WZ Sagittae as a DQ Herculis star

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Accepted 1999 January 8. Received 1998 December 30; in original form 1998 October 20

ABSTRACT

We argue that quiescent WZ Sge is a rapidly spinning magnetic rotator in which most of the matter transferred from the secondary is ejected from the system. Assuming that the observed 27.87-s oscillation period results from the spinning white dwarf, we propose that the other observed principal period of 28.96 s is a beat caused by reprocessing of the rotating white dwarf beam on plasma blobs in Keplerian rotation at the outer disc rim. The weaker, transient, 29.69-s period is identified as a beat with the Keplerian period of the magnetosphere. WZ Sge evolves through a cycle of spin-up and spin-down phases. During the spin-up phase it is a DQ Her star; during the spin-up phase it should be an ER UMa star.

Key words: stars: individual: WZ Sge – stars: magnetic fields – novae, cataclysmic variables – stars: oscillations – stars: rotation.

1 INTRODUCTION

WZ Sagittae is a remarkable dwarf nova binary system. Dwarf novae are a subclass of cataclysmic variables (CVs) which show more or less regular outbursts. In CVs, a late-type, Roche-lobe filling secondary star loses mass which, in general, is accreted by a white dwarf primary. Usually, when the white dwarf is not too strongly magnetized, the accreting matter forms an accretion disc. It is well established that such discs are the sites of dwarf nova outbursts. WZ Sge outbursts are very rare (the recurrence time is around 30 yr) and they are of the ‘superoutburst’ type, i.e., they are long (∼ 30 d), high-amplitude (∼ 7 mag) outbursts during which a ‘superhump’ in the optical light curve is observed at a period slightly longer than the orbital one. WZ Sge is the prototype of a class of systems with similar outburst properties (sometimes also called ‘TOADs’). The orbital period of WZ Sge is 81.63 min, close to the minimum period (∼ 80 min), below which no (non-degenerate) CV has been observed.

It is generally believed that dwarf nova outbursts result from a thermal–viscous instability present in accretion discs at temperatures corresponding to hydrogen partial ionization. In the standard version of the disc instability model (DIM) (see Hameury et al. 1998 for the most recent version of this model, and Cannizzo 1993 and Lasota & Hameury 1998 for reviews) one assumes that mass transfer from the secondary is constant prior to, and during, the outburst. Properties of outbursts depend on the viscosity mechanism which transports angular momentum and heats the disc. In the DIM the kinematic viscosity coefficient is taken as ν = a c s H, where c s is the (adiabatic) speed of sound, H the disc semithickness, and the ‘viscosity coefficient’ a < 1 (Shakura & Sunyaev 1973). The DIM requires different values of a in outburst and in quiescence (Smak 1984b). The quiescent value for the great majority of systems should be a ≈ 0.01 (Livio & Spruit 1991). Smak (1993) showed, however, that the outburst cycle of WZ Sge cannot be described by such a version of the DIM (see also Osaki 1996). The main reason is that for a ≈ 0.01 the disc is able to accumulate during quiescence only ∼ 10^{-3} g, whereas during outbursts ∼ 10^{23} g is accreted by the white dwarf. Assuming that a ≈ 10^{-5} solves this inconsistency, and the resulting very low viscosity may also account for the long recurrence time which is difficult to reproduce with the ‘standard’ a values. The difficulty of this version of the model is that one must find an explanation for the very low value of the viscosity coefficient in this particular system. The low value of the mass transfer rate (∼ 10^{-15} g s^{-1}, invoked as an explanation; Osaki 1996) is not much lower than in other systems with similar orbital periods, so it is doubtful that it can explain a difference in a by three orders of magnitude. In any case, the relation between accretion rate and viscosity, if any, is unknown (see Gammie & Menou 1998).

A different modification of the standard DIM was suggested by Lasota, Hameury & Huré (1995) and Hameury, Lasota & Huré (1997). They assume that the value of the a-parameter in WZ Sge is not different from the ‘standard’ one. They noticed that at low mass transfer rates outer regions of accretion discs in CVs are cold enough to be stable with respect to the thermal instability believed to be responsible for dwarf-nova outbursts. They proposed that the WZ Sge accretion disc does not extend down to the surface of the white dwarf, but is truncated at a radius corresponding to a stable outer disc. An enhancement of mass transfer would bring such a disc into an unstable state and thus trigger an outburst. One should

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stress that such an outburst would still be due to the thermal–viscous instability, the enhanced mass transfer serves only as a trigger. The recurrence time is then related to the characteristic time-scale of mass transfer fluctuations. One can also obtain long recurrence times if the truncated disc is marginally unstable (Warner, Livio & Tout 1996), but in practice such a disc is indistinguishable from a marginally stable one proposed by Lasota et al. (1995). There is still, however, a problem to be solved: since \( \alpha \sim 0.01 \), there would be only \( \sim 10^{17} \) g in the disc available. In this case, therefore, mass must be added to the disc during outburst. There is observational evidence (Smak 1997; see also Hessman et al. 1984) that irradiation of the secondary during outburst increases mass transfer from the secondary (see Hameury et al. 1997 for a simple model).

