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Laboratory Experiments on Solute Transport in Non-Homogeneous Porous Media

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Solute transport in groundwater is a process which has become of major importance during the last decades due to increasing contamination of ground water. This process usually occurs in a medium heterogeneous with respect to hydraulic conductivity and porosity, properties that affect the dispersion of the solutes.

The present paper describes an experimental investigation of the solute transport process in heterogeneous porous media, especially the connection between the statistical properties of their hydraulic conductivity distributions and the dispersion parameters governing the spreading of the solutes. The experimental results are compared to theoretical solutions derived for the same case of a solute pulse in an average uniform flow through a heterogeneous porous medium. Generally there is good agreement between the theory and the experiments.

In field applications this means that the dispersion parameters can be more readily determined from the soil properties. Furthermore, the deviations between dispersivities determined in laboratory columns and dispersitivies found under field conditions can be explained quantitatively by the differencies in the length scales and in the variances of the hydraulic conductivity distributions.

Introduction

In Denmark there has been a large increase in the number of groundwater pollution cases, and the prediction of the sizes of the contamination cases is of vital importance. The dispersion of solutes due to the spatial variability of the soil

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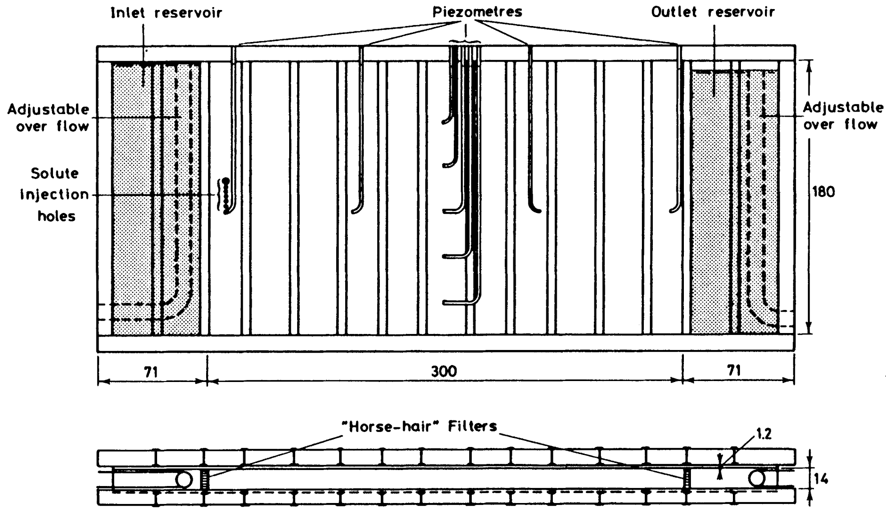

Fig. 1. Sketch of the experimental set up. All measures in cm.

hydraulic properties, i.e. hydraulic conductivity and porosity plays an essential role in many applications within groundwater hydrology. In this study only the variability of the hydraulic conductivity, being the most important of these two, is considered. Chemical reactions such as decay, adsorption etc. are also neglected. Hydraulic conductivity exhibits a large degree of spatial variability which cannot be included when modelling groundwater flow and solute transport. Stochastic modelling attempts to overcome this lack of knowledge by using the effective statistical properties of the hydraulic conductivity distribution to describe the porous media. In the study by Freeze (1975), in which a large number of field measurements of hydraulic conductivity are examined, it is concluded that the hydraulic conductivity for most applications can be considered as log-normally distributed. Assuming this and adopting a covariance function, for instance an exponential one, the entire statistical structure of the logarithmically transformed conductivity field can be described by three constants; the mean, μ_y, the variance, σ_y^2 and the length scale, l_y. The length scale characterizes the maximum distance over which the conductivities are correlated.

A quantification of the relationship between the spreading of a solute plume, described by the effective parameters, the so-called apparent dispersivities, has recently been investigated theoretically by Dagan (1982) and by Gelhar and Axness (1983), and they conclude that the apparent dispersivities are closely related to σ_y^2 and to l_y.

