen (1988) suggested an upper limit of  $(2i + 1)$  hidden layer neurons, where  $i$  is the number of input neurons. Recently Xu & Chen (2008)

reviewed several mechanisms of hidden layer neurons and proposed a novel approach based on some mathematical evidence. They suggested the relations for the optimal number of neurons ( $n_o$ ) in the hidden layer based on their experiments on a small or medium-sized dataset (with less than 5,000 training pairs) can be expressed as

$$n_o = n_m \quad \text{for } n_r \leq 30$$

$$n_o = (N(d \log N))^{1/2} \quad \text{for } n_r > 30$$

where  $n_r = N/d$ ,  $n_m = n_r$  = maximum number of neurons,  $N$  = number of training pairs and  $d$  = the input dimensions of the target function.

The general practice for selecting an appropriate number of neurons in the hidden layer is by a trial-and-error procedure (Azimathullah *et al.* 2005). Hence, the number of neurons in the hidden layer was selected by a trial-and-error method based on the minimum mean square error and the maximum correlation coefficients. The influence of the number of hidden neurons on the performance in the case of a feed-forward back-propagation algorithm has been shown in Table 5. Table 5 indicates that architecture of 4–16–1 is more appropriate for both the phases of training as well as validation, and is considered to be an optimal structure.

The above procedure was adopted to get the final optimal structure for other training algorithms. Table 6 summarizes the optimal network architecture for various algorithms of the neural network based on a trial-and-error procedure.

The training of the network was then carried out and the progress of the training was monitored with a useful

**Table 5** | Effect of neurons in hidden layer on the performance of FFBP

Network structure	Performance during training			Performance during validation		
	$r$	$\beta$	$\gamma$	$r$	$\beta$	$\gamma$
4-3-1	0.94	10.08	0.0079	0.96	10.91	0.0079
4-4-1	0.91	12.56	0.0100	0.93	15.59	0.0100
4-5-1	0.95	9.61	0.0075	0.95	14.11	0.0087
4-6-1	0.96	8.17	0.0063	0.97	10.57	0.0063
4-10-1	0.96	7.37	0.0063	0.97	9.23	0.0063
4-16-1	0.97	6.17	0.0054	0.98	9.08	0.0061
4-18-1	0.97	6.25	0.0057	0.98	9.83	0.0067
4-20-1	0.97	6.77	0.0058	0.98	9.23	0.0059

**Table 6** | Details of structure of different ANN networks for the scour depth prediction

Training algorithm	Network configuration		
	I	H	O
FFBP	4	16	1
FFCC	4	15	1
RBF	4	25	1

I, H, O represent nodes in input, hidden and output layers, respectively. FFBP stands for Feed-Forward Back-Propagation, FFCC for Feed-Forward Cascade Correlation and RBF for Radial Basis Function.

diagnostic tool available in MATLAB to plot the training and validation errors. The network training was stopped only when the validation error was more than the training error. These conditions are considered to be essential to avoid any significant over-fitting.

Table 7 shows the various parameters of performance during training and validation, respectively. It may also be observed that RBF performs the best among all the prediction models for the training set of data but not for the validation dataset. However, the FFBP algorithm of ANN performs relatively satisfactorily both for training and validation. It may also be observed that the performance of ANN with all training algorithms under consideration is better than that of the regression method. Figure 5(b) shows the scatter diagram of observed values of scour depth versus predicted results.

The FFBP model may, therefore, be recommended in general for the prediction of the scour depth due to its simple architecture. The weights and biases for FFBP have been provided in Table 8. Figure 1 shows the network configuration with FFBP that may be used for the prediction of the maximum equilibrium scour depth around bridge piers in non-uniform sediments along with the weights and biases in Table 8.

**Table 7** | Performance of various ANN prediction models for the scour depth prediction

ANN prediction models	Training stage			Validation stage		
	$r$	$\beta$	$\gamma$	$r$	$\beta$	$\gamma$
FFBP	0.97	5.78	0.0054	0.98	7.57	0.0061
FFCC	0.96	6.93	0.0049	0.97	11.43	0.0065
RBF	0.97	5.59	0.0053	0.97	9.23	0.0080

**Table 8** | Weights and biases for FFBP (Feed-Forward Back-Propagation algorithm)

Weights				Biases			
$W_{j,i}$				$W_{j,k}$	$b_j$	$b_k$	
1.255	-0.941	0.973	0.106	-0.602	-3.4875	-1.0862	
0.128	-1.255	0.933	-2.176	0.470	-0.0059		
1.211	1.334	-0.038	1.081	0.775	-3.4430		
-1.366	-0.597	1.629	-0.423	-0.506	1.5279		
1.377	-0.326	-0.831	0.188	-0.712	-1.7234		
-1.143	0.141	-0.085	1.933	-1.645	-0.6544		
-0.806	-0.938	-0.504	-0.031	-0.567	1.2535		
0.277	-2.045	0.075	-0.142	-1.627	-2.2539		
1.351	-1.204	-0.848	-1.661	-0.327	1.3222		
-1.43	0.147	0.594	0.115	0.414	-1.4777		
-1.130	0.122	-1.160	0.355	-0.566	-1.3411		
1.535	0.44	0.338	0.111	0.912	-0.9151		
-0.855	0.683	0.682	-0.445	0.197	-2.0871		
-0.326	-0.693	1.232	0.624	-0.829	-3.2256		
0.636	-0.081	0.989	0.712	-0.259	1.2068		
-0.838	0.647	0.118	-1.254	0.607	-1.9963		

