A pulsational approach to the luminosity of horizontal branch stellar structures

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ABSTRACT

We discuss an alternative approach to constrain the absolute bolometric luminosity of zero-age horizontal branch (ZAHB) structures by using the observational pulsational properties of ab-type RR Lyrae stars, theoretical relations connecting these properties to the evolutionary ones, i.e. luminosity, mass and effective temperature, and also the location in the Hertzsprung–Russell diagram of the instability strip boundaries for fundamental pulsators. As the main goal of this work is to obtain an evaluation of the ZAHB bolometric luminosity as independent of stellar evolution theory as possible, we have minimized the use of evolutionary prescriptions, the only adopted evolutionary input being the allowed mass range for fundamental pulsators. Nevertheless, the effects on our final results related to these evolutionary prescriptions have been carefully checked.

In order to test the accuracy of the current framework, we have carefully investigated the effective temperature scale provided by De Santis and adopted in the present work. It has been found that the pulsational colour–effective temperature scale fixed by this temperature scale and the relation between intrinsic $(B-V)_0$ colour, blue amplitude and metallicity given by Caputo & De Santis appear fully consistent with both theoretical colour transformations, given by Castelli, Gratton & Kurucz and Buser & Kurucz, and a semi-empirical one, provided by Green. In contrast, there is a large discrepancy between this pulsational colour–temperature scale and the one based on the Kurucz model atmospheres. The effects of adopting the temperature scale of Catelan, Sweigart & Borissova have been investigated.

The reliability of the method is shown by applying it to a selected sample of globular clusters (GCs), the heavy element abundances of which cover almost all the complete GC metallicity range. In order to verify the accuracy of the results obtained by using ab-type RR Lyrae stars, a similar analysis has also been performed by using both observational and theoretical evidence for first-overtone variables. The results obtained for the ZAHB bolometric luminosities have been critically analysed and a comparison with recent evolutionary prescriptions has also been performed. The existence of evident mismatches between current results and some evolutionary models has been verified and discussed.

Finally, our investigation has been extended to field variables to check for a luminosity difference between cluster RR Lyrae stars and field ones as suggested by Gratton. The comparison between the pulsational properties of field and cluster variables does not show the existence of any significant difference in their intrinsic luminosity, thus providing further support to the results obtained by Catelan.

Key words: stars: distances – stars: evolution – stars: horizontal branch – stars: variables: other – globular clusters: general.

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1 INTRODUCTION

The distance scale of low-mass, metal-poor stars plays an important role in stellar astrophysics, because the distance evaluations of old stellar systems – such as the globular clusters – are a fundamental step for providing accurate age determinations (see e.g. the detailed discussion by Renzini 1991) and in turn for providing a firm lower limit to the age of the Universe. The traditional distance ladder for metal-poor populations is the luminosity of horizontal branch (HB) stars, and in particular RR Lyrae variables, which are currently considered the natural Population II standard candles.

However, even though large observational and theoretical efforts have been devoted to estimating their ‘true’ luminosity, this quantity is still affected by large uncertainties. In particular, as recently suggested by Catelan (1998, and references therein) it seems that the luminosity of HB stars presents a ‘dichotomy’ between a faint and a bright distance scale. This problem has been widely discussed in the recent literature because of the release of the Hipparcos data base. However, no firm conclusion has been obtained till now. In fact, accurate analysis of the globular cluster (GC) distances, based on main-sequence fitting through Hipparcos parallaxes, seems to support both the bright RR Lyrae distance scale (Gratton et al. 1997; Reid 1997, 1998; Chaboyer et al. 1998) and the faint one (Pont et al. 1998). In addition, the long RR Lyrae distance scale seems to be supported by recent analysis on different groups of variable stars: SX Phoenicis (Mcnamara 1997), Cepheids (Feast & Catchpole 1997; Madore & Freedmann 1997) and Mira (van Leeuwen et al. 1997). Similar conclusions were also reached from investigations based on the red giant branch tip (Salaris & Cassisi 1998) and the SN1987a ring (Panagia 1998; but see also Gould & Uza 1998). This notwithstanding, the faint RR Lyrae distance scale seems to be supported by the investigations performed by Gratton (1998), Fernley et al. (1998) and Tsujimoto, Miyamoto & Yoshii (1998).

According to Gratton (1998), the discrepancy between the different conclusions obtained by adopting the Hipparcos results, could be explained if there were an intrinsic difference of the order of 0.2 mag in the luminosity of field and cluster RR Lyrae variables. However, this suggestion is not supported by the investigation recently performed by Catelan (1998).

The pulsational properties of RR Lyrae variables provide a unique opportunity for testing the prescriptions of both stellar evolution and pulsation theories. In fact, fundamental constraints on the evolutionary properties of HB stars can be derived by adopting the pulsation relation, i.e. the relation that supplies the pulsational period as a function of stellar mass, luminosity and effective temperature. The pulsation relation for RR Lyrae variables originally derived by van Albedo & Baker (1971) on the basis of linear, radiative, non-adiabatic models has been recently revised by Bono et al. (1997) and by Caputo, Marconi & Santolamazza (1998) by adopting full-amplitude, non-linear, convective models. Other interesting properties of RR Lyrae pulsation behaviour have been brought out by Sandage, Katem & Sandage (1981), who suggested the existence of tight correlations between temperature and pulsational amplitudes, and by Caputo & De Santis (1992, hereinafter CDS92), who supported the evidence of a clear correlation between period, blue amplitude and light–mass ratio.

