Einstein, Bell, and Nonseparable Realism
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ABSTRACT
In the context of stochastic hidden variable theories, Howard has argued that the role of separability—spatially separated systems possess distinct real states—has been underestimated. Howard claims that separability is equivalent to Jarrett's completeness: this equivalence should imply that the Bell theorem forces us to give up either separability or locality. Howard's claim, however, is shown to be ill founded since it is based on an implausible assumption. The necessity of sharply distinguishing separability and locality is emphasized: a quantitative formulation of separability, due to D'Espagnat, is reviewed and found unsatisfactory, in that it basically conflates separability and locality in a single notion. Finally, the possibility of an 'Einsteinian' nonseparable realism, envisaged by Shimony, is reviewed and found also to be implausible.

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1 Introduction
Among the widespread views concerning hidden variable theories is the idea that, given the Einstein–Podolsky–Rosen (EPR) argument for the incompleteness of quantum mechanics (EPR [1935]), the introduction of such theories would be a natural consequence of realism. Although the actual relationship between Einstein's realism-oriented view of physical theories and the hidden variable research program is a controversial matter (Jammer [1992]), the hidden variable theorists have found their main motivation in an argument for the incompleteness of orthodox QM, according to which the quantum mechanical wave function would only prove to be a provisional description of the physical reality (Bell [1971]).

As is widely acknowledged nowadays (Butterfield [1990]), there are three incompleteness arguments for QM. The first two are variants of the EPR argument, whereas the third is the incompleteness argument that Einstein
formulated *on his own* in some of his contributions after 1935. The first argument takes into account a pair of quantities for each particle, in such a way that the members of each pair are mutually incompatible. This is the argument in the original EPR version. The second argument, a simplified variant of the first, deals with just one quantity for each particle; this variant is formulated in the context of spin correlation and it exploits in an essential way the property of strict correlation (a straightforward exposition of this version can be found in Redhead [1987]). Finally, there is Einstein's own incompleteness argument which, although employing the same background of the EPR argument, issues a sharp distinction between a principle of separability and a principle of locality. According to the principle of separability, spatially separated systems possess *distinct real states* whereas according to the principle of locality the state of the system can be modified only by local influences or interactions.

The importance of Einstein's own argument has been emphasized quite recently, after the discovery of a letter from Einstein to Schrödinger (the date is 19 June 1935, a month later than the appearance of the EPR paper in *Physical Review*). In this letter Einstein manifests his unhappiness about how the EPR paper failed to express his real point of view, since its main point was, according to him, 'buried by the erudition'. Thus the EPR argument cannot be considered as a faithful representation of Einstein's own views. Arthur Fine was the first to stress the importance of this letter (Fine [1981]). However, it was Don Howard who carried out the most interesting analysis of the relations between the EPR argument and Einstein's own argument. In addition, Howard has also drawn interesting conclusions on the relations between Einstein's own argument and the debates on hidden variable theories and the Bell theorem (Howard [1985, 1989]).

In Howard's view the distinction between separability and locality, as is formulated in Einstein's own incompleteness argument, is valuable also from a theoretical point of view, in that it implies a broader interpretation of the Bell theorem. According to Howard, the role of separability in hidden variable theories has been underestimated. The lack of due consideration for separable hidden variable theories has, in Howard's view, considerably affected the way of looking at the derivation of the Bell inequalities. He argues that locality is not the only prerequisite for the derivation of Bell inequalities (Howard [1985]). The crucial point in Howard's argument is the proof that his separability condition is equivalent to Jarrett's 'completeness'. This equivalence, which is proved in Howard [1992], 'enables us now to argue ... that the physical circum-

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1 For a useful survey of the literature concerning the differences between the EPR argument and Einstein's own argument, see Deltete and Guy [1991].
stance that explains both experimental and quantum mechanical violations of the Bell inequality is nothing more nor less than the nonseparability at the heart of the quantum mechanical interaction formalism’ (Howard [1992], p. 307).

The aim of the present work is twofold. On the one hand, I show that Howard’s claim concerning the equivalence between separability and Jarrett’s completeness is ill founded, since it rests on an assumption which is implausible in several respects. The distinction between separability and locality, although adequate, by no means implies that the Bell theorem forces us to renounce either separability or locality. The original Bell formulation did not require the hidden variable theory to be separable. Thus what the Bell theorem undermines is not separability but rather locality. From this point of view, the question posed by Shimony, as to whether Einstein’s realism is compatible with nonseparability, is answered in the negative. On the other hand, I wish to emphasize the need for maintaining the conceptual distinction between separability and locality and for counteracting the enduring tendency to conflate the two notions or to consider the former as a prerequisite of the latter.