In order to evaluate these theoretical results a comprehensive laboratory measurement programme has been carried out in the flow channel shown in Fig. 1,

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including various combinations of σ_y^2 and l_y. This paper begins with a short discussion of the mathematical equations describing solute transport and a description of the experimental set-up. The results from two of the experiments, in the following denoted EX1 and EX2, are presented, and the main conclusions of the investigations are discussed.

Mathematical Framework

The basic equations governing saturated flow of water and solutes in a porous medium are well-known. Assuming two-dimensional, average uniform flow in an isotropic aquifer the advection-dispersion equation on which solute transport calculations are traditionally based can be written

$$\frac{\partial c}{\partial t} \equiv -U\frac{\partial c}{\partial x} + D_L\frac{\partial^2 c}{\partial x^2} + D_T\frac{\partial^2 c}{\partial y^2} \tag{1}$$

where

c – concentration
U – uniform flow velocity
D_L, D_T – longitudinal and transverse dispersion coefficients

Assuming the solute transport to take place in a completely homogeneous medium the dispersion coefficients are calculated as

$$D_L = \alpha_L U \tag{2}$$

$$D_T = \alpha_T U \tag{3}$$

where α_L, α_T – pore scale dispersivities.

The pore scale dispersivities are of the order of the mean grain size diameter of the soil, the longitudinal being about 3 times the transverse. However, natural aquifers have a complex composition, i.e. stratifications or other heterogeneities which are products of the geologic formation process, hence the spreading of solutes is much larger in these aquifers and cannot be adequately described by the pore scale dispersivities.

Nevertheless, assuming the velocity field, still on the average, to be uniform (often the velocity distribution is unknown, and this approximation is necessary) the larger spreading in natural aquifers can be accounted for simply by increasing the dispersivities. Then the terms "apparent dispersion coefficients" and "apparent dispersivities" are introduced

$$D_L^* = \alpha_L^* U \tag{4}$$

$$D_T^* = \alpha_T^* U \tag{5}$$

where

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D_L^*, D_T^* – apparent dispersion coefficients
α_L^*, α_T^* – apparent dispersivities.

The apparent dispersivities have traditionally been considered constants, i.e. the dispersion process is Fickian, but both field studies (Cherry *et al.* 1983) and theoretical studies (Dagan 1982) have shown that they are dependent on the displacement from the solute input zone, hence they cannot be considered constants. The apparent dispersivities are determined in terms of the second moments (the spatial variances) of the solute distribution. Assuming this distribution to be approximately normal the variances in the two directions can be written

$$\sigma_L^2 = 2D_L^* t \tag{6}$$

$$\sigma_T^2 = 2D_T^* t \tag{7}$$

where

σ_L, σ_T – standard deviation of the solute distribution in the two directions.

Combining Eqs. (4), (5), (6) and (7) yields

$$\alpha_L^* = \frac{1}{U} \frac{d\,\sigma_L^2}{dt} \tag{8}$$

$$\alpha_T^* = \frac{1}{U} \frac{d\,\sigma_T^2}{dt} \tag{9}$$

Experimental Set-Up and Computational Procedure

Various heterogeneous porous media with known hydraulic conductivity distributions were established in the flow channel shown in Fig. 1. The porous media were created by inserting porous concrete blocks in a bed of sand, the statistical properties of the conductivity distribution being dependent on the number and the location pattern of the blocks. The variance of the conductivity $\log_e$ ranged from 0.08 to 0.4, wheras the length scale ranged from 2.2 cm to 4.5 cm.

To measure the solute transport process, i.e. concentrations as a function of time and space, a radioactive indium isotope, In^{113m}, was used as tracer. A pulse injection of the indium in aqueous solution, uniformly distributed over the whole width of the box, formed an almost rectangular distribution in the vertical plane corresponding to the initial conditions outlined below

$$c = \begin{cases} c_0 & (x, y) \in \Delta A_0 \\ 0 & \text{elsewhere} \end{cases}$$

c_0 – initial concentration
ΔA_0 – initial area of the solute plume.