Both periods and amplitudes can be measured with high accuracy, because they are affected neither by distance uncertainties nor by interstellar reddening evaluations. As a consequence, the use of these observables and the comparison between theory and observations can supply independent and useful constraints on the intrinsic luminosity of RR Lyrae variables. A similar approach was adopted by Castellani & De Santis (1994), who showed that the absolute visual magnitudes predicted by theoretical models are, within an accuracy of 0.1 mag, in satisfactory agreement with observed values.

Quite recently, De Santis (1996, hereinafter DS96) provided an accurate revision of the RR Lyrae temperature scale and suggested a slight change in the zero-point of the magnitude scale for these variables. However, Caputo et al. (1999), by investigating the pulsational and evolutionary properties of RR Lyrae variables in M5, found a significant discrepancy between recent theoretical prescriptions and observations. According to these authors, this discrepancy could be explained, without changes in the adopted evolutionary scenario, by slightly shifting the position in the Hertzsprung–Russell diagram of the instability strip toward higher effective temperatures.

The evaluation of HB luminosity based on stellar models has recently been reviewed by Cassisi et al. (1998, 1999), by discussing in detail the dependence of HB luminosity on the input physics adopted in stellar computations. However, the current observational scenario does not supply sound constraints on HB magnitude and in turn on theoretical predictions based on different physical assumptions (De Boer, Tucholke & Schmidt 1997; Cassisi et al. 1998, 1999). As a consequence, new and independent approaches are necessary for testing the accuracy of the theoretical scenario for low-mass helium burning stars and for assessing their intrinsic properties.

The main goal of this investigation is to analyse the intrinsic luminosity of HB stars by adopting the pulsational characteristics of fundamental RR Lyrae variables (hereinafter RR$_ab$) in a sample of Galactic GCs. As we estimate the ZAHB bolometric magnitude directly, this approach overcomes the uncertainties that affect the bolometric corrections based on static atmosphere models. The theoretical framework and the method adopted for deriving the ZAHB bolometric magnitude are outlined in Section 2.

In Section 3, we briefly review our approach for estimating the effective temperatures of RR$_ab$ stars on the basis of their pulsational properties. In this section we also compare our results with the most recent theoretical colour–temperature relations and discuss the evaluation of the interstellar reddening for the selected clusters. The results of the application of our method are presented in Section 4. Finally, these results are compared with theoretical predictions and a critical analysis of the aftermath of this investigation is presented.

2 THE THEORETICAL FRAMEWORK AND THE METHOD

As we are interested in obtaining an independent measurement of the intrinsic luminosity of the ZAHB for providing firm constraints on the reliability of the current evolutionary scenario, the use of evolutionary models has been reduced as much as possible.

The pulsation relation given by van Albedo & Baker (1971) has generally been adopted in several analyses. However, recent theoretical investigations strongly support the evidence that sound estimates of both the pulsation period and the modal stability of RR Lyrae stars can only be obtained in a non-linear regime (Bono & Stellingswerf 1994). Therefore, we have adopted the pulsation...
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relation that has been derived by Bono et al. (1997) by using an extensive grid of RR Lyrae full-amplitude, non-linear and time-dependent convective models:

$$\log P = 11.627 + 0.823 \log L - 0.582 \log M - 3.506 \log T_e,$$  \hspace{1cm} (1)

where $P$ is the fundamental period (days), $T_e$ is the effective temperature and $L$ and $M$ are the luminosity and the stellar mass in solar units.

In the present work, the ZAHB luminosity is defined as the one at the effective temperature $\log T_e = 3.85$ ($L_{ZAHB}^{3.85}$). We write the luminosity of each variable as:

$$\log L = \log L_{ZAHB}^{3.85} + \Delta \log L_{ZAHB},$$

where $\Delta \log L_{ZAHB}$ is the difference in luminosity between an individual variable and the ZAHB at $\log T_e = 3.85$. With simple algebraic substitutions, equation (1) can be rewritten (see also Sandage 1981) as follows:

$$\log P - 0.823 \Delta \log L_{ZAHB} = \log P + 0.33 \Delta M_{ZAHB}^{Bol}$$

$$= 11.627 + 0.823 \log L_{ZAHB}^{3.85} - 0.582 \log M - 3.506 \log T_e,$$  \hspace{1cm} (2)

where the symbols have their usual meaning, and the quantity $\log P + 0.33 \Delta M_{ZAHB}^{Bol}$ corresponds to the fundamental reduced period ($\log P_{red}$) (Sandage 1981).

