Non-adiabatic linear pulsation models for low-mass helium stars

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\section*{ABSTRACT}
Non-adiabatic linear pulsation models have been calculated for low-mass stars with effective temperatures between 16 000 and 35 000 K, and with surface gravities in the range $3 < \log g < 5$. The radial pulsation models assume a homogeneous stellar envelope which is deficient in hydrogen and display the well-known Z-bump instability to radial pulsations. The aim of this paper has been to explore the behaviour of the Z-bump instability as a function of mass and composition around a reference model with $M = 0.5 \, M_\odot$, $X = 0.00$, $Z = 0.02$. It is shown that the Z-bump instability persists to low masses ($M \sim 0.4 \, M_\odot$) but is suppressed either by a reduction in metallicity $Z$ or by a selective enhancement of the carbon abundance. An unexpected result is the discovery that Z-bump instability persists at hydrogen abundances $X > 0.3$, although the position of the red edge is sensitive to $X$. We have found that non-radial pulsations are also excited in the same instability region as radial pulsations.

The implications of these results for individual low-mass helium stars are discussed. It is concluded that Z-bump driven pulsations (radial and/or non-radial) may be excited in some helium-rich subdwarf B stars, representing a possible major extension to the class of variable stars represented by the prototype V652 Her.

\textbf{Key words:} stars: oscillations.

\section{1 INTRODUCTION}
As observations have become more sensitive and theoretical models have become more sophisticated, the study of pulsations in stars has extended to all areas of the Hertzsprung–Russell (HR) diagram. Pulsations have been used to provide standard candles for measuring distances and as probes of stellar structure and evolution. Zones of stability and instability against pulsation in many modes have been identified as functions of mass, effective temperature and luminosity (Gautschy & Saio 1995, 1996). A further factor in determining pulsation instability is chemical composition; the presence or absence of material with the right opacity to excite pulsations is crucial, for example, in the excitation of $\beta$ Cepheid (Z-bump opacities, Kiriakidis, Eid & Glatzel 1992; Moskalik & Dziembowski 1992) or PG 1159 pulsations (CO-bump opacity: Saio 1996; Gautschy 1997).

In the case of highly evolved stars, the occasional absence of hydrogen in the stellar envelope creates additional possibilities for the excitation of pulsations. At low effective temperatures, RCrB stars show radial pulsations excited by the classical $\kappa$ mechanism also seen in $\delta$ Cepheids (Saio, Wheeler & Cox 1984). Radial pulsations in hotter helium stars are also excited, providing the luminosity-to-mass (L/M) ratio is sufficiently high (Saio & Jeffery 1988). In this case the driving mechanism appears to be strange-mode instability (Gautschy & Glatzel 1990). Pulsations in extreme helium stars such as PV Tel (masses in the region of $0.8–1.0 \, M_\odot$) and in hydrogen-deficient binaries such as $\nu$ Sgr (masses $1.0–3.0 \, M_\odot$) could both be accounted for in this way (Saio 1995).

As was the case for $\beta$ Cep pulsators, theoretical models were unable to account for pulsations in the helium star V652 Her until improved opacity calculations correctly included contributions from high ionization stages of iron-group and other elements (Rogers & Iglesias 1992). The iron-group opacities (Z-bump) lead to the excitation of pulsations ($\kappa$-mechanism) in helium stars with $T_{\text{eff}} \sim 25 000 \, \text{K}$, providing the metallicity and luminosity are sufficiently high (Saio 1993). The identification of this finger of instability led to the prediction and identification of radial pulsations in a second helium star LSS 3184 (Saio 1995; Kilkenny & Koen 1995).