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The radiation emitted by the indium was measured and processed by a scintillation detector and a counter. However, the measurements were spatially integrated as well as discrete in time and space, and these characteristics required further calculations, before measurements of radiation were converted to equivalent concentrations. A double-quadratic spline interpolation-extrapolation routine ensured that the measurements became quasi-continuous in time and space (known in all "points" every 5 minutes), and a deconvolution scheme transformed integrated measurements to "point" measurements. A "point" in these experiments is an area of 5 cm × 5 cm, which was the highest degree of resolution that could be established. From the concentration measurements the spatial variances were calculated, and from smooth curves drawn by hand on the basis of these calculations the apparent dispersivities were determined as outlined above.

Experimental Results and Discussion

Statistics of the Hydraulic Conductivity Distributions

The statistics of the conductivity distributions were calculated on the same scale as the concentration measurements, i.e. averaged over an area of 5 cm × 5 cm. Figs. 2 and 3. show how the length scales were determined by fitting an autocorrelation

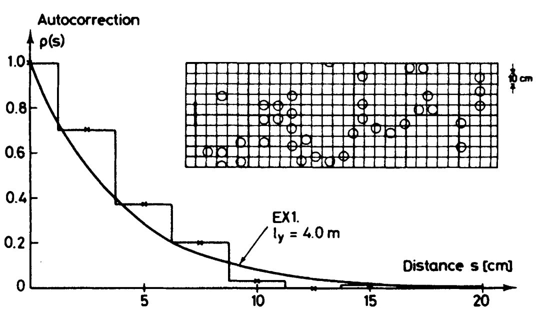

Fig. 2.
Sample and approximated autocorrelation functions for the log conductivity field. EX1.

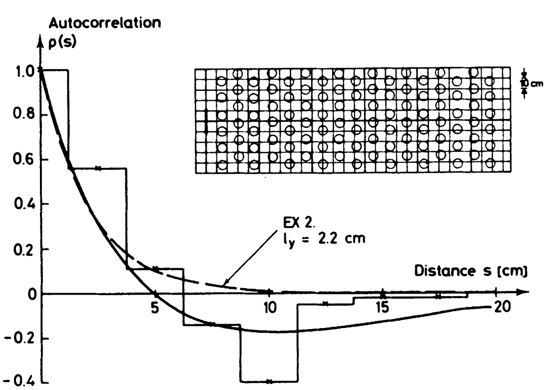

Fig. 3.
Sample and approximated autocorrelation functions for the log conductivity field. EX2.

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function of exponential type to each sample autocorrelation function. In the figures the configurations of the porous concrete blocks are indicated; the blocks in EX1 being randomly distributed, whereas the blocks in EX2 were symmetrically distributed. The variances and the length scales are listed in Table 1.