The method we have developed for estimating the bolometric ZAHB magnitude, outlined in more detail in Section 4, is based on the use of equation (2) in order to evaluate the relation between the fundamental reduced period and the effective temperature, as a function of the mass of the variable stars (see below) and $L_{ZAHB}^{3.85}$. However, before comparing theory and observations, we need to transform equation (2) into the observational plane, so we adopt

$$\Delta M_{ZAHB}^{Bol} = \Delta V_{ZAHB}^{Bol} + \Delta BC,$$

where we have defined the difference in bolometric correction between the individual variable and the ZAHB at $\log T_e = 3.85$ as $\Delta BC = BC_{log T_e} - BC_{3.85}$. Obviously the bolometric correction has to be evaluated by adopting the surface gravity and effective temperature values of the variable star and of the fictitious star located on the ZAHB at $\log T_e = 3.85$. However, in the colour–magnitude diagram (CMD) region populated by RR Lyrae stars, the allowed gravity range is quite narrow, as can be easily tested by using any set of evolutionary models available in the literature and a theoretical evaluations of the instability strip boundaries. In addition, a test performed by adopting several sets of bolometric corrections (see below) has clearly shown that the bolometric correction is sensitive to surface gravity and metallicity but the parameter $\Delta BC = BC_{log T_e} - BC_{3.85}$ presents a negligible dependence on both quantities in the whole range of effective temperatures and luminosities typical of RRab stars.

Finally we have adopted the following relation for $\Delta BC$ as a function of the effective temperature, derived by adopting the
\[ \Delta BC = -5.252 \log^2 T_e + 41.636 \log T_e - 82.454. \] (3)

The standard deviation of this relation is \( \sigma = \pm 0.004 \), while the correlation coefficient \( r \approx 1.0 \) within the ranges \( 6000 \leq T_e (K) \leq 7500, 2.5 \leq \log g \leq 3.0 \) and metallicity \( 0.002 \leq Z \leq 0.002 \). It is important to notice that the use of a different \( BC \) scale does not substantially change the dependence of \( \Delta BC \) on temperature as given by equation (3).

In order to estimate the cluster ZAHB visual magnitude, we have adopted the same operative approach described by Sandage (1990) by taking into account the lower envelope of the HB star distribution. In the previous relations we adopted the visual magnitude of the ZAHB at \( T_e = 3.85 \), while now we adopt the ‘generic’ lower envelope of the HB star distributions. Even though these definitions seem to be substantially different, a test performed by adopting various grids of evolutionary models (Dorman, Rood & O’Connell 1993; Caloi, D’Antona & Mazzitelli 1997; Cassisi et al. 1998), and sets of colour–temperature relations and bolometric correction scales, discloses that within the instability strip the dependence of \( M_V (\text{ZAHB}) \) on the effective temperature is negligible. In fact, we found that \( M_V (\text{ZAHB}) \) changes by only 0.006 mag when moving from \( T_e \approx 6700 \) K to \( \approx 7100 \) K.

As a relevant point, we note that our method is not affected by uncertainties in stellar photometry such as the zero-point and the calibration procedure, because we use only the difference in the \( V \) magnitude between each ab RR Lyrae star and the ZAHB at approximately \( \log T_e = 3.85 \).

In order to apply equation (2) for deriving the ZAHB bolometric magnitude, we have to estimate the evolutionary mass of the \( \text{RR}_{ab} \) variables. It is well known that the mass of an RR Lyrae variable cannot be directly evaluated unless the star is a double pulsator (Kovacs, Buchler & Marom 1991). For a long time, a large discrepancy existed between the pulsational and evolutionary determinations of the mass of double mode RR Lyrae stars, but recently it has been largely reduced by the use of the most accurate OPAL opacities (Rogers & Iglesias 1992).

In the present work, we adopt evolutionary determinations of the masses (Bono, Cassisi & Castellani, in preparation). It is worth noticing that this is the only evolutionary prescription adopted in the current analysis. As it is evident that we cannot assign a mass to the whole sample of variables in a GC, we use only an evaluation on the lower and upper mass (\( M_{\text{min}}^{\text{RR}} \) and \( M_{\text{max}}^{\text{RR}} \) respectively) producing ab RR Lyrae stars. The dependence on the metallicity of these parameters has been analysed in quite a large range: \( 0.0001 \leq Z \leq 0.003 \).

For each fixed metallicity, we have adopted as \( M_{\text{min}}^{\text{RR}} \) and \( M_{\text{max}}^{\text{RR}} \) the smaller and larger masses that spend at least 5 per cent of their central He burning lifetime in the instability strip as fundamental pulsators. The boundaries of the instability strip were fixed according to the prescriptions by Bono et al. (1997), i.e. the same set of RR Lyrae models adopted for the pulsational relation (equation 1).

Fig. 1 shows, for various metallicities, the fraction of the total HB evolutionary lifetime spent by HB stars at various effective temperatures. This figure shows that the decision to take into account only models that spend at least 5 per cent of their HB lifetime in the fundamental instability strip does not affect our final results. We assume (Bono, Caputo & Stellingwerf 1994) that inside the ‘OR’ region of the instability strip – i.e. the region

where both fundamental and first-overtone pulsators attain a stable limit cycle – the pulsation mode is governed by the previous evolutionary history of the variable (hysteresis mechanism).

It is worth noting that the dependence of periods on pulsator masses (see equation 1) is very weak, and indeed an uncertainty of \( \approx 0.01 M_\odot \) implies an uncertainty in \( \log L_{\text{ZAHB}} \) of the order of 0.003, when the value of the other quantities is fixed.