In Lasota et al. (1995) and Hameury et al. (1997) the truncation was assumed to be due either to evaporation (Meyer & Meyer-Hofmeister 1994) or to the presence of a magnetic field strong enough to disrupt the quiescent disc (<10^4 G would be sufficient for a low-mass white dwarf). The second hypothesis seemed to be favoured by the presence, before the last, 1978 outburst, of a 27.87-s coherent oscillation which could be attributed to the rotation of an accreting white dwarf (Patterson 1980, hereafter P80). There were two problems with this interpretation. First, in addition to the 27.87-s period, several other (some of them transient) `satellite period' oscillations have been observed (Robinson, Nather & Patterson 1978; P80). The interpretation of these periods in terms of beat frequencies resulting from reprocessing of the white dwarf’s pulsed light on various features in the accretion flow encountered several difficulties (P80). Second, all the oscillations disappeared during the outburst. After the outburst the principal, 27.87-s, oscillation has been absent for 16 yr (although the 28.96-s oscillation was seen when WZ Sge was at twice its pre-outburst brightness). Although it was easy to understand that during the outburst the increased accretion rate could suppress the magnetosphere, the lack of the principal pulse afterwards was difficult to understand, and it cast a shadow of doubt on the presence of a rapidly rotating white dwarf in WZ Sge.

In 1995 the 27.87-s oscillation reappeared in the company of the previously present 28.96-s pulsation (Patterson et al. 1998, hereafter P98). Weak, transient `satellite period' pulses are also seen at 28.2 and 29.69 s. The most important news, however, was the detection of a 27.86 ± 0.01-s period in the 2–6 keV ASCA energy band (P98), which confirmed the presence of a rotating, magnetized white dwarf in WZ Sge. The presence of a rapidly rotating white dwarf was confirmed by UV spectral observations (Cheng et al. 1997). WZ Sge is therefore a DQ Her star, a CV containing a rapidly rotating magnetized white dwarf.

In a recent article Meyer-Hofmeister, Meyer & Liu (1999) (see also Mineshige et al. 1998) argue that the inner disc `hole' in WZ Sge is due to evaporation and not to the presence of a magnetic field. These authors, however, underestimate by a significant factor the coherence of the observed pulsations, and their model seems to be contradicted by observations. For example, their model predicts the precession of superhumps well before the beginning of the superoutburst, whereas in WZ Sge this feature appeared only 10–12 d after the maximum (Patterson et al. 1981). Also, the remarkable constancy of the X-ray luminosity after the 1978 outburst is in contradiction with their model.

In this article we argue that Patterson’s (P80, also P98) hypothesis that WZ Sge contains a rapid, oblique magnetic rotator is consistent with most of the observed properties of this system. We show that sideband oscillations observed in WZ Sge can be interpreted as resulting from reprocessing of the white dwarf’s pulsed light on various features of the transferred flow. In Section 2 we present the observed properties of WZ Sge, and identify various oscillations with orbital sidebands. In Section 3 we discuss possible configurations of WZ Sge as a system containing a magnetic rotator, and show that only one is consistent with the system properties at outburst. In Section 4 we discuss the accretion history of WZ Sge and its relation to other CVs. We summarize our model in Section 5.

## 2 WZ SGE AS A MAGNETIC ROTATOR

If WZ Sge is an oblique magnetic rotator, one should, in principle, observe in its optical light curve the fundamental spin frequency and some sidebands resulting from the beat between this and the orbital frequencies (Warner 1986). In what follows we will assume that all characteristic periods observed in WZ Sge have a rotational origin. The possibility that these oscillations (or some of them; Warner 1995a) are due to white dwarf pulsations cannot be excluded (see Wood 1999 for recent arguments in favour of this model). Such a model is, however, hard to test, because it does not make any predictions (P98). In what follows we will try to show that all observed periodicities observed in WZ Sge can be explained in the framework of a magnetic rotator model.

### 2.1 General properties and parameters

Following Warner (1995a), we classify WZ Sge as a DQ Her star. DQ Her stars are rapidly rotating intermediate polars (IPs), i.e., magnetic CVs in which the white dwarf rotation is not synchronous with the orbital motion (synchronous systems are called `polars' or ‘AM Her stars’). Warner (1995a) adds also that DQ Her stars are characterized by the absence of hard X-rays in the sense that, contrary to ‘usual’ IPs, their X-ray temperature is much lower than 10 keV. In WZ Sge the X-ray temperature is ~4.5 keV (P98), so it is a hard X-ray emitter, even if its temperature is lower than that in ‘usual’ IPs (see below).

