This situation has a number of consequences. In contrast to the case first discussed where both bearings have misaligned ball sets, for the semimisaligned case the average preload does not change as the bearings rotate. Therefore, the spin-axis driving torque requirement will not have a sinusoidal modulation; it will be "straight line." Moreover, in bearing A, the retainer rate and the ball-group rate will be different; in bearing B, they will be equal and equal to the retainer rate of A:

$$\dot{\alpha}_A = \dot{\alpha}_B = \dot{\beta}_B \neq \dot{\beta}_A \tag{9}$$

A line joining the retainer *CG*'s will remain parallel to the spin axis and trace out a cylinder about it as the bearings rotate. The same is true of the ball-group *CG*'s. Thus no cross-torque, either retainer or ball-group-induced, can arise in this case.

Conclusions

A pair of ball bearings will produce torques at right angles to their spin axis as well as along it. These torques result from the eccentric rotations of the ball retainer and ball-group *CG*'s and their associated centrifugal forces. The torques are troublesome in control gyroscopes since the system cannot distinguish them from gyroscopic torques. They may be controlled by various techniques including indirect methods which take advantage of the couplings occurring in and between spin-axis bearings. The most interesting control results when one bearing is given a ball-group misalignment. In this case, the cross-torque level is zero and the spin-axis driving torque is constant. All of the effects mentioned in this paper have been verified in the laboratory.

Finally, it must be noted that a retainerless bearing which has been designed for small residual ball-gap will suffer from neither ball-group nor retainer cross-torque.

DISCUSSION

A. S. Irwin[3]

The author is to be complimented and thanked for this clear and concise treatment of one of the vagrancies of gyroscope design, one which is often charged, by devious logic, to deficiencies in bearing quality.

Probably the major cause for displacement of the bearing cage[4] from its neutral position results from the load system caused by differing velocities of the balls around the pitch circumference. These differences in velocities are due to the different planetary speeds in the various quadrants. The differing planetary ratios result from the charges in contact angle due to the application of varying radial and thrust loads. For example, let us assume a uniform thrust load on a bearing. The balls will operate, in this condition, with the same contact angle and at the same speed. Actually, the cage could be removed from such a bearing and the balls would stay in position.

Let us now consider that a radial load is applied, in the absence of a thrust load. In the radially loaded quadrant, the ball velocity will vary but slightly. The small variation is due to the changes in diameters resulting from deflections under radial ball loadings varying from 0 to a maximum in the center of the radial load zone and back to 0 in the remaining portion of the bearings; the balls are free to readjust and, therefore, cannot impose an appreciable cage load.

Under combined radial and thrust loading (Fig. 7), assuming all balls maintain tractive contact with the races, the ball velocities vary and the balls therefore move in and out of their proper spacing position. In the radially loaded zone, the inner race contact point is moved radially inward on the race, thus decreasing