### 3 The Effective Temperature Scale

We adopt the analytical relation between effective temperature, period and blue amplitude (\( A_B \)) provided by DS96, which corresponds to an updated version of the relation given by Castellani & De Santis (1994):

\[ \log T_e = -0.1094 \log P + 0.0134 A_B + 3.770, \] (4)

with a probable error equal to \( \pm 0.003 \). This relation has been derived by relying on the RR Lyrae data base of Lub (1977) and by correcting its zero-point in order to achieve agreement with the Baade–Wesselink data for field variables (Carney, Storm & Jones 1992, hereinafter CSJ92; see DS96 for more details on this topic).

The accuracy of this or similar relations for deriving the effective temperature of RR Lyrae stars has recently been questioned by Walker (1998) because of the large number of parameters that are involved and the small number of calibrating stars. However, this is not the case for equation (4), because it has been obtained by using a quite large sample (about 70 objects) of RR Lyrae stars.

As both the reliability and accuracy of this effective temperature scale have already been extensively discussed by Castellani & De Santis (1994) and DS96, this topic will not be reviewed again here. However, it is worth remembering that CDS92, through an accurate analysis of Lub’s (1977) data for field RR Lyrae stars with well-determined reddening, have shown the existence of a tight correlation between the intrinsic colour, blue amplitude and metal abundance of the variables:

\[ (B - V)_0 = -0.0775 A_B + 0.005 [\text{Fe/H}] + 0.434, \] (5)

with a standard deviation equal to \( \sigma = \pm 0.015 \) mag.

It is clear that the coupling of equations (4) and (5) provide a colour–temperature scale, so it is interesting to check the consistency between these two equations by comparing this scale with the colour–temperature relation provided by theoretical model atmospheres.

Being aware of the problems still existing with theoretical model atmospheres (Castelli, private communication), we have tried to perform this comparison by using several independent sets of transformations: Buser & Kurucz (1978, hereinafter BK78), Kurucz (1992, hereinafter K92), Castelli, Gratton & Kurucz (1997a,b, hereinafter CGK97) and the Yale semi-empirical transformations (Green 1988), which are an empirical \( UBVRI \) recalibration of Vandenberg & Bell (1985) and Kurucz (1979) synthetic colours and \( BCV \) based on various observational constraints.

The check has been performed by using the observational data for ab RR Lyrae stars in three globular clusters, namely M3, M15 (Sandage 1990) and M68 (Walker 1994). The procedure is quite simple: (i) we derive the effective temperatures of the variable stars by using their period and \( B \) amplitude (equation 4); (ii) their intrinsic \( (B-V) \) colours are then estimated by adopting an average value for the gravity (\( \log g = 2.75 \)) and by using a colour–
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temperature relation (see below) for the appropriate metal abundance ([Fe/H] = −1.3 for M3 and [Fe/H] = −2.0 for M15 and M68). These colour estimations are then compared with the ones provided by equation (5) in the (B − V)0−Aβ plane, as shown in Figs 2–4. The results of these comparisons support the following conclusions.

(i) The colour (B − V)−effective temperature relation provided by equations (4) and (5) appears fully consistent with both the theoretical scales based on model atmospheres by BK78 and CGK97 and the semi-empirical one by Green (1988). In fact, the average difference is equal to 0.005 mag in the worst case (panel a in Fig. 3). This occurrence suggests that Lub’s data for field RR Lyrae stars and, indeed, his interstellar reddening scale are quite consistent with the previous quoted colour–T eff scales.

(ii) A large discrepancy exists between the colour–T eff scale provided by K92 and our ‘pulsational’ scale. In this case the average difference is of the order of ≈ 0.04 mag. This result is clear evidence that the reddening scale of Lub (1977), on which equations (4) and (5) rely, is not consistent with the colour–temperature scale of K92. It is evident that such an occurrence could be explained as a drawback of the K92 model atmospheres or, alternatively, as resulting from a problem in Lub’s reddening scale.

On this subject, it is worth pointing out that several investigations (Bono et al. 1997; Silbermann & Smith 1995) of RR Lyrae stars have recently provided clear indications about the existence of significant problems in comparing theory and observations when using the colour–temperature relation given by K92. In addition, an analysis of the possible drawbacks in model atmosphere computations and, in particular, in the K92 calculations, has been provided by CGK97 and Castelli (private communications).

Taking into account such evidence, we suggest that the present result is a further indication against the use of the K92 colour–temperature scale, at least when working with RR Lyrae variables.

In passing we note that the satisfactory agreement achieved between the results provided by equations (4) and (5) and the Yale, BK78 and CGK97 colour–temperature scales can be considered as plain evidence of the reliability of the pulsational effective temperature scale adopted in the present work.

The relation between effective temperature, period and blue amplitude (equation 4) has been obtained by adopting for each variable in Lub’s sample the mean (B − V) colour, i.e. the time average along a full pulsational cycle of the colour curve (see DS96 for more details). At the same time, the effective temperature adopted by Bono et al. (1997) for deriving the pulsational equation (1) is the ‘static’ equivalent temperature. As a consequence, it could be claimed that we are dealing with two ‘different’ and inconsistent effective temperature definitions. However, DS96 has already shown the fine agreement between the effective temperature evaluations provided by equation (4) and
the static temperatures from CSJ92. Concerning this point, it is
important to notice that the comparison (Bono, Caputo &
Stellingwerf 1995) between static colours and mean (along the
pulsational cycle) ones cannot be adopted at all to check the
consistency between static and mean effective temperatures. This
is because of the evidence that the obtained results are strongly
dependent on the adopted colour–temperature relation. This effect
is shown in Fig. 5, where we have plotted the static colours
obtained by using the K92 (open circles) and CGK97 (full circles) colour–
temperature scales, corresponding to the pulsational models of Bono et al.
(1997) for different assumptions regarding the luminosity level but for the
same pulsator mass 0.65 M⊙ and metallicity Z = 0.001. The CDS92
relation is also displayed.

