DISCUSSION

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The most important contribution made by the authors of the paper is the modal feedforward controller designed to compensate for measured disturbances in a least squares fashion. The remaining portion of the work appears to be simply an extension of the work done by Takahashi, et al., reference [2] in the subject paper.

The method presented in the paper is nearly identical to the pseudoinverse technique examined by Takahashi, et al. in reference [2]. The only difference is that the former takes the right and left minimum inverses of the input and output matrices (B and D) in the original state space and then transforms these into modal space while the latter transforms B and D into the modal state space, and then performs the pseudoinverse operations. All other operations are identical. I am surprised to find, therefore, that one method requires greater computer time for determining the control algorithm as suggested by the authors.

While the authors are correct in pointing out the bulkiness of the recursive procedure, they should also be aware of the fact that this procedure conditions all poles exactly as specified. The pseudoinverse procedure provides a least squares fit and does not guarantee a stable feedback controller. A simple example will demonstrate a difficulty with the method.

Example: Let

$$\mathbf{A} = \begin{pmatrix} -3 & 4 & 4 \\ 1 & -3 & -1 \\ -1 & 2 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{d}^T = (0 \quad 1 \quad 0)$$

$$\mathbf{T} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad \mathbf{T}^{-1} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -2 & -3 \\ 1 & -2 & -2 \end{pmatrix},$$

$$\Lambda = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}.$$

If the feedback controller in the modal domain is chosen to be

$$\mathbf{K}_{fb} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

then the system will become unstable. No provisions are made for this in the paper. This is one danger of ignoring the conditions on measurement and manipulation.

I would finally like to point out to the authors that the so-called "imperfect control" which allegedly weakens the partitioning method of Takahashi, et al., is identical to the cross-coupling which exists in the pseudoinverse method. Only the magnitudes are different because of the different approaches. Furthermore, the recursive procedure was not an attempt to compensate for the cross-coupling problem, but an effort to extend the partitioning technique.

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The modal control approach is primarily meaningful when a control object is characterized by space modes, and it is desired to speed up recovery responses of only the dominant modes while conserving spatial modal patterns and to leave the motion of trivial modes unaffected by the application of the control. To achieve the design goal of modal control the control engineer must carefully choose the locations where the measurements are to be made and controlling inputs are to be applied. The "partitioning method" will offer the engineer an explicit guide to select or modify the input and output matrices (B and D in equation (1)) for an effective implementation of modal control. The pseudo-inverse method does not yield such a direct guide to instrumentation design.

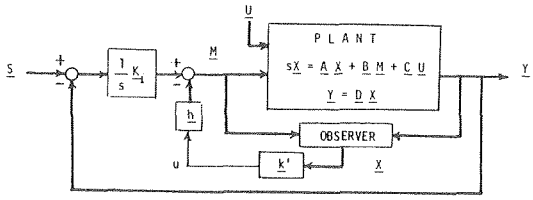

Fig. 11

If design goals call for closed loop placement rather than a preservation of the original natural spatial modes, any state vector feedback method provides a direct approach.[6] Luenberger's observers[7] or Loscutoff's method[8] serve to get an estimate of the state vector for this purpose. These will also apply to eliminate an imperfection of measurement ($\Delta F \neq 0$ in equation (25)).

Modal control is inadequate to match a set of controlled variables with a set of reference inputs (set points) when the latter vary with time, unless the controlled variables are expressed only in terms of controlled modes. In general the integral mode of control action is desirable to eliminate offset error and thus match a controlled variable precisely with a corresponding reference input in the equilibrium state. As shown in Fig. 11, the pole assignment feedback may constitute a minor loop in a control system where the major loop is closed via a set of I-control laws. The total order of the control system is then $(n + m)$ when a plant is n-th order and S and Y are both m vectors. Additional lags involved in the observer are normally negligible, and their eigenvalues are decoupled from the rest anyway. The gain parameters for the n-element row vector $\mathbf{k}'$ and the m-element diagonal matrix K_I are uniquely determined by a set of specified $(n + m)$ poles. In this figure $\mathbf{h}$ is an arbitrarily chosen column matrix for the purpose of adequately distributing a scalar variable among the elements of the vector input to the plant. The pole assignment theory [8,9] for a multi-input plant may be used for the same purpose.

The writer would like to congratulate the authors for making extensive use of the pseudo-inverse technique. It is often difficult to prove optimality of an outcome when the pseudo inverse is employed in the process. Nevertheless it is a most convenient technique to deal with engineering problems for which idealizing conditions that are necessary for a rigorous optimal solution seldom hold.

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[6]pp. 421–429 of the authors' Reference [2], and Brockett, R. W., "Poles, Zeros and Feedback: State Space Interpretation," *IEEE Trans. on AC.*, Vol. AC 10, No. 2, 1965, pp. 129–135.

[7]Luenberger, D. G., "Observers for Multivariable Systems," *IEEE Trans. on AC*, Vol. AC 11, No. 2, Apr. 1966, pp. 190–197.

[8]Loscutoff, W. V., "Arbitrary Pole Placement With Limited Number of Inputs and Outputs," Journal of Dynamic Systems, Measurement, and Control, TRANS. ASME, Series G, Vol. 96, No. 3, Sept. 1974, pp. 322–326.

[9]Chidambara, M. R., Broen, R. B., and Zaborsky, J., "A Simple Algorithm for Pole Assignment in a Multi-Input Linear Time-Invariant Dynamic System," Journal of Dynamic Systems, Measurement, and Control, TRANS. ASME, Series G, Vol. 96, No. 1, Mar. 1974, pp. 13–18.

Authors' Closure

Dr. Loscutoff's comment about our feedforward controller design is appreciated, and while he is correct about our technique being an extension of the work done by Takahashi, et al. (reference [2] in the subject paper), we believe that it is more than a simple extension which is identical to the pseudoinverse method. We feel that the construction offered by equation (12) of the paper is essentially a new way of viewing the design of the feedback matrix which may yet lead to additional insights (see, for example, the letter to the editor by Guerin and Rabins on this same paper which discusses the possibility of simultaneous pole-placement and optimal design).

The question of the differences in computer time (albeit small) may be accounted for by differences in the computing algorithms actually utilized and their respective efficiencies. Dr. Loscutoff is, of course, correct in stating that the two approaches are essentially equivalent. His counter-example involving an unstable feedback controller arises, as he points out, from ignoring the conditions on measurement and manipulation. We hope this oversight is partially remedied by the aforementioned letter to the editor by Guerin and Rabins. Finally, with regard to Dr. Loscutoff's comments on imperfect control and cross-coupling via-a-vis the partitioning method, the pseudoinverse method and the recursive procedure, we are in agreement with him.

We believe that Professor Takahashi's discussion on our paper constitutes an excellent survey of the subject and offers an important addenda to the paper. We are in complete agreement with his evaluation of the efficiency of modal control and when it should be applied. His congratulations at the end of his discussion are appreciated and we hope that his final comment regarding the difficulty of proving the optimality of control when utilizing the pseudoinverse approach may soon yield to further investigation. We thank both discussors for the obvious care with which they considered our paper.