The paper is organized as follows. First, a brief summary of Einstein’s own incompleteness argument will be given; it will be seen that the notion of completeness involved in this argument is very similar to the notion of completeness von Neumann refers to in his no-hidden-variables theorem. This similarity is far from clear when the EPR paper alone is taken into account (Section 2). This fact of this similarity deserves some attention, since it has been little investigated in the literature. However, it is conceptually independent of the main point of the paper, which is addressed in the Section 3. Here I show that the assumption that underlies Howard’s equivalence proof is untenable. Finally, in the last two sections, the possibility of an ‘Einsteinian’ nonseparable realism and the plausibility of a quantitative formulation of separability are briefly discussed. Both proposals, due to Shimony and D’Espagnat respectively, are found to be unsatisfactory.

2 Einstein’s incompleteness argument

The ideal physical situation under scrutiny in Einstein’s own argument, as reconstructed in Howard [1985], comprises a physical system $S_{12}$, a composite system made up of two noninteracting subsystems $S_1$ and $S_2$. A general condition that is assumed at the outset is a condition of separation (or separateness). This condition can be assumed to be the conjunction of two distinct principles, viz. the principle of separability and the principle of locality. According to separability, spatially separated
systems possess distinct real states whereas according to locality the state of the system can be modified only by local influences or interactions. In Howard's words '... the separability principle operates at a more basic level as, in effect, a principle of individuation for physical systems, a principle whereby we determine whether in a given situation we have only one system or two' (Howard [1985], p. 173.) The condition of completeness can then be formulated as follows: the quantum mechanical wave function $\Psi$ is associated one-to-one with the real state of the system. This formulation of the completeness condition differs from the completeness condition in the EPR paper. Let us call EPR-completeness the latter condition and E-completeness the former.

Now suppose that $S_1$ consists of a single particle. Then we can choose whether to measure its position or its momentum: with reference to this choice, we will obtain for $\Psi_2$ (the wave function for $S_2$) different representations, since from the choice of the measurement performed on $S_1$ different statistical predictions concerning the successive measurements to be performed on $S_2$ can be derived. The particular physical situation under scrutiny requires then that different wave functions be ascribed to the state of the second system, depending on the type of measurement that one chooses to perform on the first system. Therefore, several wave functions $\Psi'_2, \Psi''_2, \ldots$ are associated with the same real state of the second system. The condition of separation embodies the relative independence of the two systems (according to separability) and prohibits immediate influences between the two systems (according to locality). The definition of completeness just formulated, however, prescribes an one-to-one association between the wave function and the real state. The ideal experiment contemplates instead the association between one real state and a plurality of wave functions $\Psi'_2, \Psi''_2, \ldots$. The quantum mechanical description, 'coded' in the wave function $\Psi$, is then E-incomplete. This then is the structure of Einstein's own argument for the incompleteness of quantum mechanics.

Among the sources of the problem of the completeness of QM, a special place is occupied by von Neumann's 'no-go' theorem (von Neumann [1955]). Von Neumann's effort to give a mathematical proof of the claim that the goal of completing QM was unattainable was consistent with his belief in the 'acausal' character of the laws concerning the microworld. The completion of QM consisted, in von Neumann's presentation, in the possibility of defining dispersion-free and homogeneous statistical ensembles (representatives of the pure states of the physical system). Von Neumann aimed to prove that such attempts at completion were bound to fail, and that such failure sanctioned, once and for all, the acausal character of the laws of nature (von Neumann [1955], p. 210.) But what
kind of completeness property did von Neumann refer to in the proof of his theorem? The relation between a quantum-mechanical state \( \psi \), associated with a statistical ensemble \( S_1, \ldots, S_m \), and the values of physical quantities is in general a statistical one. QM rejects the universal validity of the ignorance interpretation and holds (von Neumann [1955], p. 302) that all \( S_1, \ldots, S_n \) are in the same state, but the laws of nature are not causal. Von Neumann then formulates an incompleteness argument, which he attributes to the defenders of the possibility of completing QM. This argument shows a substantial coincidence between von Neumann’s notion of completeness and Einstein’s own one, outlined above.