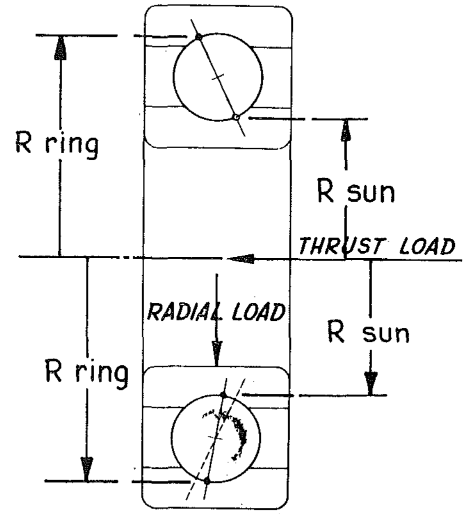

Fig. 10

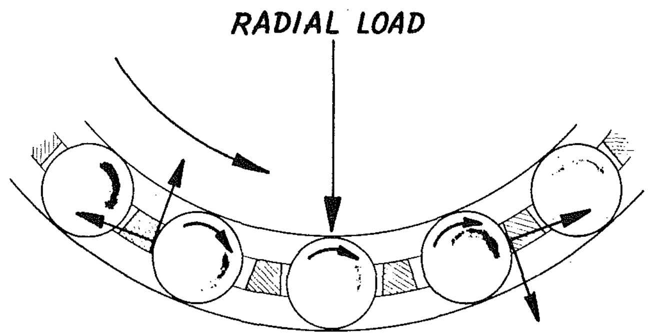

Fig. 11

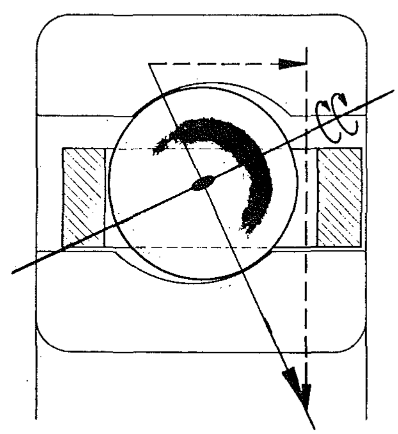
Fig. 12

the ball speed. As the ball enters the radially loaded zone, therefore, it will slow down or drop behind its proper position, Fig. 8. It is therefore pushing back on its cage pocket. At the same time, there is a ball coming out of the area of maximum load which is speeding up and is therefore pushing the cage ahead. The frictional components of these two forces impose a moment on the cage about a center somewhere between the two balls in question. This moment, acting in the plane in the bearing, tends to rotate the cage out of the bearing. This moment is resisted, for a ball-riding separator, by the contacts of the cage with the other balls; and, in the case of a land-riding retainer, by pressure on the re-

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[4] Excerpted from Panel Presentation "Bearings for Power Plant Accessory Drives," ASME 1965 Winter Annual Meeting.

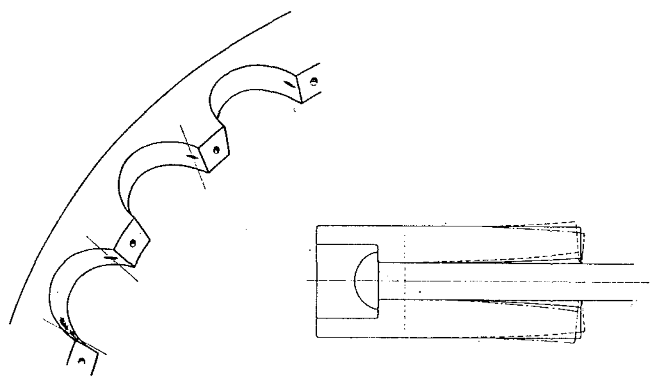

Fig. 13

tainer guiding land. It is interesting to note that, for a bearing with an inner land-riding retainer operating under combined loading in which the radial load rotates in direction with the inner race, one sector of the inner race is subjected to continuous pressure from the cage. In the case of an outer land-riding retainer under these conditions, the entire circumference of the outer race land is subjected to this loading and the wear is therefore distributed over a considerably larger area. For a bearing subjected to combined loading in which the radial load is fixed in direction with respect to the outer race, with an outer land-riding retainer, the retainer contact is again concentrated in one sector of the outer race land; and, in the case of an inner land-riding separator, the contact is distributed around the circumference of the inner race land.

The geometric relationships discussed previously are distorted for high speed bearings in which the contact angle on the outer race is reduced and the contact angle on the inner race is increased by the centrifugal force of the ball. Under these conditions, the contact angle changes into and out of the radially loaded zone are more drastic and therefore the cage loads are higher than in the case of lower speed bearings.

In a bearing operating under thrust load, the contact angle is inclined, with respect to the normal, to the shaft centerline, Fig. 9. The net cage to ball load therefore has a component parallel to the shaft which causes the cage to be axially displaced, and the contact area between the cage and ball to be located to one side of the retainer center faces. On the basis of the analysis discussed earlier, for combined radial and thrust load, one half of the retainer is therefore subjected to contact pressures and frictional components of a reversing nature, Fig. 10.

In addition, the fact that the cage is axially displaced reduces the effective ball pocket clearance in the plane of the balls.

Contrary to the assumption of the author, the ball pocket clearance is not always made equal to the retainer diametral clearance. In fact, one "rule of thumb" for determining these clearances indicates that the maximum ball pocket clearance in the plane of the balls should be equal to twice the minimum land clearance.

In future research of the type reported, the author should consider the effect of matching the contact angles of the two bearing rows.

This work should be encouraged as it has significance in other fields other than gyroscopes. For example, in machine tool grinding spindles, it has been demonstrated that, in a pair of matched preloaded bearings, placing the high points of face runout 180 deg apart minimizes radial and axial runout of the spindle. More analytical work needs to be done along these lines.

C. T. Walters[5]

The author is to be congratulated for his continuing pioneering efforts in the detailed study of instrument ball bearing kinematics and the effects of rather subtle motions and couplings on gyroscope performance. With regard to the measured cross torque presented in Fig. 5, it would be enlightening if the author could reveal the level of agreement of amplitude predicted by the simple theory and that of the experimental results. It is interesting to note that all four mechanisms the author mentions which might contribute to disagreements in amplitude and frequency (whirl motions, bouncing, ball spacing shifts, axial retainer center of mass motion) have been observed by the discusser in BASDAP[6] calculations of gyro-bearing dynamics. A fifth mechanism observed which would affect cross torque arises from the execution of retainer wobble motions in which the area of contact of a race guided retainer shifts from one side of the ball groove to the other.