$W_{j,i}$  = weight between hidden layer neurons and input layer neurons;  $W_{j,k}$  = weight between hidden layer neurons and output layer neurons,  $b_j$  = biases of hidden layer neurons;  $b_k$  = biases of output layer neurons.

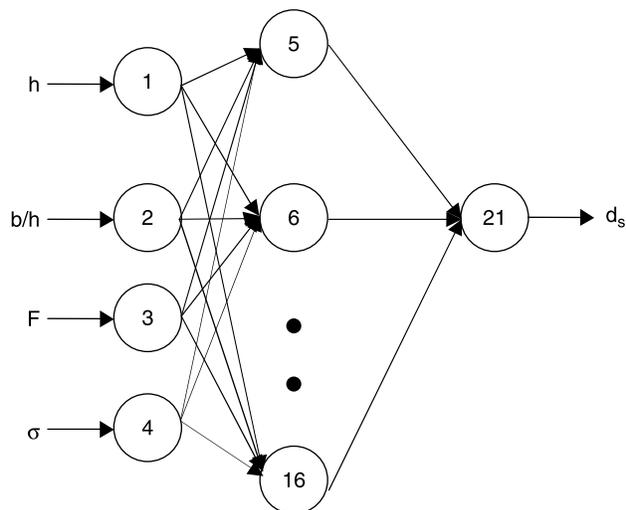
Some drawbacks of the FFBP algorithm have also been reported in the literature. The FFBP are sensitive to the selected initial weights and may provide performances differing from each other significantly. Another problem faced during the application of FFBP is the local minima issue. During the training stage the networks are sometimes trapped by the local error minima, preventing them from reaching the global minimum (Cigizoglu & Murat 2006).

## ANFIS APPROACH

The fuzzy logic system is good at knowledge acquisition and handling such fuzzy information as an expert's experience with respect to the observed input-output data. The fuzzy logic system has been widely applied to modeling, control, identification, prediction, etc. But the fuzzy model lacks a self-learning and adaptive ability. The neural network has been shown to possess a learning and adaptive ability to input-output data. It is proved to have a good approximate capability for a wide range of nonlinear function and has been modeled for nonlinear dynamic systems. But in system

modeling, network training results in a black-box representation. The model developed is difficult to interpret through human language (Sun & Cheng 2005).

The adaptive neuro-fuzzy inference system (ANFIS) is basically an integration of the techniques of fuzzy

**Figure 1** | Structure of ANN with FFBP.

systems and artificial neural networks (ANN). The ANN provides connectionist structures and learning abilities to the fuzzy systems whereas the fuzzy systems offer ANN a structured framework with high level IF-THEN rule thinking and reasoning.

There are two types of fuzzy inference systems: (i) Mamdani-type (Mamdani 1974) and (ii) Takagi-Sugeno (TS)-type (Takagi & Sugeno 1985). Mamdani's fuzzy inference method is the most commonly seen fuzzy methodology and was among the first control systems built using fuzzy set theory. Mamdani approach provides the outcome of the fuzzy rule as a fuzzy set for the output variable and hence the step of defuzzification is essential to get a crisp value of the output variable, whereas the TS approach does not require a classical defuzzification procedure and the outcome of the fuzzy rule is a scalar rather than a fuzzy set for the output variable. The main problem associated with the TS fuzzy logic modeling is related to the selection of the parameters. An effective method is, therefore, required to tune the membership functions so as to minimize the error measures. Jang (1993)

proposed the ANFIS approach to optimize the parameters of the membership functions and the consequent part by using a hybrid learning algorithm. The fuzzy model parameters may be estimated by various approaches such as clustering techniques, genetic algorithms, gradient decent algorithms and numerical analysis. The neural network back-propagation learning algorithm and the least squares method are, however, simple and efficient methods and generally employed to estimate the membership function parameters and the consequent part parameters, respectively (Uyumaz *et al.* 2006).