According to von Neumann’s incompleteness argument, QM violates what he calls the ‘principle of sufficient cause’, which he says is nothing but a definition of identity:

...two identical objects \( S_1, S_2 \)—i.e. two replicas of the system \( S \) which are in the same state—will remain identical in all conceivable interactions... For if \( S_1, S_2 \) could react differently to the same intervention in their interaction (i.e. if they gave different values in the measurement of a quantity \( R \)), then we would not have called them identical. Therefore, in an ensemble \([S_1, \ldots, S_n]\) which has dispersion relative to a quantity \( R \), the individual systems \( S_1, \ldots, S_n \) cannot (by definition) all be in the same state... Since one will obtain different values in the measurement of the same quantity \( R \) in several systems, which all are in the state with the wave function \( \phi \)—if \( \phi \) is not an eigenfunction of the operator \( R \) of \( R \)—therefore these systems are not equal to one another—i.e. the description by the wave function is not complete (von Neumann [1955], p. 303, our emphasis).

As can be seen, the condition of completeness singled out above by von Neumann is identical to the condition of completeness in Einstein’s own incompleteness argument, although expressed in a somewhat involved way. Von Neumann’s completeness (cf. the italicized sentence above) prescribes an one-to-one correspondence between the real state of the systems belonging to the ensemble and the wave function, which is the mathematical element entering the statistical algorithm for determining the probability distributions of values for the physical quantities. The presence of a nonzero dispersion for the ensemble, relative to the quantity \( R \), determines different values of \( R \) for the several elements of the ensemble: but the elements of the ensemble are—by definition—replicas of a system \( S \), that remain ‘identical in all conceivable interactions’. There being a single wave function associated with the totality of the elements of the ensemble, it follows that the description provided by the wave function is not complete.

The connection between von Neumann’s notion of completeness and Einstein’s own notion can be further clarified by recalling a passage
contained in 'Quanten-Mechanik und Wirklichkeit', that we previously quoted. In this passage Einstein speaks of the plurality of representations for $\Psi_2$, the wave function for the system $S_2$, corresponding to the choice of the physical quantity to be measured on the system $S_1$. Translating this EPR-type physical situation in von Neumann's language, let us suppose we introduce the ensemble of the replicas of $S_2$, where each replica is associated with one of these possible representations of $\Psi_2$ for $S_2$. The elements of this ensemble are identical, since they are by definition replicas of $S_2$, but they are associated with different representations of $\Psi_2$: this means that the description of the $\Psi$ is not von Neumann-complete.

Two remarks are in order. First, there is an asymmetry in the completeness-violating way of associating the $\Psi$-representation with the real state of the system. In Einstein's argument, the real state of the system comes to be associated with a plurality of wave functions $\Psi_1, \Psi_2, \ldots$, whereas in von Neumann's argument a plurality of values in the measurement of a physical quantity comes to be associated with a single wave function $\phi$, describing an ensemble of replicas of the given system. Furthermore, an assumption of determinism is required for von Neumann's argument to work. However, although the differences between the two arguments are not to be overlooked, von Neumann's strategy is very close to Einstein's: the agreement between Einstein and von Neumann on the idea of completeness (an one-to-one correspondence between the real state of the system and the wave function) is substantial.

### 3 Is nonseparability the focus of Bell's theorem?

The formulation of Einstein's own incompleteness argument allowed Howard not only to distinguish sharply the principles of separability and locality in the argument but also to analyse the consequences of the Bell theorem for hidden variable theories with reference to this distinction. It is to this analysis that we now turn our attention. The hidden variable framework we will refer to in the sequel is the class of stochastic hidden variable theories for correlation experiments.

In a schematic representation of a correlation experiment, we can suppose that there is a source from which pairs of particles, denoted by 1 and 2 respectively, are emitted. We denote by $\lambda$ the complete state of the pair $1 + 2$; $\lambda$ belongs to a space of complete states on which we assume we can define probability measures. The two particles are supposed to move away in opposite directions after the emission, and are supposed to enter an analyser each characterized by a certain adjustable parameter: we denote by $a$ and $b$ the parameters of the analysers the particles 1 and 2 enter, respectively. The possible outcomes of the analyser measurement are
supposed to lie in the interval \([-1,1]\); we will denote such outcomes by \(m, n, \ldots\). Stochastic hidden variable theories are characterized by the fact that the complete specification of the parameters and of the state suffices to give only the probability of the single and joint measurement outcomes (that are assumed to be well-defined). Formally,