Even though our approach for estimating the intrinsic ZAHB
luminosity does not depend on the reddening, we now wish to
adopt the relation between intrinsic colour and pulsational
properties in order to evaluate the interstellar reddening for the
same sample of clusters adopted for further investigation on the
HB luminosity level (see below).

To derive the reddening we adopt a fairly simple procedure: the
observational data for the sample of ab RR Lyrae stars – in each
cluster – is compared in the (B–V)–A′ diagram with the reddening-
free colours provided by equation (5) by using the observational
blue amplitudes and the cluster metallicity [Fe/H] as given by
Carretta & Gratton (1997, hereinafter CG97). The amount of
colour shift that is needed in order to achieve a satisfactory
agreement between observations and the CDS92 relation provides
a fine evaluation of the cluster reddening. In Figs 6–8, we show
the comparison between the observed colour of the RR Lyrae stars
and the dereddened colour as provided by the CDS92 relation for
the clusters M3, M15 and M68, respectively.

The reddening evaluations and their maximum errors are listed
for all clusters in our data base in Table 1 together with other
relevant cluster parameters. We note that our evaluation of the
M68 reddening is in good agreement with the estimate recently
provided by Gratton et al. (1997) on the basis of their accurate
Strömgren photometric analysis of field stars projected on the sky near a selected sample of GCs.

4 THE ZAHB LUMINOSITY LEVEL

4.1 Fundamental pulsators

In order to apply the method for estimating the ZAHB luminosity we selected a sample of Galactic GCs for which homogeneous photometric observations for both non-variable HB stars and RR Lyrae variables are available, and which cover a wide range of metal contents. On the basis of these requirements we selected the following clusters: M3, M15, M68, NGC 6171, NGC 1851, NGC 6362 and NGC 6981. The data for M3, M15, NGC 6171 and NGC 6981 have been collected from the work of Sandage (1990, and reference therein); for M68, NGC 1851 and NGC 6362 we have used the data from Walker (1994), Walker (1998) and Brocato et al. (1999), respectively. Unfortunately, for some of these clusters only photographic photometry of static and variable HB stars is available in the literature. However, because our analysis relies on the difference in magnitude between the RR Lyrae stars and the ZAHB, the lack of accurate CCD data is not a strong limit to the application of our method.

For each cluster in our sample, in order to apply the method outlined in Section 2, we have to estimate first the ZAHB visual magnitude, and also the expected range of fundamental pulsator mass. In Fig. 9, we have plotted for each cluster in our sample the observed distribution of HB stars and marked the visual magnitude of the ZAHB (see also Table 1) estimated by adopting the lower envelope of the observed distribution. In the following paragraphs, we discuss for each individual cluster the values adopted for these important parameters.

NGC 1851: CG97 do not provide any metallicity evaluation for this cluster, so we have adopted the estimate provided by Zinn & West (1984). However, in order to be consistent with the CG97 metallicity scale adopted for all other clusters, the relation relating the Zinn & West (1984) scale to the CG97 one (equation 7 in CG97) has been used, so finally we adopted [Fe/H] = −1.08, which corresponds to Z = 0.002. By using the theoretical prescriptions shown in Fig. 1, we obtain a mass interval for ab RR Lyrae stars in this cluster equal to 0.61 ≤ MRRab/M⊙ ≤ 0.67.

We estimate for NGC 1851 a value for the visual magnitude of the ZAHB (VZAHB) equal to 16.13 ± 0.025 mag.

M68: the metallicity measurement provided by CG97 is equal to [Fe/H] = −1.99, i.e. Z = 0.0002. From data in Fig. 1, this mass range for ab RR Lyrae stars has been evaluated: 0.70 ≤ MRRab/M⊙ ≤ 0.80. By analysing the photometric data from Walker (1994), we estimate a value for the ZAHB visual magnitude equal to 15.70 ± 0.02 mag.

M3: the heavy element abundance of this cluster according to the CG97 metallicity scale is equal to [Fe/H] = −1.34, which corresponds to about Z = 0.001. Therefore, we estimate that the most suitable fundamental pulsator mass range is 0.64 ≤ MRRab/M⊙ ≤ 0.70. The adopted value for VZAHB is 15.73 ± 0.02 mag.

NGC 6171: by adopting [Fe/H] = −0.87 from the stellar models for Z = 0.003, one obtains a RRab mass range equal to 0.60 ≤ MRRab/M⊙ ≤ 0.635. From the data plotted in Fig. 9, the most suitable estimate for VZAHB is about 15.85 ± 0.10 mag.

NGC 6362: CG97 report a metallicity value equal to −0.96 dex, (Z = 0.002). Therefore we adopt for this cluster a RR Lyrae mass range equal to 0.61 ≤ MRRab/M⊙ ≤ 0.66. For the visual magnitude of the ZAHB, we derive a value of 15.33 ± 0.03 mag.