WZ Sge has been one of the best observed CVs, but the values of its fundamental parameters are still uncertain. Smak’s (1993) photometric solution gives \( M_1 = 0.45 \) (where \( M_1 \) is the white dwarf mass in solar units) and \( q = M_2/M_1 = 0.13 \), where \( M_2 \) is the secondary’s mass, whereas Spruit & Rutten (1998), who model the ‘hotspot’ at which the mass transfer stream interacts with the outer disc regions, get \( M_1 = 1.2 \) and \( q = 0.075 \). As we shall see, Smak’s value is too small if the white dwarf in WZ Sge is spinning at 27.87 s, but the Spruit & Rutten (1998) value might be too high.

In Fig. 1 we show various mass–radius relations relevant for WZ Sge. The white dwarf mass–radius relation is that of Nauenberg (1972), which is suitable for helium white dwarfs (using other mass–radius relations gives very similar results; e.g. P98). Also plotted is the corotation radius

\[
R_a = \left( \frac{G M_\odot M_1 P_1^2}{4 \pi^2} \right)^{1/3} = 1.5 \times 10^8 P_1^{1/3} M_1^{1/3} \text{ cm},
\]

where \( P_1 \) is the white dwarf spin period. The corotation radius corresponds to a distance at which a free particle in circular Keplerian orbit corotates with the white dwarf. In particular, the white dwarf radius must satisfy \( R_1 < R_a \). If one assumes that \( P_1 = 27.87 \text{ s} \), Fig. 1 shows that for Smak’s \( M_1 = 0.45 \) the white dwarf would be rotating just below the break-up speed, so \( M_1 \) must be larger than 0.45 (see also P98).
oscillations are present, sometimes with additional periods up to 30 s (i.e., 28.19, 28.52, 29.69 s). A 28.2-s oscillation is also seen in the UV (Welsh et al. 1997) [strictly speaking, the UV oscillation at 28.2-s (28.19-s) period].

2.2 Orbital sidebands

In addition to the 27.87-s oscillation, several other optical oscillations between 27.8 and 30 s have been observed in WZ Sge (P98, and references therein). In general, either the 27.87- or 28.96-s oscillations are present, sometimes with additional periods up to 30 s (i.e., 28.19, 28.52, 29.69 s). A 28.2-s oscillation is also seen in the UV (Welsh et al. 1997) [strictly speaking, the UV oscillation is 28.09 ± 0.12 s, but we will follow P98 in identifying it with the optical 28.2-s (28.19-s) period].

We will denote the white dwarf angular rotation frequency as \( \omega = 2\pi \dot{P}_w \) and the orbital frequency as \( \Omega = 2\pi \dot{P}_{orb} \). As shown by Warner (1986), in IPs the rotating white dwarf’s beam and its reprocessing on the surface layers of the disc, hotspot and/or the secondary produces peaks in the power spectra of optical light curves not only at \( \omega \) and \( \omega - \Omega \), but there also exist sidebands at \( \omega + \Omega \) and \( \omega - 2\Omega \). (Wynn & King 1992 analysed power spectra of X-ray light curves in IPs in the case of discless accretion.) For a white dwarf rotation period of 27.87 s and an orbital period of 81.63 min (see P98) this gives sidebands at 28.03, 27.71 and 28.19 s. Patterson (1980) already noted that the 28.2-s period is consistent with \( \omega - 2\Omega \). He also noted that significant peaks in the power spectra exist in the sidelobes close to the main peak at 27.87 s. These periods were, however, not discussed by him; visual inspection of his fig. 3(b) shows that the peaks of these sidelobes are at \( \sim 27.96 \) and \( \sim 27.99 \) s. These are very close to \( \omega - \Omega \) and \( \omega + 2\Omega \) respectively. We therefore propose that the three optical sideband frequencies are present, making stronger the suggestion that an appreciable magnetic field exists in WZ Sge, truncating the inner parts of an accretion disc and making it similar to an IP. It also confirms that the 27.87 s period is the white dwarf spin period; checks made by assuming the white dwarf spin period is at the other observed frequencies show that although the expected sideband frequencies match the observed frequencies, there is no explanation for the 27.87 s period. For example, assuming 28.19-s as the white dwarf spin period gives sidebands at 28.03, 28.35 and 28.52 s. These are all observed in the power spectra of P80 and P98. In addition, \( \omega - 3\Omega \) gives 29.86 s. \( P_1 = 28.09 \text{-s gives sidebands at 28.25, 27.93, 28.42 s, which in principle could match the observed peaks in the power spectrum, as well as } \omega - 3\Omega \text{ which gives 28.58 s. In both cases, however, the 27.87-s oscillation remains unexplained.}