\[
p_1^1(m \mid a, b) \text{ is the } \lambda\text{-probability of the outcome } m \text{ for the measurement on particle } 1, \text{ given the parameters } a \text{ and } b,
\]

\[
p_2^2(n \mid a, b) \text{ is the } \lambda\text{-probability of the outcome } n \text{ for the measurement on particle } 2, \text{ given the parameters } a \text{ and } b,
\]

\[
p_{12}^1(m, n \mid a, b) \text{ is the } \lambda\text{-probability of the joint outcomes } m \text{ and } n \text{ for the measurement on particle } 1 \text{ and } 2 \text{ and given the parameters } a \text{ and } b,
\]

\[
p_1^3(m \mid a, b, n) \text{ is the } \lambda\text{-probability of the outcome } m \text{ for the measurement on particle } 1, \text{ given the parameters } a \text{ and } b \text{ and the outcome } n \text{ for the particle } 2,
\]

\[
p_2^4(n \mid a, b, n) \text{ is the } \lambda\text{-probability of the outcome } n \text{ for the measurement on particle } 2, \text{ given the parameters } a \text{ and } b \text{ and the outcome } m \text{ for the particle } 1.
\]

The usual locality condition for stochastic hidden variable theories, often called \textit{factorizability}, is expressed as follows:

\[
P_1^{12}(m, n \mid a, b) = p_1^1(m \mid a)p_2^2(n \mid b) \text{ (LOC)}
\]

In 1984 Jarrett showed that LOC is actually the conjunction of two independent conditions, that Jarrett called \textit{locality} and \textit{completeness} respectively (Jarrett [1984]). We will adopt the more neutral terms introduced by Shimony, namely \textit{parameter independence} and \textit{outcome independence} respectively. The two conditions are defined as follows:

\textbf{Parameter Independence}

\[
p_1^1(m \mid a, b) = p_1^1(m \mid a) \text{ (PI-1)}
\]

\[
p_2^2(n \mid a, b) = p_2^2(n \mid b) \text{ (PI-2)}
\]

\textbf{Outcome Independence}

\[
p_1^3(m \mid a, b, n) = p_1^3(m \mid a, b) \text{ (OI-1)}
\]

\[
p_2^4(n \mid a, b, m) = p_2^4(n \mid a, b) \text{ (OI-2)}
\]

The assumption of factorizability allows one to derive an inequality for stochastic hidden variable theories (Clauser and Horne [1974]). This inequality turns out to be violated by quantum probabilities. For this class of hidden variable theories, Bell's theorem can be then summarized as follows: no factorizable stochastic hidden variable theory can reproduce all statistical predictions of quantum mechanics.
In Howard’s view, Bell’s theorem can be given a broader interpretation if the distinction between separability and locality is adequately taken into account. According to Howard, Bell himself underestimated the importance of formulating a separable hidden variable theory:

The problem is that when one writes down the hypothetical hidden state of the joint system (the pair of previously interacting systems), one writes down a single state for the joint system, not a product of separate states... It is not that the possibility of a separable hidden-variable theory has been ignored in the investigations inspired by Bell’s theorem. In fact, Bell mentions the possibility at the start of his original paper, but he dismisses the need for explicit consideration of separable hidden-variable theories (Howard [1985], p. 195).

The passage Howard refers to is the following:

Some might prefer a formulation in which the hidden variables fall into two sets, with $A$ dependent on one and $B$ on the other; this possibility is contained in the above, since $\lambda$ stands for any number of variables and the dependences thereon of $A$ and $B$ are unrestricted (Bell [1964], in Bell [1987], p. 15).

In Howard’s view, the lack of consideration for separability in hidden variable theories has had relevant consequences on the meaning that is usually attached to the Bell inequalities. Therefore, he aims to show that locality is not the only prerequisite for the derivation of the Bell inequalities:

In another paper I show that we can derive the Bell inequality from two independent assumptions—the separability principle and the locality principle... I also show that any hidden-variable theory whose predictions satisfy the Bell inequality is separable, at least with regard to those aspect of the hidden state which are at issue in the Bell experiments (Howard [1985], p. 196).