With regard to the author's comment that ball-race coupling is not well understood, the discusser would like to point out recent advances in the theory of ball motion with realistic elastohydrodynamic lubrication presented in the paper by Walters, et al.[7] Some of the important conclusions are:

- Strict race control of the ball does not occur
- Increasing cross-race curvature does not necessarily decrease ball-race coupling
- Cross-race slip is always present to some extent due to gyroscopic moment
- Ball excursions are highly transient in the presence of a retainer and make stroboscopic data and static force balances of questionable value.

Since the discusser used Newton's laws of motion in the ball studies, he is indeed reassured by the author's experimental results "that a transverse force applied to a bearing ball results in a small change in the orbital velocity of the ball in the direction of the transverse force."

Under *Retainer Lock-In* it would be helpful if the author would clarify why the torque demand varies so much in Fig. 9 when the ball groups are supposedly locked in step and the preload presumably constant.

The ball-jump retainer control method appears very promising and the discusser looks forward to the author's forthcoming paper to see if the simple theory of the mechanism is indeed adequately justified by the experimental results.

Finally, the author is to be praised for his continuing campaign to win converts for the full-complement ball bearing. The concept is promising if long-term meager lubrication can be effected.

Author's Closure

Mr. Irwin is concerned mostly with bearings which support a combined radial and thrust loading, a case not mentioned in this paper. He does say, for uniform thrust load, "... the balls will operate ... with the same contact angle and at the same speed ... the cage could be removed ... and the balls would stay in position." As pointed out in this paper, if there is a ball group misalignment interaction between the bearings, and there usually is, the loads on the balls vary under a pure thrust load, and individual ball orbit rates are *not* the same, or constant. The speed variations are small, but they operate over times of the order of the beat period of the two ball groups which is large, and hence they have large effects.

The case of exactly matched contact angles was explicitly taken up in the section of the paper labeled "Retainer Lock-In."

Mr. Walters wonders about the level of agreement between the theory presented in this paper and measured cross-torque levels. In general, the cross-torque predicted from retainer CG excursion

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[6] BASDAP is a comprehensive computer program developed at Battelle for the detailed calculation of ball and separator dynamics under realistic lubrication conditions.[7]

[7] Walters, C. T., et al., Final Report on, "Study of the Behavior of High-Speed Angular-Contact Ball Bearings Under Dynamic Load," Contract NAS 8-21255, May 12, 1969.

is measured. The ball group CG excursion calculation is for the worst possible (but unlikely) condition, and gives larger values than are commonly measured.

I am gratified that some of the effects described in this paper can be calculated with a computer program. It is dangerous to write a program without some idea of what to look for, as has been demonstrated historically. In my opinion, anyone making a calculation relating to ball control via cross-race curvature and based on Coulomb friction, is wasting his time. The level of understanding of ball-race coupling has progressed beyond dry friction but is not yet at the point where coefficients in the differential equations can be calculated rather than measured.

With regard to Newton's laws, perhaps for clarity I should have written "... a transverse force applied to a bearing ball results in a small change in the *terminal* orbital velocity of the ball in the direction of the transverse force." Surely Newton was concerned with the effects of forces upon accelerations, rather than upon velocities. One hopes that BAS DAP recognizes this distinction.

With regard to variations in spin axis torque during retainer lock-in, as shown in Fig. 9, this trace shows only the ac part of the orque demand. The variations in Fig. 9 represent changes on he order of 100 dyne centimeters out of a total demand of 10,000 dyne centimeters. There are two types of variation during lock-in: a drift, which is due to an overall temperature change in the laboratory of perhaps 0.5 deg C; and a repeatable variation due to changes in the convective heat transfer coefficient of the test housing in its different positions. These changes are reflected in a temperature change *at the bearings* which has been estimated at $^1/_{20}$ deg C. It is such changes which produce the small long-term torque changes in Fig. 9 (the table rotated at 10 times earth rate in this experiment, 150 deg/hr).

A paper giving experimental proof of cross-torque control by ball group coupling is in press.

Reasons given by management for including a ball retainer in *an instrument bearing boil down to two: to provide an oil reservoir*; and to keep the balls apart. Both of these reasons are based on conjecture, and neither is confirmed by experiment. An order of magnitude improvement in stability and a factor of eight decrease in torque demand at high speed are easily available with a full-complement bearing.[8] Life does not appear to be impaired in those experiments I have run, raising some interesting questions as to the assumptions commonly made in EHD analysis.

[8] Kingsbury, E., *Experimental Observations on Instrument Ball Bearings*, MIT/DL Report E-2316, Sept. 1968.