We identify the 28.2-s optical and UV oscillations with \( \omega - 2\Omega \), which would result from reprocessing on the hotspot. Welsh et al. (1997) identify this period with the white dwarf spin period, but the presence of the 27.87 s period in X-rays does not support this identification. If the 28.2-s oscillation results from reprocessing on the hotspot, one must then explain why the fundamental spin period (27.87 s) is not seen in the UV. In order to understand this absence, one should, presumably, understand the structure of the accretion flow. As we shall see below, the spin period of WZ Sge implies that this system is in an ‘ejector’ phase, ejecting most of the transferred matter. Such a case would be totally different from the usual IPs (see, e.g., Hellier 1998). No model of such a flow seems to exist at present.

It is also interesting to note that a reprocessing projected area might also vary with the \( 2\Omega \) component (Warner 1995a). This will introduce components at frequencies \( \omega + 2\Omega \) and \( \omega - 3\Omega \) in the power spectrum. The latter produces a peak near 28.35 s, which is visible (at \( \sim 28.34 \) s) as a shoulder of the peak near 28.19 s in the ‘grand average’ power spectrum of the observations prior to the outburst end of 1978 (P80; see also P98). We note that the observed oscillation at 28.52 s is consistent with \( \omega - 4\Omega \) (P80), but we have no interpretation for this fact.

Having explained to our satisfaction various sidebands, we are still left with the 28.96- and 29.69-s periods which are not commensurable with the orbital period. The 28.96-s signal appears as one of the two main peaks in the power spectrum and is generally present. The 29.69-s signal is weak and transitory.

3 WZ SGE AS A MAGNETIC EJECTOR

3.1 A discless system?

P80 suggested that the 28.96-s period could result from the beat between the white dwarf spin frequency and the Keplerian frequency at the magnetosphere. The required Keplerian period is \( P_K = 733.47 \) s and could correspond to the rotation period of plasma blobs at the disc inner edge. The problem with such an interpretation is that, if true, not much of a disc would be left. The corresponding magnetospheric radius would be

\[
R_M(28.96) = 1.22 \times 10^{10} M_1^{1/3} \text{cm.}
\]  

According to Smak (1993), for \( M_1 = 0.45 \), the outer disc radius is \( R_0 = (1.07 \pm 0.19) \times 10^{10} \) cm, whereas Spruit & Rutten (1998) obtain, for \( M_1 = 1.2, R_0 = (1.75 \pm 0.14) \times 10^{10} \) cm so that in both cases we would rather have a ring, then a disc, or no disc at all, depending on the details of plasma interaction with the magnetic field (see, e.g., King 1993). In addition, if the Keplerian radius \( R_K \) corresponding to the 28.96-s period were the magnetospheric radius, the so-called ‘fastness parameter’ (see, e.g., Frank, King

\[ \frac{28.96}{P_K} \]
& Raine 1992) would be
\[ \omega_s = \frac{P_s}{P_c} = 26.3. \]  

Since \( \omega_s > 1 \), matter transferred from the secondary could not be accreted on to the white dwarf and would rather be ejected (as noticed by P98). WZ Sge could then be similar to another DQ Her star, AE Aqr, which is a discless ‘ejector’ (Wynn, King & Horne 1997). Such a solution would be rather attractive: two, of three known, DQ Her stars would be discless magnetic ejectors; apparent similarities between WZ Sge and AE Aqr have already been pointed out in P80.

We think, however, that such a model cannot describe WZ Sge. First, Wynn et al. (1997) show that in AE Aqr the Hα Doppler maps of the system are consistent with their simulations of a discless flow. Similar maps for WZ Sge (Spruit & Rutten 1998) look completely different, and clearly show the presence of a disc and not just a ring, although the authors point out that the brightness distribution is less ‘disc-like’ than in other CVs. One should bear in mind, however, that, as pointed out by Spruit & Rutten, the Hα brightness may not be a good tracer of matter distribution. On the other hand, Menickent & Arenas (1998) find that the accretion flow in WZ Sge forms a ‘ring’ with a ratio of the inner to the outer radii \( \sim 0.3 \).