From this alleged proof, Howard draws the following conclusions:

If my derivation of the Bell inequality is sound, then the interpretation of the results of the Bell experiments is simple. We must give up either separability or locality. And those two alternatives correspond, respectively, to accepting either non-separable quantum mechanics or non-local hidden-variable theories. But if these are our only alternatives, then most of us would likely prefer the former alternative, on the grounds that special relativistic locality constraints are too much a part of our physics to be sacrificed to the cause of saving separability, all the more so because we have ready at hand a highly successful non-separable quantum mechanics, but no well-developed non-local hidden-variable theory ([1985], p. 197).

In order to discuss Howard’s claims, let us first define Howard’s notion of separability in the context of stochastic hidden variable theories. Two systems 1 and 2 will be defined state separable if, for every $\lambda$, there exist
separate states $\alpha, \beta$ for these systems such that

$$p^1_{\lambda}(m, n \mid a, b) = p^1_{\alpha}(m \mid a)p^2_{\beta}(n \mid b) \quad (\text{SEP})$$

We italicized 'for every $\lambda$' because this clause is missing in Howard's definition but is nevertheless essential for the argument. Its inclusion in the definition will help us to understand better the circumstance in which the systems are not state separable. With regard to this condition, Howard explicitly says:

With states defined as probability measures, (SEP) corresponds directly to the more recognizable quantum mechanical definition of separable systems, according to which the joint state is a tensor product of the separate states ... One can regard the separability condition defined here as asserting that the joint state contains no information about joint measurement outcomes not already contained in the separate states, or, conversely, that the separate states contain all of the information necessary to determine completely all possible joint outcomes. In a more philosophical idiom, one might say that the joint reality of [1 and 2] is exhausted by their separate realities (Howard [1992], p. 311).

Howard rightly stresses the fact that in the definition of SEP, in the right-hand side both the parameters appear. That is to say, SEP does not in itself appeal to Jarrett's locality. It is not difficult, then, to prove that SEP is equivalent to outcome independence, provided we make the following identifications:

$$p^1_{\alpha}(m \mid a, b) = p^1_{\lambda}(m \mid a, b) \quad (1)$$

$$p^2_{\beta}(n \mid a, b) = p^2_{\lambda}(n \mid a, b) \quad (2)$$

We prove first that SEP implies 0I-1. From the standard probability definitions we can write LOC as

$$p^2_{\lambda}(m, n \mid a, b) = p^2_{\lambda}(m \mid a, b, n)p^2_{\lambda}(n \mid a, b) \quad (3)$$

(3) in turn can be written as

$$p^1_{\lambda}(m \mid a, b, n) = p^2_{\lambda}(m, n \mid a, b) / p^2_{\lambda}(n \mid a, b). \quad (4)$$

According to SEP, we have

$$p^1_{\lambda}(m \mid a, b, n) = p^1_{\alpha}(m \mid a, b)p^2_{\beta}(n \mid a, b)/p^2_{\lambda}(n \mid a, b); \quad (5)$$

according to the identification (2) we can simplify and obtain

$$p^1_{\lambda}(m \mid a, b, n) = p^1_{\alpha}(m \mid a, b) \quad (6)$$

and according to the identification (1) we finally obtain

$$p^1_{\lambda}(m \mid a, b, n) = p^1_{\lambda}(m \mid a, b) \quad (7)$$

that is, 0I-1. In a similar way, 0I-2 can be obtained.
Conversely, from (3) and OI-1 we obtain

\[ p_{12}(m, n \mid a, b) = p_{\lambda}(m \mid a, b) p_{\mu}(n \mid a, b) \]  

so that, according to the identifications (1) and (2), SEP follows.

Howard’s proof of equivalence rests in an essential way on the identifications (1) and (2). These identifications, however, are implausible. In order to see why it is necessary to examine more in detail the notion of state in this context.

In a different framework, namely that of the development of a modal interpretation of QM, van Fraassen has emphasized the necessity of distinguishing two concepts of state: the value state and the dynamic state. Although formulated in a different context, the distinction can be employed for our purposes. The value state is ‘fully specified by stating which observables have values and what they are’, whereas the dynamic state is ‘fully specified by stating how the system will develop if isolated, and how if acted upon in any definite, given fashion’. It is important to note that, since measurement is an interaction, ‘the prediction of measurement outcome probabilities belongs to the role of the dynamic state’ (van Fraassen [1991], p. 275). Given a quantum state \( \phi \), we can conceive a value component and a dynamic component of \( \phi \). Therefore, in the local hidden variable theories for QM, the contribution of the hidden variable to the quantum state \( \phi \) will include a ‘value’ contribution and a ‘dynamic’ contribution.