NGC 6981: because the metallicity is equal to ≈ −1.30, we adopt the same mass range adopted for M3 and, by analysing the CM diagram, we obtain a value for VZAHB of about 17.07 ± 0.03 mag.

M15: the metallicity of this cluster is quite similar to the M68 one, so the same mass range for the fundamental pulsators has been adopted. From the data in Fig. 9, we estimate a value for VZAHB to equal 15.92 ± 0.05 mag.

In the following, we compare, in a log P$_{red}$−log T$_{e}$ diagram, the observational data for the ab RR Lyrae variables in each individual cluster with the prescription provided by equation (2). As these theoretical expectations depend on the bolometric magnitude of the ZAHB at log T$_{e}$ = 3.85 (according to our definition), this comparison supplies a straightforward evaluation of the ZAHB luminosity. In more detail, the adopted procedure is as follows.

(i) As equation (2) depends on the mass of the pulsators, by adopting both the lower and upper limit of the RRab mass range we obtain two different solutions for the behaviour of the reduced period as a function of the effective temperature, both depending on the ZAHB luminosity level.

(ii) The ZAHB luminosity is then estimated by properly fitting the lower and upper boundaries of the RR Lyrae distribution in the reduced period−temperature plane. This approach is shown in Fig. 10.

The results listed in Table 1 support the following conclusions.

(i) The bolometric ZAHB luminosity decreases significantly: Δ log L$^*$ZAHB ≈ 0.15, when increasing the metallicity from the most metal-poor cluster (M15) to the most metal-rich one (NGC 6171) in our sample.

(ii) The ZAHB luminosity evaluations for the two metal-poor clusters M15 and M68 appear to be in good agreement. The same outcome applies for the intermediate-metallicity clusters such as NGC 6981 and M3.
The suggested method allows us to estimate the bolometric ZAHB luminosity at a fixed effective temperature with high accuracy, and indeed the uncertainty on \( \log L_{\text{ZAHB}} \) is of the order of 0.04 in the worst case.

4.2 First-overtone pulsators

In order to provide an independent test on the accuracy of the method adopted in the previous section, we undertake a similar investigation but by adopting first-overtone variables (\( RR_c \)).

Bono et al. (1997) have provided a relation similar to equation (1), but suitable for first-overtone pulsators:

\[
\log P = 10.789 + 0.800 \log L - 0.594 \log M - 3.309 \log T_e. 
\]

where the symbols have their usual meaning and the units are the same as in equation (1). By adopting the same definitions used in Section 2, one obtains

\[
\log P + 0.32 \Delta M_{\text{bol}}^{\text{ZAHB}} = 10.789 + 0.800 \log L_{\text{ZAHB}} - 0.594 \log M - 3.309 \log T_e. 
\]

As relation (4) is valid only for \( RR_ab \) variables, we have adopted a different approach for estimating the effective temperature of \( RR_c \) Lyrae stars.

(i) By using the reddening evaluations obtained in Section 3 (see data in Table 1), we have derived the true \((B-V)\) colour of the first-overtone pulsators in the selected clusters.

(ii) The effective temperature of the variable has then been estimated by using the \((B-V)_0\) estimation and a theoretical colour--temperature relation (such as those given, for instance, by CGK97, BK78 or Yale).

This method has been applied to the variables in the clusters M15 and M68. In the case of M15 we have taken into account 26
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$10^5$ RR Lyrae stars (Sandage 1990), while for M68 we have excluded from the analysis all variables affected by the Blazko effect (Walker 1994), so we have recovered only 15 variables.

The mass range for first-overtone pulsators has been obtained from data in Fig. 1. As these clusters have quite similar metallicity, a mass interval equal to $0.67 < M_{RRc}/M_\odot < 0.75$ has been adopted for both clusters. The results of this investigation are shown in Fig. 11.

We have obtained ZAHB luminosity levels equal to $\log L_{ZAHB}^{3.85} = 1.67$ and 1.69 for M15 and M68 respectively. These estimations appear to be in good agreement, within the associated uncertainties, with the previous evaluations based on the $RR_{ab}$ pulsational properties.

5 DISCUSSION AND CONCLUSIONS

In the previous sections, it has been shown that the method we suggest for deriving an estimate of the ZAHB bolometric magnitude from the pulsational properties of RR Lyrae stars in GCs allows us to evaluate this quantity with a high accuracy: the average uncertainty in $\log L_{ZAHB}^{3.85}$ is equal to $\sim 0.02$. Therefore, it can be safely considered one of the most reliable approaches to constrain the HB luminosity level in GCs.

Nevertheless, it is important to provide a deeper insight into this method by showing the main possible error sources in order better to quantify its accuracy. The most important uncertainties can be derived from the evaluation of the visual magnitude of the ZAHB in GCs, the estimate of a suitable mass range for fundamental pulsators, the adopted relation between pulsational period and evolutionary properties, the cluster metallicity and the RR Lyrae temperature scale. On this subject, it is quite easy to verify the following indications.

(i) An uncertainty in $V_{ZAHB}$ of about 0.025 mag produces an error in $\log L_{ZAHB}^{3.85}$ of about 0.01.