### ANFIS architecture

ANFIS was first put forward by Jang (1993) and is classified into three types according to the types of fuzzy reasoning and fuzzy if-then rule employed. A third of them are Takagi-Sugeno ANFIS. The selection of the fuzzy inference system (FIS) is the major concern in the design of ANFIS. The present study is based on Takagi and Sugeno's fuzzy if-then rules representation, wherein the consequent

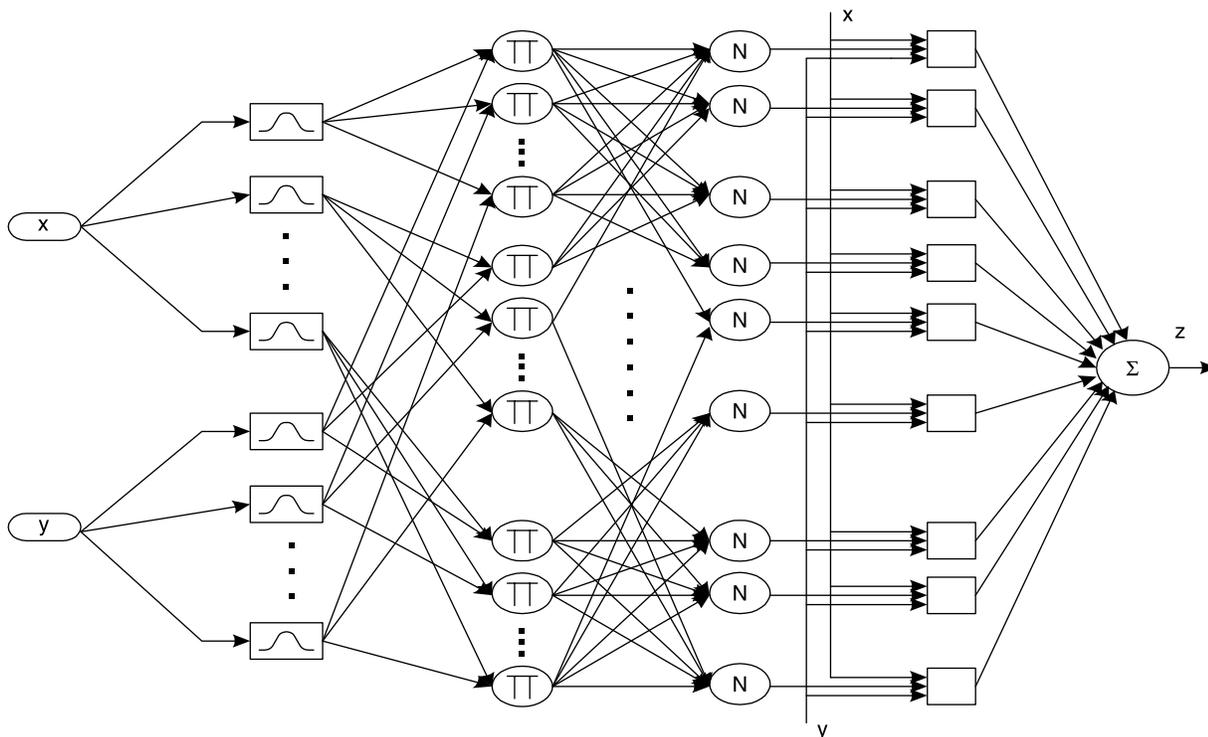


Figure 2 | Structure of two input and single output ANFIS.

**Table 9** | The two passes of ANFIS hybrid learning

Premise parameters	Forward pass fixed	Backward pass gradient descent
Consequent parameters	LS estimator	Fixed
Signal	Node output	Error signal

part of the rule is a linear function of input variables and the parameters may be estimated by a simple least-squares error method.

Figure 2 depicts a typical architecture of ANFIS for a system with two inputs ( $x$  and  $y$ ) and single output ( $z$ ). The inference mechanism of ANFIS is mathematically expressed by the set of rules. These rules are generated through the experience of the system operator, design engineer or expert.

The  $k$ th rule is generally expressed in the form (If *premise* THEN *consequence*) and is given by

If ( $x$  is  $A_i^k$ ) and ( $y$  is  $B_j^k$ ) Then (3)  
 ( $z$  is  $f_k = p_kx + q_ky + r_k$ )

where  $A_i^k$  and  $B_j^k$  are the  $i$ th and  $j$ th fuzzy term sets of representing  $x$  and  $y$ , respectively. The consequent function

$f_k = p_kx + q_ky + r_k$  has parameters  $p_k$ ,  $q_k$  and  $r_k$ , which are adjustable and are tuned in the training phase. A bell-shaped or Gaussian membership function is commonly considered for each fuzzy subset that has three/two parameters. We have considered a Gaussian membership function in the present study, which has only two parameters. Each fuzzy subset (say  $A_i$ ) is defined by a membership function  $\mu_{A_i}(x)$  as in (4):