Given this distinction, it is perfectly possible that \( p_{\alpha}(m \mid a, b) \) differs from \( p_{\lambda}(m \mid a, b) \). The probability \( p_{\lambda} \) is actually related to the dynamic component of \( \lambda \), which is the (complete) state of the composite system at the source. This probability, being induced by \( \lambda \), could well be different from \( p_{\alpha} \); the latter is the probability induced by \( \alpha \) and the dynamic component of \( \alpha \), which \( p_{\alpha} \) is related to, is in general different from the dynamic component of \( \lambda \). Therefore, for a stochastic hidden variable theory, there need not exist, for every \( \lambda \), two separate states \( \alpha \) and \( \beta \) such that \( p_{\lambda}^{12}(m, n \mid a, b) \) be equal to \( p_{\alpha}^{1}(m \mid a, b) \cdot p_{\beta}^{2}(n \mid a, b) \) Nevertheless, since in general \( p_{\alpha}^{1}(m \mid a, b) \) differs from \( p_{\lambda}^{1}(m \mid a, b) \), it still makes sense to define—for such a theory—locality as factorizability, namely as conjunction of parameter independence and outcome independence. This implies that we can have stochastic hidden variable theories that are SEP-violating and factorizable. Therefore the violation of Bell’s inequalities, occurring when the probabilities of the local stochastic hidden variable theory are replaced by the QM probabilities, does not concern separability in the Howard sense (i.e. SEP) but locality, defined as the conjunction of parameter independence and outcome independence.

One can argue that the violation of SEP would imply the nonseparability
of the two systems *tout court*, and that it is hard to accept that in correlation experiments we do not deal with *two* systems but really with just *one*. Howard himself, however, acknowledges that nonseparability of states need not imply nonseparability of systems:

There are two ways to deny the separability principle. The more modest concerns the individuation of states; it is the claim that spatio-temporally separated *systems* do not always possess separable *states*, that under certain circumstances either there are no separate states or the joint state is not completely determined by the separate states. I call this way of denying the separability principle the *nonseparability of states*. The more radical denial may be called the *nonseparability of systems*; it is the claim that spatio-temporal separation is not a sufficient condition for individuating *systems* themselves, that under certain circumstances the contents of two spatio-temporally separated regions of space-time constitute just a single system ([1989], p. 226).

The violation of the separability of systems implies, according to Howard, that in the correlation experiments we might have, under certain circumstances, a *single* system that exhibits different properties at spacelike separated regions. Recall the distinction between the value component and the dynamic component. A sufficient condition for two particles in a state $\Phi$, in a correlation experiment, to be treated as one single system would be for them to share the same value component: the same observables are relevant for the two particles and these observables have the same values for the two particles in $\Phi$. On the other hand, Howard’s nonseparability of states refers to the dynamic component of the states themselves. Such components induce just the measurement outcome probabilities for the two particles and the coincidence on these probabilities is certainly *not* a sufficient condition for the two particles to be treated as one single system. Nonseparability of states is indeed envisaged in Bell’s original treatment, since the complete specification of the source, at the level of complete states, was not necessarily assumed to distinguish between two hypothetical separate complete states, one referring to particle 1 and the other referring to particle 2 (see also Jarrett [1989], pp. 65–6).²

² By not requiring that the single joint state be necessarily the product of the separate states of the two systems, Bell and his followers have meant to achieve a greater generality. In Howard ([1989], p. 231) it is argued that ‘... this generality turns out to be spurious: any theory whose predictions satisfy the Bell inequalities tacitly assigns separate physical states to the two systems, such that the joint state is the product of the separate states, whether or not that fact is explicitly recognized in the formalism of the theory’. We hold that such ‘tacit assignment’ only follows from Howard’s identifications (1) and (2), which are just the identifications we object to. If these identifications are rejected, the theory is not forced to express any joint state as the product of two separate states (and, although Howard rapidly dismisses the point, the recognition of this fact in the formalism of the theory is highly relevant).
Therefore, if the identifications (1) and (2) are questioned—and according to what we have just argued there are good reasons why they should be—the state separability and the outcome independence are not equivalent. As a result, the negation of SEP does not yet tell against local hidden variable theories that, however, are at variance with the Bell inequalities. The latter can still be considered as the test of nonlocality in nature, not at all of nonseparability.\footnote{In a 1989 paper, Steven French argued that Howard’s conclusion—the EPR/Bell situations do not imply nonlocality but rather nonseparability as a loss of individuality—is not justified, since ‘...it is perfectly possible to maintain that quantal particles are individuals, in a classical sense if you like, but subject to state accessibility restrictions over and above the classical ones’ (French [1989], p. 7). The possibility of considering the quantal particles as individuals, notwithstanding the nonclassical nature of their statistical correlations, is defended by French by appealing to the notion of ‘non-supervenient’ relations holding between the particles, a notion first introduced in the philosophy of QM by Teller [1986].}