In linear calculations carried out since the introduction of new opacities, and in the non-linear calculations by Fadeyev (1990) and Fadeyev & Lynas-Gray (1996), the lowest-mass helium star considered has been $0.7 \, M_\odot$, corresponding to the value derived for V652 Her (Lynas-Gray et al. 1984). It was assumed, because the lower limit for pulsational instability is proportional to the L/M ratio (Saio & Jeffery 1988), and in shell-burning stars L/M increases with $M$ (Jeffery 1988, Saio 1988), that lower masses would not be of interest. Implicit was the assumption that all pulsating helium stars are structurally analogous to post-AGB helium-shell burning stars with CO degenerate cores. That
assumption was probably wrong. There are several reasons why it is appropriate to examine pulsation in helium stars with masses as low as 0.4 \( M_{\odot} \). These will be examined in detail in a later section, but briefly they include a postulated revision of the mass for V652 Her (now withdrawn, Jeffery et al. 1999), evidence for helium star pulsations outside the \( Z \)-bump instability finger (Jeffery 1998), the existence of helium-rich subluminous stars between the extended horizontal branch and the \( Z \)-bump instability finger (Jeffery 1996), and the discovery of \( Z \)-bump driven pulsations in subluminous B stars of normal composition (EC14026 variables: Kilkenny et al. 1997, Charpinet et al. 1996). In this paper we explore the stability of low-mass helium stars against pulsations. We discuss our results with respect to current observations of helium stars and the possibility of detecting these pulsations.

2 RADIAL PULSATION MODELS

Linear non-adiabatic models of radial pulsations in stars with helium-rich envelopes have been constructed following the method described by Saio (1995), but including the most recent OPAL95 opacities (Iglesias & Rogers 1996). The parameter range we have considered is \( M/M_{\odot} = 0.4 - 0.9 \), \( \log T_{\text{eff}}/K = 4.20 - 4.55 \), \( \log g = 3.0 - 5.0 \), \( Z = 0.004 - 0.02 \). This includes all possible combinations of mass, effective temperature, and metallicity.

![Diagram](https://example.com/diagram.png)

**Figure 1.** The boundaries for pulsational instability of low-mass helium stars. The model parameters are shown in each panel, the solid line represents the case for \( M = 0.50 M_{\odot} \).
low-mass helium stars between the helium main sequence and the Eddington limit within this temperature range. In general, we have considered a hydrogen-free mixture $X = 0.00$, where $X$ is the hydrogen abundance by mass, but have investigated the effect of raising the hydrogen content, and of including additional carbon in the models. These correspond to observations of hydrogen and carbon in many helium-rich stars.

For each model, we have calculated the period ($P$) and growth rate ($\gamma$) of the non-adiabatic pulsation mode with a period closest to that of the fundamental radial mode given by adiabatic theory ($Q = \Pi \dot{\rho}$). In most cases this mode corresponds to the fundamental radial mode, as shown by Saio (1995) for pulsations around the $\nu_1$-bump instability finger. However, in the most luminous models, strange mode pulsations are excited and the mode with the period closest to that of the adiabatic fundamental mode is an overtone pulsation with one or more radial nodes.

Table 1. Growth rates ($\gamma$) and pulsation periods ($P$) for low-mass helium stars. Growth rate exponents are shown in parentheses. Periods (in days) are shown for unstable modes only.