Second, contrary to AE Aqr, WZ Sge is a dwarf nova so that we expect an accretion disc to be present prior to, and during, the outburst (this is independent of the validity of the DIM – there is ample observational evidence that dwarf nova outbursts require the presence of accretion discs around white dwarfs). One could imagine that enhanced mass transfer prior to outburst could squeeze the magnetosphere and allow the reappearance of an accretion disc. However, even if this happened, the superoutbursts observed in WZ Sge would not occur. This is shown in Figs 2–4, where we have plotted the magnetospheric radius as a function of the accretion rate for different primary masses. For the magnetospheric radius we use the approximate formula (see, e.g., Frank et al. 1992)
\[ R_M = 9.8 \times 10^8 M_1^{12/7} M_1^{17/7} \mu_{30}^{4/7} \text{cm}, \]  

where \( M_1 \) is the accretion rate in \( 10^{15} \text{g s}^{-1} \), and \( \mu = \mu_{30} (10^{30} \text{g cm}^{-3}) = B R_1^3 \) is the magnetic moment of a white dwarf with a surface field \( B \). Magnetic moments corresponding to the 29.69-s oscillations are: \( \mu_{30} = 34 \) for \( M_1 = 0.45, \mu_{30} = 55 \) for \( M_1 = 0.80 \), and \( \mu_{30} = 77 \) for \( M_1 = 1.2 \) (see also Section 3.4).

Near maximum of the 1978/1979 outburst most of the disc material was accreted on to the white dwarf. According to Smak (1993) the maximum accretion rate during the 1978 outburst was \( 3.2 \times 10^{18} \text{g s}^{-1} \). For primary masses larger than Smak’s \( M_1 = 0.45 \), this maximum would be lower (Smak, private communication). As the mass accretion increases during the rise to outburst, the magnetospheric radius moves inward. Accretion occurs only when this radius moves inside the corotation radius. Since no clear oscillations are seen during most of the outburst (Patterson et al. 1981), this suggests that in WZ Sge the disc was able to reach the white dwarf surface (P98). It can be seen, however, from Figs 2–4 that at outburst maximum the magnetospheric radius given by equation (2), i.e., the inner disc radius, would just reach the corotation radius. For a given maximum accretion rate this is the case for all white dwarf masses (because the relevant radii are Keplerian), but in reality, for higher masses, the magnetospheric radius would not even get there, since in this case the maximum accretion rate would be smaller than the value plotted in Figs 2–4. It is obviously impossible to have the maximum of accretion luminosity at the very moment at which accretion on to the white dwarf just begins.

**Figure 2.** Shown are the critical radius (equation 6, with \( \alpha = 0.01 \); dotted-dashed line) and the magnetospheric radius (equation 4) as a function of the mass accretion rate for a white dwarf mass \( M_{\text{wd}} = 0.45 M_{\odot} \). The magnetospheric radius relations are normalized so that the magnetospheric radii in quiescence are equal to the Kepler radii which are derived from the observed 28.96- and 29.69-s pulsations. Also indicated are the white dwarf radius (\( R_{\text{wd}} \); dashed line), the outer disc radius (\( R_{\text{disc}} \); dash-dot-dotted line) as derived by Smak (1993), and the corotation (\( R_{\text{c}} \)) and Kepler radii at which the beat between the white dwarf spin frequency and the Kepler frequency gives rise to the observed oscillations at 28.96 and 29.69 s (dashed lines). The uncertainties in \( R_{\text{wd}} \) and \( R_{\text{disc}} \) are given by the grey hatched areas. The mass accretion rates of WZ Sge during quiescence (\( 2 \times 10^{17} \text{g s}^{-1} \)) are indicated with dotted lines.

We conclude therefore that WZ Sge is not in a discless ejector phase. As we will show below, WZ Sge is most probably in a state intermediate between those of the other two DQ Her stars: AE Aqr, a pure discless ejector (Wynn et al. 1997), and DQ Her itself which seems to have a steady accretion disc (Warner 1995a).

### 3.2 Blobs at the outer disc edge

As can be seen in Fig. 2, the Keplerian radius corresponding to the frequency giving the 28.96-s beat with the white dwarf spin frequency is very close to the outer disc radius as determined from modelling observational data. For \( M_1 = 0.45 \) it coincides, within the error bars, with \( R_{\text{D}} \), and one can expect that this will be true also for white dwarf masses smaller than \( 1 M_{\odot} \). We propose, therefore, that the 28.96-s beat frequency results from reprocessing of the spinning, magnetized, white dwarf beam on plasma blobs orbiting at, or close to, the outer disc radius. According to Spruit & Rutten (1998) the ‘tail’ of the observed hotspot’s Hα emission is probably due to material orbiting at the outer edge of the disc at the Keplerian velocity. This material had crossed the stream–disc interaction region undergoing a sequence of heating and cooling events that are likely to form a non-uniform flow. In a stationary accretion disc the outer radius is well defined and constant in time, so that one can expect the motion of plasma blobs there to give coherent oscillations. Warner (1995b) suggested that QPOs observed during some dwarf nova outbursts could be due to blobs orbiting at the outer disc.
radius. During outbursts, however, the outer disc radius moves (in and out; see, e.g., Smak 1984a), and so coherent oscillations should not be expected in such a case.