(ii) As already discussed in Section 2, a shift of the whole mass range suitable for fundamental pulsators of $\Delta M = 0.01 M_\odot$ gives

Figure 10. Comparison, in a fundamental reduced period–temperature diagram, between the observational data for RR Lyrae stars in selected GCs and the prescriptions of the pulsational theory as given by equation (2) (see text for more details). In each panel, the two lines correspond to the evaluations provided by equation (2), when adopting either the upper (solid line) or lower (dashed line) limit for the allowed mass range for fundamental pulsators.
heavy-element abundances. We have investigated whether the use of this revised relation could significantly affect our results: it has been found that for the metal-poor clusters there are no significant effects on the ZAHB luminosity, and for metal-rich GCs the change in the log $L_{ZAHB}^{3.85}$ estimate is equal to $\approx -0.01$ in the worst case.

(iv) Concerning the cluster metallicity, it is worth remembering that the heavy-element abundance plays a role only in the choice of the $RR_{ab}$ mass range. However, bearing in mind the previous discussion and accounting for both the theoretical prescriptions shown in Fig. 1 and the mean uncertainty in the CG97 metallicity scale ($\approx 0.10-0.15$ dex), one can conclude that the uncertainty in the cluster metallicity does not affect significantly the evaluation of $L_{ZAHB}^{3.85}$. However, we note that in all previous discussions we have used the [Fe/H] spectroscopical measurements as representative of the true cluster metallicity but, as is well known (see e.g. the review by Wheeler, Sneden & Truran 1989), there is clear observational evidence that the $\alpha$-elements are enhanced in GCs stars. It is therefore interesting to evaluate the effect on present analysis, related to an $\alpha$-element enhancement. Owing to the lack of $\alpha$-element enhancement measurements for the selected clusters, we have adopted for all clusters (irrespective of their [Fe/H] value) a mean enhancement $[\alpha/Fe] = 0.30$ (Gratton et al. 1997), obtaining an average decrease of log $L_{ZAHB}^{3.85}$ of the order of 0.004–0.005.

(v) The accuracy of the adopted $T_{\text{eff}}$ scale has been discussed in detail by DS96 and briefly reviewed in Section 3. However, we now address an important question concerning the possibility that our $T_{\text{eff}}$ scale can be dependent on the stellar luminosity. It is evident that this is a quite important question, because if the adopted temperature scale were to be dependent on the luminosity of RR Lyrae stars, this could produce an inaccurate estimate of the ZAHB luminosity.

On this last point, Catelan (1998); Catelan, Sweigart & Borissova (1998, hereinafter CSB98) have recently claimed that a relationship involving only the equilibrium temperature, blue amplitude and metallicity would be safer to adopt in period shift analysis than CSJ92's (depending also on the pulsational period like our equation 4) because period shifts caused by luminosity variations could easily be misinterpreted as being caused by temperature variations'. Therefore, we have verified whether the adopted temperature scale could be dependent on the luminosity. To do this, we have taken into account the sample of 17 field ab-type RR Lyrae stars studied using the Baade–Weesselink method by CSJ92, and investigated whether the difference between the temperature provided by equation (4) and that given by CSJ92 (their table 4) depends on the measured luminosity of these stars. The results, shown in Fig. 12, show the absence of any correlation between the effective temperature residuals and the luminosity. This allows us to be confident in the use of DS96’s temperature scale in the present analysis. However, in order to provide a more deep investigation of the accuracy of our method, we have taken into account the warning from Catelan (1998) and CSB98 and repeated our analysis by adopting the period-independent CSB98 scale.

The results of this investigation for the cluster M3 (the same results have been achieved for the other clusters) have been plotted in Fig. 13. From this figure, one can obtain the following relevant indications.

(i) The estimated ZAHB luminosity level is equal to the one obtained by adopting the $T_{\text{eff}}$ given by equation (4).
(ii) The observational points, corresponding to the stars at lower effective temperatures inside the instability strip, as given by the CSB98 temperature scale, are not in satisfactory agreement with the pulsational theory prescriptions. In fact, one can see that the slope of the observational data is different from the theoretical one. In our belief, this occurrence has to be related to the fact that the calibrating stars used by CSB98 for deriving their relation do not cover the full expected range of RR Lyrae effective temperatures.

5.1 Comparison of field and cluster RR Lyrae stars

In an accurate analysis on field RR Lyrae stars, the parallaxes of which have been recently provided by the Hipparcos mission, Catelan (1998) has shown that no evidence exists for a mean luminosity difference between field RR Lyrae stars and GC variables. As the main goal of the present investigation is to evaluate the ZAHB luminosity level from the pulsational properties of RR Lyrae stars, it is natural to extend our investigation to field variables in order eventually to provide further support to Catelan’s (1998) result. By courtesy of Dr M. Catelan, we have been provided with his original list of field RR Lyrae stars and related pulsational properties, which corresponds to a subsample of the field variables belonging to the original list adopted by Tsujimoto et al. (1998) in their Hipparcos-based investigation.

The procedure adopted to perform the comparison between GC RR Lyrae stars and field variables is similar to the one used by Catelan (1998): we have split the data for field variables into two different subsamples corresponding to two different metallicity ranges, with an average metallicity equal to $-2.0$ and $-1.3$, respectively. Then we have compared the pulsational properties of field variables in each range of metallicity with the properties of RR Lyrae stars in GCs of suitable metallicity. In Fig. 14(a), the comparison between variables in the cluster M3 and field RR Lyrae stars filling the ‘metal-rich’ range is shown, while Fig. 14(b) shows the same comparison, but between RR Lyrae stars in M15 and M68 and field variables in a different metallicity range (as labelled).