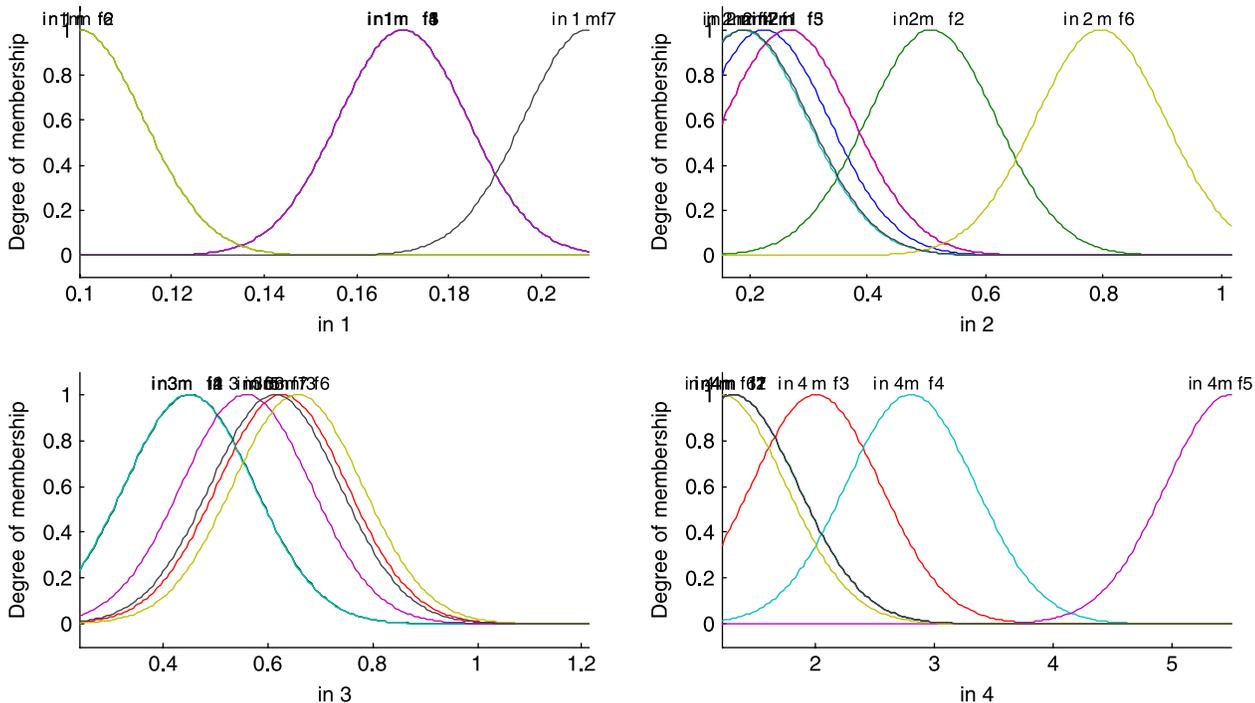
$$\mu_{A_i}(x; a_i, c_i) = \exp\left(-0.5\left(\frac{x - c_i}{a_i}\right)^2\right) \tag{4}$$

where  $a_i$  and  $c_i$  are the parameters of the antecedent fuzzy set  $A_i$  of the  $i$ th membership function. These parameters control the shape of the Gaussian membership function.

The architecture of the ANFIS in Figure 2 has five layers. The functional details of these layers are as follows:

**Layer 1**

This layer calculates the degree to which the given input  $x$  (or  $y$ ) satisfies the term set  $A_i$  (or  $B_j$  for input  $y$ ).



**Figure 3** | Details of initial membership functions (in 1 =  $h$ , in 2 =  $b/y$ , in 3 =  $F$  and in 4 =  $\sigma_s$ ).

The membership function for each term set  $A_i$  (or  $B_j$ ) may be a generalized bell function or Gaussian function, as described above.

The parameters  $(a_i, b_i, c_i)$  or  $(a_i, c_i)$  for a generalized bell or Gaussian function respectively are termed as premise parameters.

**Layer 2**

The nodes in this layer are fixed nodes labelled  $\pi$ . The output of each node is the product of the incoming signals. Thus the output of the  $k$ th node of the layer is given by

$$w_k = \mu_{A_i}(x)\mu_{B_i}(y) \tag{5}$$

The output of each node represents the strength of the corresponding rule.

**Layer 3**

The nodes of this layer are fixed nodes labelled  $N$ . The  $i$ th node of the layer calculates the normalized firing

strength of the corresponding rule. The normalized firing strength ( $\bar{w}_k$ ) of a rule ( $k$ th) is the ratio of the strength of that rule ( $w_k$ ) and the sum of the strengths of all rules, i.e.