<table>
<thead>
<tr>
<th>log $g$</th>
<th>log $R$</th>
<th>$P$ (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.00</td>
<td>0.899</td>
<td>2.1(-1)</td>
</tr>
<tr>
<td>3.10</td>
<td>0.894</td>
<td>4.2(-1)</td>
</tr>
<tr>
<td>3.20</td>
<td>0.799</td>
<td>3.4(-1)</td>
</tr>
<tr>
<td>3.30</td>
<td>0.749</td>
<td>2.7(-1)</td>
</tr>
<tr>
<td>3.40</td>
<td>0.699</td>
<td>1.8(-1)</td>
</tr>
<tr>
<td>3.50</td>
<td>0.669</td>
<td>1.3(-1)</td>
</tr>
<tr>
<td>3.60</td>
<td>0.599</td>
<td>1.1(-1)</td>
</tr>
<tr>
<td>3.70</td>
<td>0.549</td>
<td>1.4(-2)</td>
</tr>
<tr>
<td>3.80</td>
<td>0.499</td>
<td>7.4(-3)</td>
</tr>
<tr>
<td>3.90</td>
<td>0.449</td>
<td>2.5(-2)</td>
</tr>
<tr>
<td>4.00</td>
<td>0.399</td>
<td>2.0(-2)</td>
</tr>
<tr>
<td>4.10</td>
<td>0.349</td>
<td>1.1(-3)</td>
</tr>
<tr>
<td>4.20</td>
<td>0.299</td>
<td>1.4(-3)</td>
</tr>
<tr>
<td>4.30</td>
<td>0.249</td>
<td>2.2(-2)</td>
</tr>
<tr>
<td>4.40</td>
<td>0.199</td>
<td>2.9(-2)</td>
</tr>
<tr>
<td>4.50</td>
<td>0.149</td>
<td>2.2(-4)</td>
</tr>
<tr>
<td>4.60</td>
<td>0.099</td>
<td>0.12(-5)</td>
</tr>
<tr>
<td>4.70</td>
<td>0.049</td>
<td>0.20(-6)</td>
</tr>
<tr>
<td>4.80</td>
<td>0.001</td>
<td>0.11(-6)</td>
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<tr>
<td>4.90</td>
<td>0.021</td>
<td>0.21(-7)</td>
</tr>
<tr>
<td>5.00</td>
<td>0.031</td>
<td>0.24(-7)</td>
</tr>
</tbody>
</table>

Fig. 1 shows the results of our analysis. Figs 1(a) and (b) show the stability boundaries in both $L-T_{\text{eff}}$ and $g-T_{\text{eff}}$ space for the $X = 0.0$, $Z = 0.02$ models and for $M = 0.4-0.7$ $M_\odot$, while Figs 1(c) and (d) show the stability boundaries for the $X = 0.0$, $Z = 0.50$ $M_\odot$ models for $Z = 0.004$, 0.01 and 0.02. Figs 1(e)–(h) show the effect of varying both the hydrogen and additional carbon abundances. The $g-T_{\text{eff}}$ diagrams provide a useful comparison with observations of helium-rich stars since, for most, no independent estimate of their mass is available. The periods and growth rates of unstable modes are shown in Table 1 for models with $M = 0.5$ $M_\odot$, $X = 0.0$, $Z = 0.02$ as a function of $T_{\text{eff}}$ and $g$. 

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To first order, the periods and growth rates in this table are independent of mass, while the periods are also independent of $Z$, because the stellar mean density is approximately fixed by $g$ and II is governed by the period mean-density relation. These values are not independent of $L$.

The extension of pulsation instability to low luminosities or high gravities at $T_{\text{eff}} \sim 23,000$ K in Fig. 1 is the result of high iron-group element opacities at temperatures around $10^5$ K, the now well-known $Z$-bump instability finger (Saio 1993). What we see from Fig. 1(b) is that $Z$-bump instability extends to very low-mass helium stars, should such objects exist. The only constraint on the extent of the instability finger, at least in $g-T_{\text{eff}}$ space, is the metallicity of the stellar envelope, which directly governs the magnitude of the $Z$-bump opacity.

Since composition plays such an important role, it is to be expected that the contribution of the $Z$-bump opacity may be modified by additional hydrogen or carbon opacity. Fig. 1 demonstrates that the addition of carbon suppresses the instability, in a manner similar to the reduction of the total heavy element fraction. Note that in both cases, the reduction of $Z$ and the enhancement of $C$, the instability finger is shifted to the red as it is suppressed. In contrast, the addition of hydrogen has the opposite effect and shifts the instability finger to the blue. Adding hydrogen in the envelope reduces the mean molecular weight and increases the opacity per unit mass. Both effects reduce density in the envelope. This reduces the driving effect of the Z-bump. Therefore, including hydrogen tends to suppress pulsations. This effect is small around the blue boundary, where the damping below the $Z$-bump becomes important. A smaller density reduces the damping effect as well as the driving effect at the $Z$-bump. Near-cancellation between both effects seems to make the blue boundary nearly independent of the hydrogen abundance. Consequently, and somewhat to our surprise, the suppression of $Z$-bump pulsations requires a considerable quantity of hydrogen. An appreciable instability finger remains when $X = 0.3$ and can still be identified at $X = 0.5$ (these mass fractions correspond to number fractions $n_\text{H} = 0.64$ and 0.81 respectively). The consequences of even modest helium enhancement in the envelopes of stars with $5.0 < \log g < 3.8$ and $T_{\text{eff}} \sim 25,000$ K are thus considerable, and a large range of stellar types may exhibit radial pulsations due to $Z$-bump instability.