5.2 Comparison with theoretical results

In this section, we compare the ZAHB bolometric magnitude for the different clusters with the theoretical prescriptions based on the most recent evolutionary models. This comparison has been performed in Fig. 15. In particular, we have considered the ZAHB
stellar models from Castellani et al. (1991), Dorman et al. (1993), Caloi et al. (1997), Cassisi & Salaris (1997), Cassisi et al. (1998, 1999), Salaris & Weiss (1998) and Vandenberg (private communication). All evolutionary computations have been developed in a canonical framework, except Cassisi et al.’s (1998, 1999) models, which have been computed for both a canonical scenario and also a scenario accounting for element (helium + heavy elements) diffusion. We refer to the quoted papers for details of the evolutionary computations. We note that the ZAHB models of Cassisi & Salaris (1997) are fully consistent with the evolutionary computations used in the previous sections.

We have already discussed the effect on the evaluation of the ZAHB luminosity caused by a possible $\alpha$-element enhancement in the heavy-element distribution and shown that it is very small. Nevertheless, when comparing observational evidence with theoretical results, one has to make sure to adopt self-consistent metallicity evaluations. For this reason, and also being aware of the lack of accurate $\alpha$-element enhancement measurements for the selected clusters, we have used two different assumptions (following the suggestion given by Gratton et al. 1997) by adopting $[\alpha/Fe] = 0.0$ (Fig. 15a) and 0.30 (Fig. 15b).

From this analysis, in the limits of the small sample of clusters, it is possible to derive the following points.

(i) The slope of $\log L_{ZAHB}^{L_\odot}$ with metallicity is in satisfactory agreement, at least for $[M/H] \leq -1.0$, with the values provided by almost all theoretical investigations, except the models provided by Caloi et al. (1997).

(ii) In the more metal-rich range, a significant discrepancy seems to exist between theory and the ‘pulsational’ estimations if the observational point corresponding to the cluster NGC 6171 is accounted for. The discrepancy appears less evident when we consider an $\alpha$-element enhancement for the GCs equal to 0.30. Nevertheless, it is worth noticing that we have only two clusters, NGC 6171 and NGC 6362, at higher metallicity, and that the evaluation of $\log L_{ZAHB}^{L_\odot}$ for NGC 6171 is affected by a large uncertainty. However, if further investigations should support these preliminary results, one is faced with the evidence that theoretical models underestimate the decrease of the ZAHB luminosity at a fixed effective temperature when the metallicity increases. Nevertheless, when excluding from the analysis the result for the cluster NGC 6171, one can safely assess that theory and observations are in fair agreement at least up to a metallicity of the order of $\approx -0.9$, $-0.8$ dex.

(iii) A satisfactory agreement has been achieved for both assumptions regarding the $\alpha$-element enhancement between the values of $\log L_{ZAHB}^{L_\odot}$ for the selected clusters and the evolutionary prescriptions as given by Cassisi & Salaris (1997) and Vandenberg.
(private communication) concerning both the slope and the absolute values.

(iv) Cassisi et al. (1998, 1999) have recently investigated the effects on various evolutionary quantities related to the use in stellar computations of different prescriptions on physical inputs, and have computed a large set of models by adopting the most updated physical scenario. The results in Fig. 15(a) show that the ZAHB luminosity levels provided by these models seem to be ruled out by the present investigation. The agreement is slightly better for the models accounting for element diffusion, and when we assume a non-scaled solar heavy element distribution for the clusters. Such an occurrence could be regarded as an indication of the fact that one (or more than one) updated physical input (such as, for instance, nuclear cross-section, neutrino energy loss, conductive opacity, etc.) adopted in these computations is still affected by a large uncertainty.

Nevertheless, we think that, because of the non-negligible uncertainties in the observational measurements of [Fe/H] and the heavy-element distribution, there is not yet clear evidence that this is the case. Although a deeper insight on this topic is clearly out of the scope of the present work, we wish to note, as a warning, the possibility that the element diffusion coefficients adopted in the evolutionary codes could be affected by a large uncertainty (as recently discussed by Fiorentini, Lissia & Ricci 1999), and that these non-determinations produce significant changes in the ZAHB luminosity (Castellani & Degl’Innocenti 1999). Therefore, in our belief the possibility still exists, in the parameter (adopted in stellar computations) space, of obtaining a better agreement between the present results and these evolutionary computations.

As a final point, we notice that the present results allow us to obtain relevant information about both the slope and the zero-point of the relation between the absolute visual magnitude of the RR Lyrae stars and the metallicity. We will address this subject in a forthcoming paper (Cassisi & De Santis, in preparation).

It is evident that firmer conclusions about the true luminosity level of the ZAHB can be obtained only when the sample of GCs with high-accuracy photometry for both RR Lyrae stars and non-variable HB structures and a fine measurement of the RR Lyrae pulsational properties becomes larger. As it has been shown in the present work that the analysis of the pulsational properties of the RR Lyrae stars allows us to evaluate the ZAHB luminosity with high accuracy and with an approach largely independent of evolutionary computations, this occurrence is strongly desired in order finally to achieve a consensus about the most important Population II distance scale.

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