$$\bar{w}_k = \frac{w_k}{\sum_k w_k} \tag{6}$$

**Layer 4**

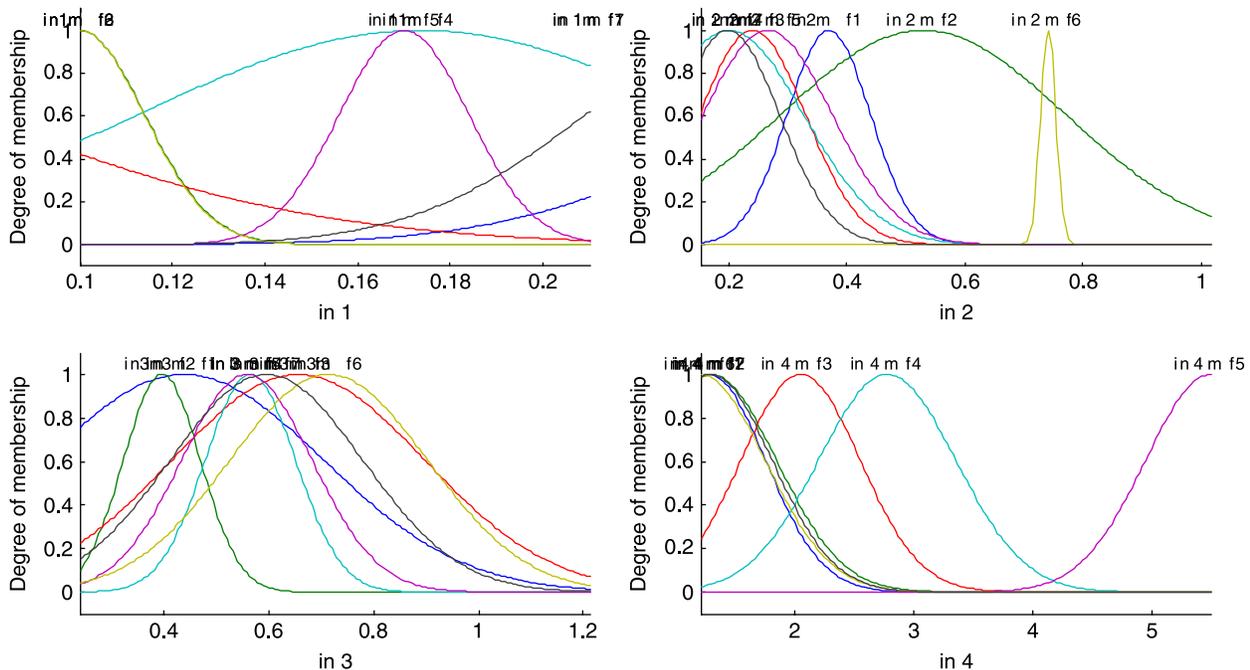
The nodes of this layer are adaptive nodes. The node function of the  $i$ th node of the layer is given by

$$z_k = \bar{w}_k f_k = \bar{w}_k (p_k x + q_k y + r_k) \tag{7}$$

Here  $\bar{w}_k$  is the normalized firing strength of the  $k$ th rule, which is obtained in layer 3, and  $(p_k, q_k, r_k)$  is the set of parameters of this layer. These parameters are referred to as consequent parameters.

**Layer 5**

It has one node for a single output, which is a fixed node labelled  $\Sigma$ . This node calculates the sum of all incoming



**Figure 4** | Details of final membership functions (in 1 =  $h$ , in 2 =  $b/y$ , in 3 =  $F$  and in 4 =  $\sigma_s$ ).

**Table 10** | ANFIS details

Number of nodes	77
Number of linear parameters	35
Number of nonlinear parameters	56
Total number of parameters	91
Number of training data pairs	104
Number of checking data pairs	26
Number of fuzzy rules	7
Radius of cluster	0.36

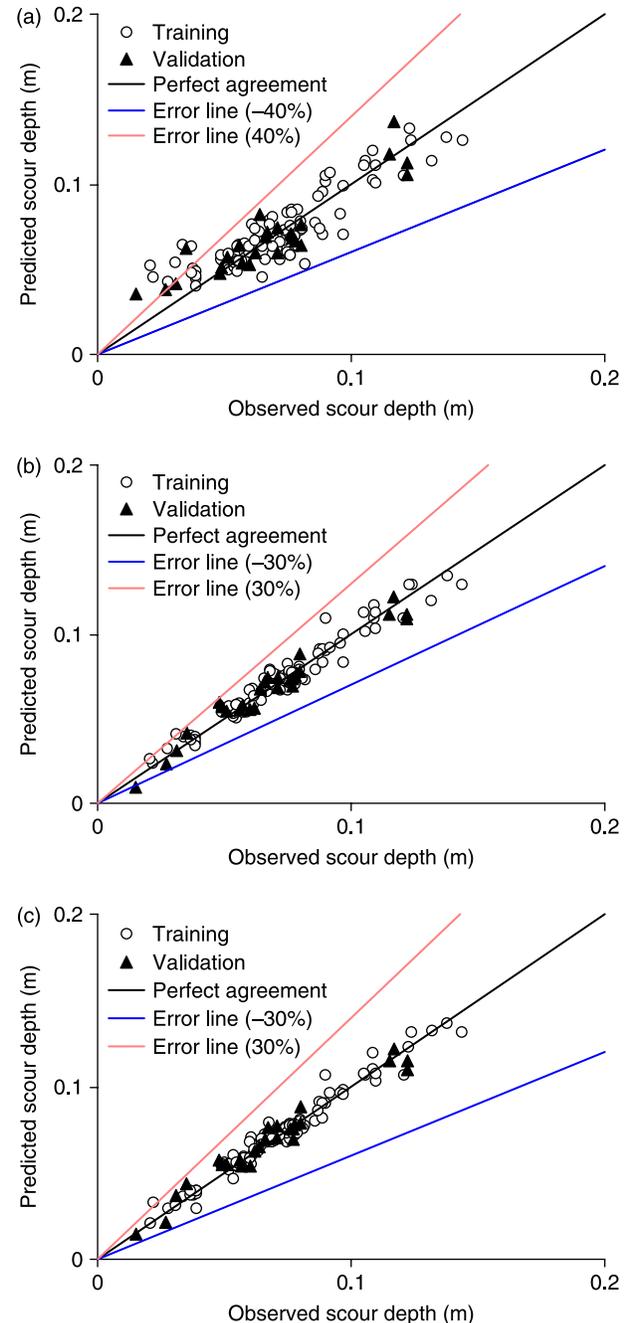
signals. Thus the overall output ( $z$ ) of the ANFIS is