3 NON-RADIAL PULSATIONS

To examine the stability of non-radial pulsations it is necessary to have whole stellar models. Two kinds of models have been examined for the stability of non-radial pulsations; zero-age helium main-sequence stars with $0.4 \geq M/\text{M}_\odot \geq 0.7$, and evolutionary models from a zero-age horizontal branch having an envelope chemical composition of $X = 0.3$. We have not found any overstable non-radial modes in our zero-age helium main-sequence models, while we have found that some non-radial modes are overstable in the evolutionary models located in the instability region on the HR diagram obtained in the previous section.

Zero-age helium main-sequence models for $Z = 0.02$ extend, in the HR diagram, from $(\log L, \log T_{\text{eff}}) = (1.569, 4.591)$ to $(0.779, 4.493)$ for masses ranging from $0.7$ to $0.4\text{M}_\odot$. The logarithmic values of the surface gravity range from $5.9$ to $6.2$. Clearly, all the models are outside the instability region obtained in the previous section for radial pulsations. For the zero-age helium main-sequence models we have examined the stability of non-radial modes from $g_{10}$ to $p_{10}$ for $\ell = 1$ and $2$ and have found no overstable modes.

The models for $Z = 0.05$ have $\log L$ and $\log T_{\text{eff}}$ lower by $0.05$ and $0.03$, respectively, than the $Z = 0.02$ ones, but are still located on the blue side of the instability boundary. We have found no overstable mode in any of these models.

For another set of models we have calculated evolutionary models from a zero-age horizontal-branch model having an envelope hydrogen abundance of $X = 0.3$. We have adopted a helium core mass of $0.473\text{M}_\odot$. A total mass $0.52\text{M}_\odot$ has been chosen so that at least a part of the evolutionary track resides in the instability region of radial pulsations. The evolutionary track is shown in Fig. 2 with the instability boundary of radial pulsations for $X = 0.3$. Model properties at selected evolutionary phases are given in Table 2. After a small increase in luminosity, the luminosity decreases and the effective temperature increases. In the early phase of evolution, the hydrogen-burning shell generates most of the surface luminosity. Depending on the envelope mass and hydrogen content, the outward-moving hydrogen-burning shell becomes inactive and the star contracts towards the helium main sequence (HeMS). The envelope mass at which bifurcation occurs, between evolution to the HeMS and evolution to the AGB, increases with $Y$.

We have examined the stability of non-radial as well as radial pulsations for the models indicated by circles in Fig. 2. We have found that non-radial as well as radial pulsations are excited only in the third and fourth models in Table 2. As shown in Fig. 2, these models are located in the instability region on the HR diagram for $X = 0.3$ and $M = 0.5\text{M}_\odot$. The excitation is a result of the $Z$-bump $\kappa$ mechanism, for which the efficiency is determined mainly by the pulsation period. In the third model, overstable pulsations have periods between 0.75 and 1 h irrespective of the degree $\ell$ of non-radial pulsations. These are $g_{12} - g_{16}$ modes for $\ell = 1$, $g_{20} - g_{26}$ modes for $\ell = 2$, and the radial fundamental mode.
the period of which is 0.78 h. In the fourth model, the period range excited is between 0.5 and 0.6 h. This is smaller than in the third model, because the fourth model is located close to the instability boundary (Fig. 2). The radial fundamental mode, which is also excited, has a period of 0.51 h. Since the Brunt–Väisälä frequency is high in the interior, the non-radial modes have mixed character and high-order g modes are excited. These modes have periods similar to the radial fundamental mode and no node appears in the p-mode propagation zone for the overstable modes given in this paper. The mode classification is based on the number of nodes in the g-mode propagation zone. This result indicates that stars in the instability region could show multiple periodicities. We note that these periods are longer than the periods of overstable modes in the more compact hydrogen-rich sdB stars studied by Charpinet et al. (1996).