Our identification of the 28.96-s period origin is therefore qualitatively consistent with the Spruit & Rutten (1998) model of WZ Sge. A quantitative agreement will be difficult to achieve, because the interpretation of the Hα ‘tail’ as due to matter in Keplerian motion leads to their high white dwarf mass (1.2 $M_\odot$). With such a mass the Keplerian period of the outer disc rim cannot be $\sim 733$ s. The value of the white dwarf mass in WZ Sge is, however, subject to controversy (see, e.g., the discussion in Spruit & Rutten 1998), but for our purpose (see below) the value of 1.2 $M_\odot$ is too high. A lower value of $\sim 0.8 M_\odot$ would be consistent with our hypothesis concerning the origin of the 28.96-s period and the estimates of P98.

3.3 A (marginally) stable accretion disc?

As shown by Lasota et al. (1995), the very long outburst recurrence time of WZ Sge can be reconciled with the ‘standard’ values of the viscosity parameter $\alpha$ if the disc is truncated so that between outbursts it is (marginally) stable. As we will see below, at present, the flow of transferred matter in WZ Sge does not, probably, form a standard accretion disc. We know, however, that in the past there was an accretion disc in WZ Sge (and we expect one to form in the future), so it is interesting to see what is the inner radius required by the stability criterion.

The critical value of accretion rate $M_{\text{crit}}$, below which a disc is in cold, stable equilibrium can be expressed as (Hameury et al. 1998; for a similar expression see, e.g., Smak 1984b and Ludwig, Meyer-Hofmeister & Ritter 1994):

$$M_{\text{crit}} = 4.0 \alpha^{-0.06} M_1^{0.89} R_1^{2.67},$$

where $M_{\text{crit}}$ is the critical $M$ in units of $10^{15}$ g s$^{-1}$, and $R_1$ the accretion disc radius in units of 10$^{10}$ cm. Note that $M_{\text{crit}}$ depends only weakly on the value of $\alpha$ (in similar formulae by other authors the critical $M$ is independent of $\alpha$ (e.g. Ludwig et al.

1994 – it should be kept in mind that equation 5 is a fit to numerical results).

It follows from equation (5) that for a given $M$, an accretion disc will be (marginally) stable for $r < R_{\text{crit}}$, where

$$R_{\text{crit, in}} = 0.6 M_1^{0.375} \alpha^{0.015} M_{\text{wd}}^{0.333}.$$  

The corresponding Kepler frequency at $R_{\text{crit}}$ is given by

$$\omega_{\text{crit}} = 0.025 M_1^{0.56} \alpha^{-0.022}.  \tag{6}$$

This equation shows that $\omega_{\text{crit}}$ is independent of the mass of the white dwarf and only very weakly dependent on $\alpha$. Since the quiescent $M$ in WZ Sge is $\sim 10^{15}$ g s$^{-1}$ (Smak 1993), this leads to a Kepler period at $R_{\text{crit}}$, i.e., $P(R_{\text{crit}}) \sim 330 - 370$ s for values of $\alpha \sim 0.01 - 1$. Therefore, in the framework of the truncated disc model, any beat period between the white dwarf spin period and a Kepler period of matter in the accretion disc in the quiescent state of WZ Sge should have periods shorter than $\sim 30.4$ s, as observed.

We notice that the recently appeared oscillation period at 29.69 s (P98) is very close to this value. We therefore suggest that the 29.69 s period is the beat between the white dwarf spin period and the Kepler period of matter near $R_{\text{crit}}$. Since the inner disc radius, $R_{\text{in}}$, is larger than $R_{\text{crit}}$ in quiescence, the 29.69-s period would mark the inner part of the disc, where matter is dominated by the magnetic field.

We propose that the magnetospheric radius is located at

$$R_{\text{in}}(29.69) = 8.87 \times 10^3 M_1^{1/3} \text{ cm},  \tag{8}$$

corresponding to the Keplerian period $P_M = 454.65$ s. The fastness parameter is now

$$\omega_s = \frac{P_K}{P} = 16.3,  \tag{9}$$

still much larger than 1, so that most of the mass transferred from the secondary cannot be accreted by the white dwarf, but is ejected from the system (see below).

Not all matter is ejected – a few per cent finds its way to the white
dwarf and accounts for the pulsed emission. The configuration we have in mind is different from disc accretors, or disc accretors with overflowing accretion stream (see, e.g., Frank, King & Lasota 1987 and Hellier 1999). It is also different from the pure, discless ejector models of King (1993) and Wynn & King (1995). It could be a truncated ‘excretion disc’. It might be a structure one obtains in the Wynn & King (1995) model for low values of the drag coefficient $k_0$. According to these authors $k_0 \propto B^2 P_{\text{orb}}$, so that $k_0$ for WZ Sge should indeed be lower than in AE Aqr.