$$z = \sum_k z_k = \sum_k (\bar{w}_k f_k) = \frac{\sum_k w_k f_k}{\sum_k w_k} \quad (8)$$

The ANFIS modelling involves two major phases: (i) structure identification and (ii) parameter estimation. The structure identification amounts to determining the proper number of rules required, i.e. finding how many rules are necessary and sufficient to properly model the available data and the number of membership functions for input and output variables. Clustering techniques have been recognized as a powerful tool to extract initial fuzzy rules from given input-output data. The purpose of the clustering is to identify a natural grouping of data from a large dataset to produce a concise representation of the system behaviour. Subtractive clustering is commonly used for the initialization of the parameters for ANFIS training.

The steepest descent (or back-propagation) method is applied for identification of the parameters. But this usually takes a long time before it converges. In the present case we find that some of the parameters are linearly related to the output (consequent parameters). Thus these linear parameters can be identified by the least-squares error (LSE) method for fixed values of nonlinear parameters (premise parameters). After identifying the consequent parameters (which are linear) we can apply the steepest descent method for identification of premise parameters (which are nonlinear parameters). This hybrid learning approach, which combines the steepest descent method and LSE method, gives fast identification of

parameters. The hybrid learning algorithm has two passes, the forward pass and the backward pass. In the forward pass the node output is calculated till layer 4. Then LSE is applied to identify the consequent parameters. After this the backward pass is done. In this, the error signal is the



**Figure 5** | Scatter diagram of observed versus predicted values of scour depth for (a) regression method; (b) ANN model and (c) ANFIS model.

**Table 11** | Performance of ANFIS model for scour depth prediction

Index	Training	Validation
$r$	0.98	0.98
$\beta$	4.78	7.49
$\gamma$	0.0046	0.0055

$r$  = correlation coefficient;  $\beta$  = mean absolute percentage error;  $\gamma$  = root mean squared error.

input and is propagated backward. From the error signal the premise parameters are calculated by applying the steepest descent method. The procedure of the hybrid learning is summarized in Table 9.

## APPLICATION OF ANFIS TO THE SCOUR DEPTH PREDICTION

The ANFIS was employed to get the fuzzy parameters for the prediction of scour depth in the case of non-uniform sediments. As in the previous two cases, here also only 80% of the available data was used for model prediction and the remaining unseen 20% of data was used for testing of the model. This was done in the MATLAB environment. ANFIS, along with a subtractive clustering method, was employed for the scour depth prediction using flow depth ( $h$ ), relative pier width ( $b/y$ ), Froude number ( $F$ ) and sediment gradation ( $\sigma$ ) as the input and scour depth ( $d_s$ ) as the output. The optimum value of cluster radius was determined by trial and error based on the criterion of the maximum correlation coefficient and minimum root mean square error. In the present analysis we got its value to be 0.36 for which the optimum number of rules obtained was only seven. The study showed that a smaller value of the cluster radius results in a large number of clusters leading to a larger number of rules, and vice versa. Larger numbers of

rules take longer for calculations and it gives better prediction. However, the improvement is seen to be not a substantial one.

Figures 3 and 4 depicts the details of the initial and final membership functions (MFs), respectively. The initial and final MFs of each of the input parameters ( $h$ ,  $b/h$ ,  $F$ ,  $\sigma$ ) may be compared with each other. It may be observed that there is a drastic change in the shape of  $h$  and  $F$ , a moderate change in  $b/h$  and a slight change in  $\sigma$ . The change in the shapes of MFs for an input after training reflects its influence on the output.

Brief details of the ANFIS parameters for the present problem have been shown in Table 10. The performance of the ANFIS was assessed qualitatively, shown in Figure 5(c), and quantitatively in Table 11, both for training and validation processes. It can be clearly observed from these that the performance for ANFIS-based modeling is satisfactory and better.