That the violation of state separability is the focus of the Bell theorem is implausible also from another point of view. If the circumstance that explains theoretical and experimental violations of the Bell inequalities were assumed to be just the violation of the state nonseparability implicit in the QM formalism for interacting systems, the comparison—on empirical grounds—between QM and a general hidden variable theory would be of little significance. For such a general hidden variable theory should necessarily—in the case of EPR–Bohm experiments for correlated spin 1/2 particles—embody the singlet-state-determined statistics and thus it should \textit{ipso facto} embody the nonseparability property for the relevant cases, namely the EPR–Bohm correlation experiments. In this way, the general hidden variable theory, local or nonlocal, is already bound to exhibit a discrepancy with the Bell inequality, so that the possibility of a significant empirical test between local hidden variable theories and QM is denied \textit{at the beginning}. There would be no point in introducing local hidden variable theories since it is state separability that would be at stake in this case. It is reasonable to abandon the state separability property and to allow for local hidden variable theories that may violate state separability: these hidden variable theories can be said ‘to inherit’ a nonseparable feature which already belongs to QM.

Let us suppose we require the inclusion of a state separability principle in hidden variable theories as a formulation of ‘Einsteinian’ hidden variable theories. These theories could not possibly be good candidates for completion of QM, since they could not reproduce all the statistics delivered by the QM entangled states. In view of this fact, it appears very reasonable not to impose separate complete states for the correlated systems at the beginning, as is implicit in the aforementioned remark of
Bell in his 1964 paper. To assume such state separability at the outset appears—on the contrary—as an unjustified restriction. It is actually true that 'with states defined as probability measures, (SEP) corresponds directly to the more recognizable quantum mechanical definition of separable systems, according to which the joint state is a tensor product of the separate states ...' (Howard [1992], p. 311). The tensor product states, however, are all Bell-inequality-satisfying states. Any hidden variable theory with a state separability condition would then capture just the statistics delivered by such Bell-inequality-satisfying states and no conflict would be even discernible.

With reference to the problem of separability in EPR situations, Ghirardi and Grassi stress that 'usually, in discussing EPR-like situations one pays little attention to separability, which is in a sense tacitly assumed as a prerequisite of the locality principle' (Ghirardi and Grassi [1994], p. 405). In order to clarify this position, which they agree with, Ghirardi and Grassi refer to Einstein [1948], where he holds separability as a necessary condition for realism: 'Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things "lie in different parts of space"' (Einstein [1948]). And in a letter to Born, Einstein says:

\[\ldots\text{that which we conceive as existing ('actual')}\text{ should somehow be localized in time and space. That is, the real in one part of space, A, should (in theory) somehow 'exist' independently of that which is thought of as real in another part of space, B}\ldots\text{ However, if one renounces the assumption that what is present in different parts of space has an independent, real existence, then I do not at all see what physics is supposed to describe (in Howard [1985], pp. 190-1).}\]

The last paragraph explains, according to Ghirardi and Grassi, not only why the problem of separability in EPR situations is in fact usually skipped but also why it \textit{should be skipped}. However, to skip the problem of separability in EPR situations implies skipping it in the hidden variable issue to which the EPR argument gave rise and, as we have shown in the case of Howard's equivalence between separability and outcome independence, this is not always without confusing consequences. For in the case of the EPR–Bohm correlation experiments the claim that the two particles possess 'an existence independent of one another' is compatible with the claim that there does not exist—for every \(\lambda\)—a pair of separate states \(\alpha\) and \(\beta\) such that SEP holds. The fact that Bell, in his 1964 seminal paper, did not impose separability on complete states suggests that some sort of state nonseparability is not in principle incompatible with the possibility of formulating
realism-oriented hidden variable theories. Whether the rejection of the assumption of separability _tout court_ is still compatible with Einstein's realism is another matter that will be briefly discussed in the next section.