Table 1 – Log conductivity valves

	EX1	EX2
l_y [cm]	4.0	2.2
σ_y^2	0.17	0.37

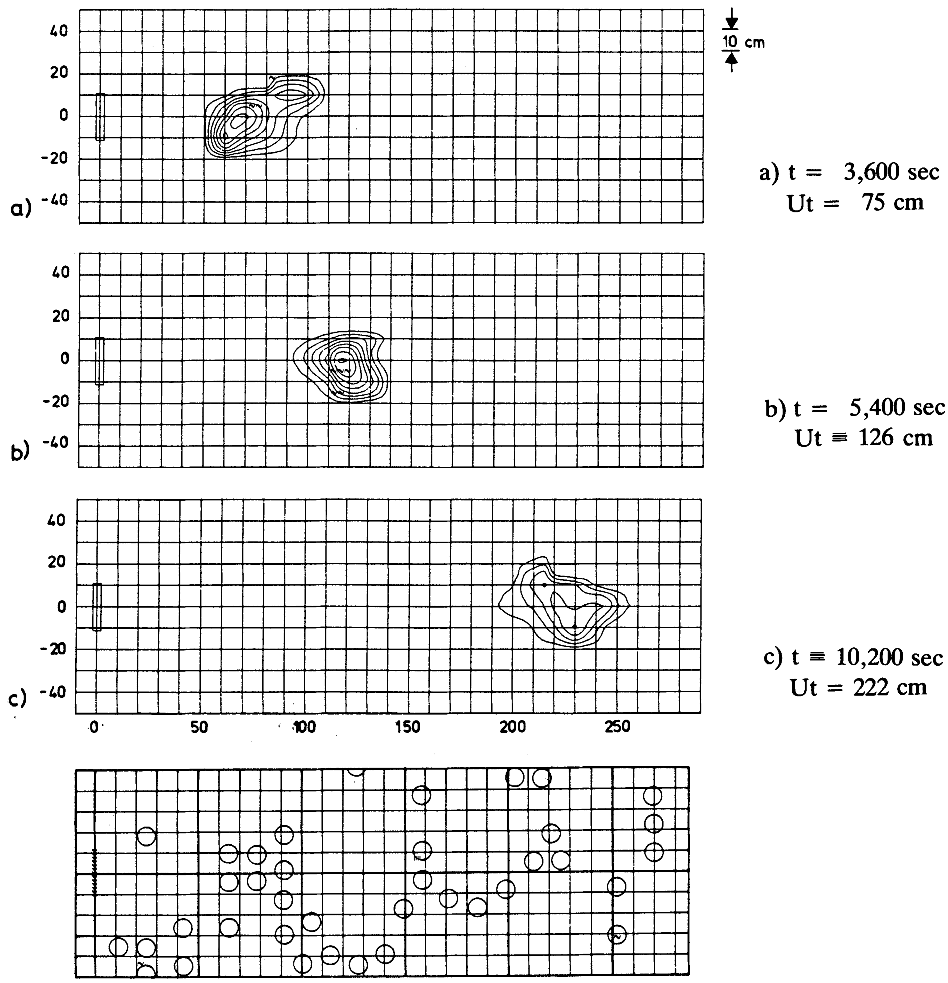

Fig. 4. Measured concentrations. EX1. Below the block configuration.

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Results

The dispersion of the solute plume in each experiment is shown in Figs. 4 and 5. Larger scale variations caused by the random block distribution in EX1 can be recognized in Fig. 4, hence the solute plume was alternately dispersed due to heterogeneous parts (60-100 cm, ~150 cm, ~225 cm), and "compressed" due to the homogeneous sand (100-150 cm, 160-200 cm). Furthermore, the dispersion process had not become Fickian at the end of the experiment. In contrast to this the solute plume was dispersed very smoothly when the distribution of the blocks were more dense as in EX2, (cf. Fig. 5), and Fickian dispersion occurred at the end.