From equations (4) and (8), and the white dwarf mass–radius relation, one can deduce that for higher ($\geq 0.7 M_\odot$) white dwarf masses, at outburst maximum the magnetosphere would not reach the white dwarf surface. Even taking into account the rather crude description of the disc–magnetic field interaction used above, one can conclude that our model is incompatible with high white dwarf masses.

### 3.4 The magnetic field of WZ Sge

If the inner disc radius is given by $R_\text{in} = R_\text{M}$, with $R_\text{M}$ set by the observed 29.69-s oscillation, we derive magnetic field strengths of $\sim 3.6 \times 10^5$, $\sim 1.6 \times 10^6$ and $\sim 1.3 \times 10^8$ G respectively for $M_\text{w} = 0.45, 0.8$ and 1.2. In the last case, the white dwarf in WZ Sge would have a magnetic field surface intensity comparable to that of most IPs. The magnetic moment of the white dwarf in WZ Sge is therefore $\mu \sim (3 \pm 8) \times 10^{31}$ G cm$^3$.

If the magnetospheric radius is given by equation (8), then, for all white dwarf masses of interest,

$$R_\text{M} > 4 R_1$$

which, according to Wickramasinghe, Wu & Ferrario (1991) (see also Warner 1995b) means that WZ Sge should be a hard X-ray emitter, as is indeed observed. In this respect, as mentioned before, WZ Sge is different from the other two DQ Her stars, but for a reason that is obvious: a much lower mass transfer rate.

On the other hand, in outburst, WZ Sge is not a DQ Her star but enters the dwarf nova oscillation (DNO) regime in which the ‘slippage of the surface’, i.e., differential rotation of the white dwarf’s outer layers is expected (Warner 1995a,b). For WZ Sge the minimum field required to maintain rigid rotation is $B \sim 2 \times 10^8 M_1^{\frac{3}{2}}$ G, so in WZ Sge $B < B_\text{eq}$ (Fig. 5). No confirmed DNOs have been seen, however, during the last outburst (Patterson et al. 1981).

### 4 The accretion history of WZ Sge

If WZ Sge is in an ejector phase, its white dwarf should be spinning down. Indeed, Patterson (1980) reports $P_\text{eq} = 8 \times 10^{-12}$. This value is very large compared with the $P_\text{eq} = 5.64 \times 10^{-14}$ observed in AE Aqr (de Jager et al. 1994). The implied spin-down power of WZ Sge would be $\geq 5 \times 10^{35}$ erg s$^{-1}$, two orders of magnitude higher than in AE Aqr. Wynn et al. (1997) find that (assuming that plasma interacts with magnetic field in the form of blobs) $L_\text{spin}$ scales with $M$. Since in WZ Sge the mass transfer rate is two orders of magnitude lower than in AE Aqr, the two orders of magnitude higher spin-down power is hard to understand. One should note, however, an interesting point: in WZ Sge the light cylinder radius $R_\text{LC} = c \omega / \Omega$ is larger than the size of the system ($a \sim 3.4 \times 10^{10}$ cm), which allows, in principle, acceleration of particles outside the system. It may well be that results for systems in which $R_\text{LC} \leq a$ do not apply to WZ Sge. For particles accelerated to speeds close to the speed of light

$$L_\text{spin} \approx M R_\text{LC} \omega \approx 9 \times 10^{35} M_1^{\frac{3}{2}} \text{erg s}^{-1},$$

so that to obtain the spin-down power that corresponds to $P \sim 8 \times 10^{-12}$ the escape velocity of particles removing rotational energy would have to be a significant (~90 per cent) fraction of the speed of light. This is, probably, not totally absurd, since TeV $\gamma$-rays have been observed from AE Aqr (but not from WZ Sge, which is closer), but we would still expect a spin-down rate in WZ Sge closer to (or lower than) that measured in AE Aqr. A reliable $P$ would be an important test of any model, assuming that WZ Sge contains a fast magnetic rotator.

In any case, we have to consider how the white dwarf in WZ Sge had been spun up to its present rate. The only way to achieve this is through accretion disc spin-up. The white dwarf spin rate increases as it accretes the Keplerian angular momentum of matter at the disc accretion radius. The white dwarf spin rate increases as it accretes the Keplerian angular momentum of matter at the inner disc edge. An equilibrium period is reached when angular momentum is accreted at the same rate as it is centrifugally expelled by the spinning white dwarf (a magnetic field is not necessary for this to work). In the case of a magnetized white dwarf the equilibrium period is equal to the Keplerian period of the magnetosphere ($\omega_\text{eq} = 1$). The equilibrium spin is then

$$P_\text{eq} = 360 M_1^{\frac{5}{3}} M_\odot^{\frac{2}{3}} P \mu_5^{\frac{2}{3}} \text{s}$$