## An overall assessment of scour depth prediction models

An overall assessment of the various scour depth prediction models has also been made. This is to compare the performance of the ANFIS model over other models. Table 12 shows the details of the performance indices of the models for the training as well as validation set of data. It may be observed that the performance of ANFIS is the best among the prediction models under consideration, as it gives maximum correlation ( $r = 0.98$ ), minimum mean absolute percentage error ( $\beta = 7.49$ ) and minimum root mean squared error ( $\gamma = 0.0055$ ) for the validation set of data. A similar conclusion may also be drawn for the data of the training set. Table 12 also shows that the ANFIS model provides the best prediction of the scour among the scour

**Table 12** | Performance of various models of scour depth prediction

Scour prediction models	Training stage			Validation stage		
	$r$	$\beta$	$\gamma$	$r$	$\beta$	$\gamma$
Regression model (Johnson 1992)	0.86	23.69	0.0240	0.89	25.50	0.0242
Regression model (present study)	0.88	15.35	0.0112	0.91	19.62	0.0114
ANN model (FFBP)	0.97	5.78	0.0054	0.98	7.57	0.0061
ANFIS model	0.98	4.78	0.0046	0.98	7.49	0.0055

depth predictors in the literature (Johnson 1992), for which the data is available to the author. It gives better prediction even with respect to the regression models developed in the present study.

The detailed calculation of the performances is also listed in Table 13. The best performance of the ANFIS model over the other models under consideration may also be observed qualitatively in Figure 5(c). Thus an overall assessment of scour depth prediction models indicates herein that the scour data are more amenable to fuzzy if-then rules rather than the crisp value processing.

The ANFIS ensures localized functioning of the transfer function as against the globalized one of a general feed-forward network (FFN) resulting in smaller number of values participating in the mapping process, and hence may work well with the limited data for training.

### ANFIS model for scour depth prediction using original dataset

The pattern of the data presented for network training is considered to be one of the important aspects of the ANFIS

**Table 13** | Performance of various scours models for the validation dataset

Observed ( $d_s$ )	Predicted scour depth ( $d_s$ )			Absolute percentage error		
	RM	ANN	ANFIS	RM	ANN	ANFIS
0.049	0.051	0.057	0.055	4.08	16.33	12.24
0.060	0.053	0.055	0.0544	11.33	8.00	9.33
0.077	0.070	0.074	0.0754	8.57	4.55	2.08
0.076	0.071	0.073	0.0757	6.18	3.82	0.39
0.051	0.057	0.055	0.0548	11.57	7.45	7.45
0.077	0.069	0.069	0.0695	10.91	10.13	9.74
0.071	0.075	0.075	0.0776	4.93	5.35	9.30
0.035	0.063	0.042	0.0437	78.86	18.86	24.86
0.064	0.082	0.067	0.0649	28.28	5.16	1.41
0.066	0.069	0.072	0.0691	5.00	8.94	4.70
0.015	0.036	0.009	0.015	137.33	38.67	0.00
0.048	0.047	0.060	0.0571	1.46	25.00	18.96
0.027	0.038	0.024	0.0217	40.37	12.22	19.63
0.080	0.064	0.078	0.0786	20.25	3.00	1.75
0.071	0.060	0.069	0.07	15.49	3.10	1.41
0.078	0.067	0.075	0.078	14.74	3.59	0.00
0.031	0.042	0.031	0.0367	34.84	1.29	18.39
0.057	0.054	0.058	0.0539	5.61	1.58	5.44
0.056	0.064	0.055	0.0571	13.75	1.61	1.96
0.062	0.059	0.056	0.063	4.19	9.68	1.61
0.080	0.076	0.088	0.0882	5.00	10.25	10.25
0.117	0.137	0.122	0.1221	16.75	4.10	4.36
0.122	0.112	0.111	0.1152	7.87	8.77	5.57
0.067	0.072	0.074	0.0762	6.72	10.75	13.73
0.122	0.105	0.109	0.11	13.69	10.49	9.84
0.115	0.118	0.112	0.1147	2.43	2.87	0.26
Mean absolute percentage error				19.62	9.06	7.49
Standard deviation of absolute percentage error				28.97	8.28	7.01

**Table 14** | Performance of ANFIS models for different combinations of data

Patterns of input data	Cluster radius	Performance index in training			Performance index in validation		
		$r$	$\beta$	$\gamma$	$r$	$\beta$	$\gamma$
Grouped data	0.36	0.98	4.78	0.0046	0.98	7.48	0.0055
Raw data	0.45	0.98	5.23	0.0053	0.98	7.14	0.0054

modeling approach. The two combinations of data have been considered in the present study. The first combination consists of dimensionless parameters (grouped data) as the inputs and the dimensional parameters as outputs. This is already discussed in previous sections. The second combination considers dimensional parameters (raw data) where both inputs and outputs are raw data. The scour prediction Equation (1) is in the form of grouped data. It can be rewritten in terms of original variables as

$$d_s = f(h, b, v, \sigma) \quad (9)$$

It indicates that the scour depth depends on the flow depth ( $h$ ), pier width ( $b$ ), the approach flow velocity ( $v$ ) and sediment gradation parameter ( $\sigma$ ). The performance of the ANFIS model in training as well as in validation is shown in Table 14 with reference to different combinations of data. It may be observed that the ANFIS predicts scour depths better when it is trained with the non-dimensional (grouped) data than the original (dimensional) data in training and comparably in the validation.