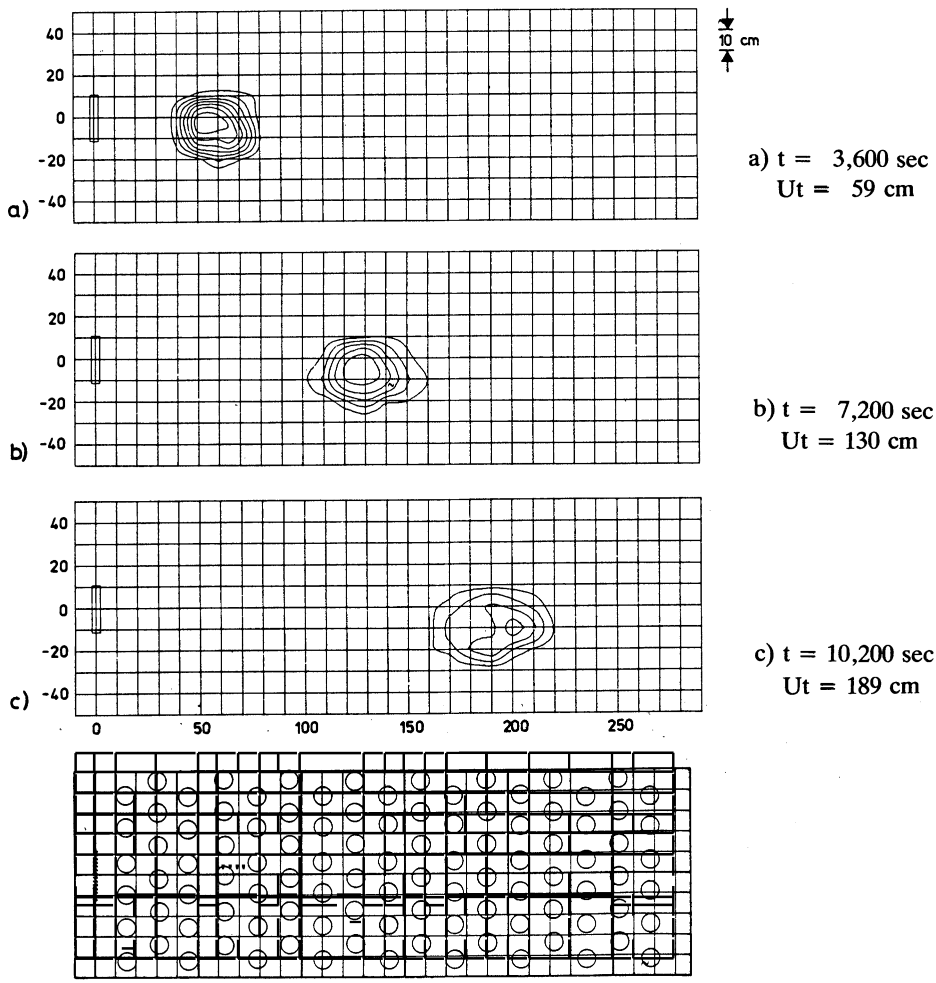

Fig. 5. Measured concentrations. Ex2. Below the block configuration.

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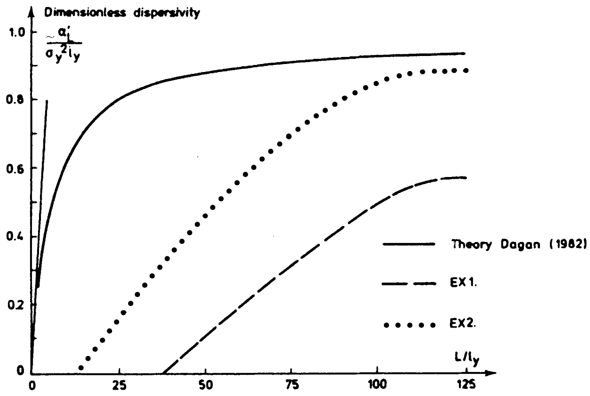

Fig. 6. Measured apparent longitudinal dispersivity compared to dispersivity predicted theoretically by Dagan (1982). The straight line through origin indicates the small time limit.

In order to extend EX1 to the Fickian zone, a numerical method, the USGS-MOC model was applied to an extended version of the experimental set-up to simulate the transport process. Since model predictions compared well to measurements it was considered reasonable to use the USGS-MOC model to support the experiments and to extend the investigations to solute transport in hypothetical heterogeneous media.

In Fig. 6 the experimental results are compared to the theoretical predictions by Dagan (1982) using a dimensionless form of the apparent longitudinal dispersivity and the travel distance. The figure shows a displacement of the experimental curves compared to the theoretical one, because in the experiments the solute plume initially moved a certain distance through homogeneous sand before it met the heterogeneous part of the media, and therefore the apparent dispersivities were zero in this zone. The experiments and the theory compared qualitatively well in several aspects, thus the apparent longitudinal dispersivity initially increased linearly with the travel distance, and after displacements larger than approximately 75 length scales it approached a constant value.