We see immediately that an accretion rate $\sim 10^{17}$ g s$^{-1}$ is required to bring WZ Sge to its present rapid rotation rate. Such accretion rates are typical of nova-like CVs at orbital periods longer than 3 h, whereas WZ Sge is very close to the minimum period for CVs where secular mass transfer rates are two orders of magnitude lower. The present mass transfer rate in WZ Sge is in good agreement with that predicted by mass-loss from the secondary being driven by gravitational radiation alone. It might even be possible that WZ Sge is already on the ‘other side’ of the minimum period (see, e.g., Patterson 1998). At orbital periods close to 80 min the secondary, which due to mass-loss is out of thermal equilibrium, stops to contract in response to mass-loss, and the binary system

starts expanding to longer orbital periods. This ‘bounce’ is helped
by the secondary becoming degenerate (see, e.g., King 1997). It
would seem, therefore, that high accretion rates are impossible at
short orbital periods.

The situation, however, is not as hopeless as it would seem. We
are interested here in time-scales of the order of the white dwarf
spin-up time, i.e., $10^3 - 10^5$ yr. Fluctuations on such short time-
scale do not modify the secular evolution of the binary.

There is at least one piece of evidence which shows that the
accretion history of WZ Sge might have had episodes of high mass
transfer. There exist, at orbital periods between 79 and 92 min,
systems which are supposed to have high ($\sim 10^{17}$ g s$^{-1}$) mass
transfer rates. These are ER UMa systems, which show extremely
short intervals between superoutbursts (19–44 d) and very short
(3–4 d) ‘normal’ outburst interval. They seem to be the high
accretion rate equivalent of the low accretion rate systems show-
ing superoutbursts: the SU UMa systems (WZ Sge is a SU UMa
system showing superoutbursts only). The idea that DI UMa (one
of the four known ER UMa systems with $P_{orb} = 79$ min), which is
more luminous than WZ Sge by a factor $\sim 50$, spends most of its
life as ‘an ordinary WZ Sge star’, but was caught during an
‘upward surge in accretion’, has been proposed by Patterson
(1998). He speculates that the short period CV mass transfer
cycles in which $M$ surges to high values ($\sim 1$–5 percent of the
cycle time) are somehow associated with classical nova eruptions
because, in addition to the four ER UMa’s, four other systems are
‘too bright’ for their orbital periods, and three of them are classical
nova remnants.

Whatever the reason, what is important for our argument is the
presence of systems which accrete at high rates but have orbital
periods close to that of WZ Sge. With such accretion rates the white
dwarf in WZ Sge can be spun up to $P_{eq} = 27.87$ s.

When the white dwarf gets to this equilibrium spin rate the mass
transfer is stable (Wynn & King 1995). After some time the mass
transfer rate will return to its secular value. This value, however, is
two orders of magnitude lower than that required for $P_{equ} \sim \text{few} \times 10^3$ s. In a very short time (viscous time of the disc
$\sim$days) the magnetosphere will start expanding and becoming
larger than the corotation radius. The system enters into the ejector
phase. As shown by Wynn & King (1995), this phase is dynami-
larger than the corotation radius. The system enters into the ejector

5 SUMMARY OF THE MODEL

We can summarize our model (or ‘scenario’) as follows: WZ Sge
contains a $\lesssim 0.8$ M$_\odot$ magnetized white dwarf spinning with a
period of 27.87 s. The magnetic moment is $\lesssim 5 \times 10^{31}$ G cm$^3$, i.e.,
a magnetic field at the surface $\lesssim 2 \times 10^9$ G.

In quiescence the mass transferred from the secondary forms a
disc-like structure with outer radius and inner radii at 1.22 and
$1 (the corotation radius is much smaller than the magnetospheric
radius), most of the matter cannot be accreted by the white dwarf.
A small fraction ($\lesssim 10$ percent) finds its way to the white dwarf
surface, where it is responsible for the observed X-ray emission.

The white dwarf is in a spin-down phase, and most of the
transferred matter is ejected from the system. The disc-like struc-
ture formed by the flow might be composed of plasma ‘blobs’. At
outburst, which is triggered by an enhanced mass transfer event, an
accretion disc is (re)created. At outburst maximum the disc reaches
down to the surface of the white dwarf.

The white dwarf in WZ Sge will spin down to an equilibrium
rotation period of a few minutes. In this phase an increase of
accretion rate will transform it into an ER UMa star. After a spin-
up phase the system will become a DQ Her star once more.

ACKNOWLEDGMENTS

We thank Jean-Marie Hameury, Henk Spruit and Joe Smak for
helpful discussions and help. Part of this work was performed
during a visit of JPL to Oxford and EK to Meudon; they thank the
respective departments for their hospitality. The visits were made
possible by the British-French collaboration grant Alliance.

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