## SENSITIVITY ANALYSIS

The sensitivity tests are commonly carried out to ascertain the relative significance of each of the independent parameters on the dependent parameters. All the independent parameters re considered in turn in the sensitivity analysis.

**Table 15** | Sensitivity analyses of the non-dimensional independent parameters

Inputs	Training phase			Validation phase		
	$r$	$\beta$	$\gamma$	$r$	$\beta$	$\gamma$
All	0.98	4.780	0.0046	0.98	7.48	0.0055
Without $\sigma$	0.89	15.58	0.011	0.92	22.13	0.0112
Without $F$	0.92	10.35	0.0096	0.96	11.30	0.0088
Without $b/h$	0.88	13.47	0.0112	0.72	31.75	0.0193
Without $h$	0.98	5.40	0.0047	0.98	7.80	0.0060

## Sensitivity analysis of the input parameters based on grouped data

The results of sensitivity analysis for the input scour parameters based on grouped data are shown in Table 15. The table indicates that  $b/h$  and  $h$  have respectively the most and the least effect on the scours depth ( $d_s$ ).

## Sensitivity analysis of the input parameters based on raw data

The results of sensitivity analysis for the input scour parameters based on raw data are depicted in Table 16. It may be observed from this table that  $b$  and  $h$  have respectively the most and the least effect on the scour depth ( $d_s$ ). In the present case, the mean value of  $b/h$  (Table 2) is 0.382, which is less than 0.7. This is the case of the narrow pier (or deep flow) where scour depth depends on the pier size and independent of the flow depth (Melville 1997). In the case of deep flow, the surface roller (bow wave) does not interfere with the down flow and its strength and, as such, the flow depth has only marginal or no influence on the scour depth. Table 3 also support this result where the highest correlation exists between  $b/h$  and  $d_s$  and the coefficient of variation of flow depth being less than that of the pier width (Johnson 1992). It is therefore in agreement with the experimental results available in the literature.

**Table 16** | Sensitivity analyses of the dimensional independent parameters

Inputs	Training phase			Validation phase		
	$r$	$\beta$	$\gamma$	$r$	$\beta$	$\gamma$
All	0.98	5.23	0.0053	0.98	7.14	0.0054
Without $\sigma$	0.88	15.21	0.0111	0.90	23.76	0.0123
Without $v$	0.91	10.56	0.0097	0.96	10.83	0.0088
Without $b$	0.88	13.94	0.0114	0.81	31.15	0.0172
Without $h$	0.97	6.07	0.0058	0.95	12.10	0.0088

## CONCLUSIONS

A reliable estimation of the scour depth is a prerequisite for the efficient foundation design of a bridge pier. The present study addressed this problem and developed a regression-based prediction model of scour depth for the piers in non-uniform sediments. The refinement in the prediction model was done through the recent alternative approaches of ANN and ANFIS.

The ANN was trained with the commonly used training algorithms such as the Feed-Forward Back-Propagation (FFBP), Feed-Forward Cascade Correlation (FFCC) and Radial Basis Function (RBF). It was found that RBF performed the best among all the training algorithms during the training phase. However, FFBP outperforms it during the validation phase. Moreover, during the training phase, the FFBP algorithm shows comparable performance with RBF. Therefore, FFBP can be considered for training of ANN for the prediction of the scour depth at the bridge pier due to its overall performance and simple structure. Further, all these training algorithms for ANN were found to give better performance when compared with the regression method.

The implementation of the ANFIS model indicated that the ANFIS with seven rules having a cluster radius of 0.36 appeared to be the best among the models considered (regression model as well as the ANN model) during the training as well as validation phases. It shows that the scour data are more amenable to fuzzy if-then rules rather than crisp value processing. The ANFIS ensures localized functioning of the transfer function as against the globalized one of a general FFN, resulting in a smaller number of values participating in the mapping process, and hence may work well with the limited data for training.

The pattern of the data presented for the network training is considered to be one of the most important aspects of the ANFIS modeling approach. It was also found that the ANFIS gives better prediction of scour depth when it is trained with grouped (dimensionless) data rather than raw data.

The sensitivity analysis of the grouped data indicated that  $b/h$  and  $h$  have respectively the most and the least effect on scour depth ( $d_s$ ), while that of raw data showed that pier diameter has a greater influence on equilibrium scour depth than the other independent parameters.

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