Non-radial pulsations with periods shorter than the above ranges are stabilized because, as for radial modes, damping in the region exterior to the Z-bump becomes stronger. Stable longer-period modes have a large number of nodes in the core. The main damping for these modes comes from radiative dissipation in the core caused by the very short spatial wavelength of the oscillations. This is one of the reasons why we need to include the whole stellar structure in non-radial pulsation analyses.

4 LOW-MASS HELIUM STAR PULSATORS

It has been established that at least, some helium stars with $3.0 < \log g < 5.0$ and $20000 < T_{\text{eff}}/K < 28000$ are unstable against radial and possibly non-radial pulsations, even with masses as low as $0.4 M_\odot$. In this section we compare these models with the observed properties of helium-rich stars close to the theoretical instability region (Fig. 3).

As discussed in the introduction, the principal examples are V652 Her and LSS 3184. In the case of the former, current data indicate a mass of $0.7 M_\odot$, whilst the latter is a carbon-rich helium star and likely to be of similar mass. However, arguments from stellar evolution currently suggest that a mass of $\sim 0.6 M_\odot$ for V652 Her may be more easily explained whilst remaining consistent with the observations. HD 144941 is included in Fig. 3 as an example of a low-mass helium-rich star in the Z-bump instability region which does not pulsate (Jeffery & Hill 1996). The simple reason is that its metallicity is too low for Z-bump pulsations to be excited (Jeffery & Harrison 1997).

The carbon-rich helium star LS IV + 6° (Jeffery 1998) is suspected of being variable, but lies outside the Z-bump instability finger as presented here. If the variability and the surface parameters are correct, we may have to consider an enhancement of Fe abundance around the Z-bump driving zone arising from radiative levitation, as considered by Charpinet et al. (1997) for hydrogen-rich sdBs. Another helium star, LSS 5121 (Heber, Jonas & Drilling 1986), probably resembles the more luminous helium star V2076 Oph, which lies above the horizontal part of the instability boundary (Saio & Jeffery 1988) and pulsates non-radially (Lynas-Gray et al. 1987).

In addition to these bona fide helium stars are the relatively unexplored helium-rich subdwarf B stars (HesdBs). These have been identified primarily in surveys by Green, Schmidt & Liebert (1986) and by Moehler et al. (1990) and have optical spectra dominated by absorption lines of neutral helium and, in some cases, C ii and C iii. These should be distinguished from the hotter helium-rich subdwarf O stars, in which the spectra are dominated by He ii lines. Since current nomenclature is confusing, efforts are being made to refine the spectral classification system for hot subdwarfs (Jeffery et al. 1997; Drilling et al., in preparation). As a

![Figure 3. Fundamental radial pulsation periods (in days; dashed lines) and observed helium-rich stars (symbols) in the log $g$–log $T_{\text{eff}}$ diagram. Solid symbols represent known variable extreme helium stars, circles are stars in the Z-bump instability finger. Open squares represent preliminary results for three helium-rich sdB stars. The solid line represents the instability boundary for models with $M = 0.50 M_\odot$, $X = 0.00$, $Z = 0.02$. The loci of the zero-age main sequence (ZAMS), zero-age and terminal-age horizontal branch (ZAHB and TAHB) are shown as dotted lines. The ECI4026 variables all lie below this figure with log $T_{\text{eff}} \sim 4.53$, 5.5 < log $g < 6.1$ (Billères et al. 1997).](https://academic.oup.com/mnras/article-abstract/308/1/221/1006795/3)
consequence a significant number of extremely hydrogen-deficient B-type subdwarfs have been identified. Unfortunately, systematic efforts to analyse individual members of this class remain incomplete.