However, the theoretical curve in Fig. 6 is a first order solution for small perturbations of the hydraulic conductivity based on several assumptions that are not fulfilled in the experiments. For instance the pore scale dispersion is assumed to be negligible, whereas in the experiments pore scale dispersion proved to be of the same order of magniturde as the dispersion due to the heterogeneity of the porous media. These deviations can explain why the experimental dispersivities generally were smaller than the theoretical.

In examining the apparent transverse dispersivity the same agreement was not found. The experimental transverse dispersivities were approximately one order of magnitude less than the longitudinal and showed less dependency on the statistical

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properties of the porous media conductivity distribution, as predicted by the theories of Dagan (1982) and Gelhar and Axness (1983). However, the determination of the transverse variances from which the transverse dispersivities were calculated was highly uncertain because of the large concentration gradients at the edges of the solute plume.

Conclusions and Applicability to Natural Conditions

It has been shown experimentally that the dispersion of solutes in a heterogeneous porous medium initially cannot be assumed Fickian with constant dispersivities, thus the longitudinal dispersivity is dependent on the displacement from a source. This dependency is theoretically found by applying a stochastic description of the soil hydraulic properties of the porous medium as well as of the governing partial differential equations. The experiments and the numerical simulations carried out in this study have qualitatively and partially quantitatively supported this theory.

For small displacements from an injected source of solute the apparent longitudinal dispersivity increased approximately linearly with the travel distance while it approached a constant value for displacements larger than about 75 length scales. Apparently this value was close to $\sigma_y^2 \times l_y$ for isotropic media.

The apparent transverse dispersivity was shown to be at least one order of magnitude less than the corresponding longitudinal dispersivity, and there seems to be less dependency on the hydraulic conductivity distribution and no dependency on the travel distance as theoretically predicted. However, uncertainties due to measurement problems should be noted.

The main problem associated with practical applications of the stochastic theories to field conditions is that almost no field data on σ_y^2 and l_y exist at the present. It is necessary to convert the geologists qualitative knowledge about the variability of the aquifers into a quantitative knowledge to give estimates of σ_y^2 and l_y either for a Monte Carlo modelling technique or for a theoretical stochastic modelling technique. This requires a few field studies where a comprehensive collection of both geological and hydrogeological data takes place. In situ tracer experiments on the same fields should be performed to investigate whether the stochastic theories can be applied to field problems. The results collected from these fields form the basis for establisheing a "data bank" containing the characteristic statistical properties for soil types which occur commonly as aquifers. Whenever a new aquifer is under examination the "data bank" is possibly able to give rough estimates of σ_y^2 and l_y, and these properties can be used to determine the first estimates of the apparent dispersivities and hence the size of the actual contamination problem.

Given information on the statistical properties of the hydraulic conductivity of an aquifer, on which modelling of a contamination problem is taking place, extent of

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the problem can be related to the length scale, l_y; if the extent is less than 50-75 times l_y, nearfield conditions apply, and if the extent is larger, farfield conditions apply (constant apparent longitudinal dispersivity).

When modelling the solute transport process in the near-field of a contamination source the experiments in this study have confirmed that a deterministic modelling approach is very uncertain in this zone. The solute transport process is non-ergodic and the dispersion process is non-Fickian, and to account for these matters a stochastic modelling approach has to be adopted. Some deviations between the theoretically predicted values of the apparent dispersivities occurred in this zone and therefore the theoretically predicted dispersivities are assumed to be unrealiable. Instead, the use of a Monte Carlo modelling technique can be adopted and this technique is able to take the spatial variability of the hydraulic conductivity into account in a physically realistic way. The concentration predictions in each point of the model area can be given as a probability density function so that access risks are easily determined.

In the far-field, however, a deterministic modelling approach can be adopted. In this zone the experimentally predicted apparent dispersivities showed very good agreement with the theoretical expressions hence these can be applied to determine estimates of $\alpha_L{}^*$ and $\alpha_T{}^*$ corresponding to the degree of knowledge of the spatial distribution of the hydraulic conductivity.

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