Preliminary data for the class prototype, PG1544 + 487, indicate $T_{\text{eff}} = 31,000 \text{ K}$, $\log g = 5.1$ (Heber et al. 1988). H and C contribute $< 0.3$ and $\sim 1$ per cent by number to the photospheric composition, respectively. A second HesdB, JL 87, is more hydrogen-rich ($X = 0.5$, Schulz, Heber & Wegner 1991), but still interesting in the present context. Whilst PG1544 + 487 and JL 87 both lie outside the theoretical Z-bump instability finger, Jeffery et al. (1997) showed that the spectra of He-rich subdwarfs form a continuum which extends towards lower gravity objects such as LS IV + 62. They have gravities lower than those for typical normal sdB or blue horizontal branch stars. It is therefore of considerable importance to make detailed measurements of $T_{\text{eff}}$, $\log g$ and composition, and to observe these stars systematically for variability.

In this context, our finding that Z-bump instability persists in the presence of a substantial hydrogen abundance is significant. Note that $X = 0.2$ corresponds to a H/He ratio $\sim 1$ by numbers. Whilst Drilling et al. (1999) have made a systematic classification of several hundred subdwarf spectra, Montañés Rodriguez, Jeffery, Drilling & Moehler (in preparation) are currently calibrating the classification system using local thermodynamic equilibrium model atmospheres. Although primarily an exercise designed to explore systematic effects in the subdwarf classification system, the results are particularly useful for estimating the basic parameters of those subdwarfs in which He dominates He II. For example, they find in a preliminary study that the expected spectrum for a model atmosphere with $T_{\text{eff}} \sim 25,000$, $\log g \sim 4.5$, He/He = $0.3$, $X \sim 0.35$ strongly resembles the observed spectrum of PG0229 + 064 (classified sdBS:He5). According to the current Z-bump models, this star would be an interesting candidate for radial Z-bump instability (see Fig. 1g).

It remains to be seen how many such stars may be found, since the identification of B-type subdwarfs and blue horizontal-branch stars becomes confused with B-type main-sequence stars at the effective temperature of the Z-bump instability.

One of the principal questions concerning HesdBs is whether all, some, or any are related to the extreme helium stars or to normal subdwarf B (sdB) stars. If the former, then little further can be said about their masses than that V652 Her has a mass $\sim 0.7 M_\odot$. If HesDbs are similar to normal sdB stars, their canonical masses would be $\sim 0.5 M_\odot$ (Heber 1986). Normal sdB stars are thought to be extremely blue horizontal-branch stars and to consist of a helium-burning core with a very thin hydrogen skin. In this case, it may be reasonable to suppose that some specimens have atmospheres contaminated with more helium than normal.

5 CONCLUSION

Previous calculations of pulsation instability in helium stars have been extended to include a range in mass, hydrogen, and carbon abundance that is far wider than any previously considered. Models for both radial and non-radial pulsation have been explored, mainly focused on the Z-bump instability finger ($20,000 < T_{\text{eff}} / K < 28,000$). Z-bump instability has been shown to extend to stars with masses $\sim 0.4 M_\odot$, thus including masses typical of the extended horizontal branch. The blue boundary of the Z-bump instability finger is insensitive to the hydrogen abundance ($X \leq 0.3$, $n_H \leq 0.64$). Such ‘hydrogen-rich’ models at the blue boundary roughly coincide with an increasing number of helium-rich subdwarfs discovered in recent years. These are currently thought to be extended horizontal-branch stars with enhanced surface helium abundances. Models of such stars are also overstable to non-radial pulsations within the Z-bump instability finger and observations may thus detect single- or multiple-period variability.

As a consequence of these calculations we have predicted that pulsations may be excited in any low-mass hot stars with moderate enhancements of surface helium and normal metallicity ($Z > 0.01$) which lie within the Z-bump instability finger. Such pulsations may be radial or non-radial. Careful photometric studies of HesDb candidates are required. Although we appear to predict a new class of variable stars, in reality these models represent an extension to the pulsations already exhibited by the pulsating helium stars V652 Her and LSS 3184. Since the driving mechanism for both radial and non-radial pulsators is the same, V652 Her should remain the class prototype.

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