After taking into account both the importance given in Einstein's perspective to separability (SEP) and locality (LOC) and the consequences of Bell's theorem, Howard [1985] considers the position of different theories with respect to SEP and LOC. The situation is the following. General relativity would be separable and local, QM would be nonseparable and local, whereas Bohm-type hidden variable theories would be separable and nonlocal. This picture, however, needs some serious qualification. Since Bohm's theory is nonlocal and deterministic, it violates parameter independence and it satisfies outcome independence. The formulation of outcome independence given above shows that in the right-hand side of OI-1 and OI-2 the two parameters appear, that is a single-wing result need not be determined _just_ by λ and that wing's setting. This feature is also present in Howard's definition of SEP:

... in both expressions on the right-hand side of (SEP) one condition-alizes on both _i_ and _j_, which is to say that the states _α_ and _β_ are defined by reference to the _global_ context, meaning both the parameter measured in the local wing and the parameter measured in the distant wing ([1992], p. 312).

In view of Howard's equivalence between outcome independence and state separability, Bohm's theory, satisfying outcome independence, should satisfy state separability. The way to explain how Bohm's theory might be separable would be to think of _α_ and _β_ as the positions, respectively, of the particles 1 and 2. On the other hand, we can think of _λ_ as including _α_, _β_ and the total quantum state, namely all that is causally relevant to the determination of the predicted probabilities for the experiment, according to Bohm's theory. In this theory, the development of the total quantum state affects the evolution of the single particles' variables via the quantum potential. Therefore, the identifications in (1) and (2) cannot be valid in Bohm's theory: they would imply that the outcome probability at each wing is causally independent from the contribution of the quantum potential, that—according to the theory—affects _both_ particles. Thus Bohm's theory, while satisfying outcome independence, cannot satisfy SEP and then it has to be classified as a nonseparable and nonlocal hidden variable theory, contrary to what Howard suggests in his classification. Furthermore, since locality amounts to the conjunction of parameter independence and outcome independence, we should rather say that QM is a nonseparable, parameter independent theory, since it is provably outcome dependent.
4 Einstein’s realism and nonseparability

The negation of locality, as a consequence of Bell’s theorem, implies—according to Jarrett’s result—the negation of either parameter independence or outcome independence. Shimony argued that a peaceful coexistence can be established between QM and relativity theory, since the quantum-mechanical violation of outcome independence does not allow signalling. Although the coexistence seems to be peaceful, the Bell theorem points out, among other things, the impossibility of interpreting the entanglement of quantum states as purely epistemic. The Bell theorem shows us that the local completion of the $Φ$ is not viable. If in the correlation experiments the $Φ$ is a complete description, say, of the polarization state of the pair of the photons, then the indefiniteness of the polarization of each photon with respect to the $x$-$y$ axes is to be accepted as a fact (Shimony [1988], in Shimony [1993], p. 177). What does the intrinsic, non-epistemic entanglement of quantum states imply for Einstein’s realism, a view grounded on both separability and locality? Einstein’s view, taken from the cited article ‘Quanten-Mechanik und Wirklichkeit’ and referred to in the last section, is illuminating if reported at length:

> ... the concepts of physics refer to a real external world, i.e., ideas are posited of things that claim a ‘real existence’ independent of the perceiving subject (bodies, fields, etc.), and these ideas are, on the other hand, brought into as secure a relation as possible with sense impressions. Moreover, it is characteristic of these physical things that they are conceived as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things ‘lie in different parts of space’ (Einstein [1948]).

According to Shimony, Einstein’s realism, as expressed in this quotation, can be made compatible both with nonlocality and nonseparability, provided that we suitably generalize such realism by the introduction of the idea of potentiality, which is the main by-product of the acknowledgement of the nonepistemic character of the entanglement. It is compatible with nonlocality since the quantum mechanical violation of only the outcome independence implies that quantum mechanics ‘peacefully coexists’ with relativity. Moreover, it is compatible with state nonseparability since the thesis that physical things claim a real existence independent of the perceiving subject ‘... is consistent with all the conclusions we drew

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4 It is important to remark that there are arguments that the lack of superluminal causation in relativity theory does not favour giving up outcome independence over giving up parameter independence (see, for instance, Butterfield [1992]; Jones and Clifton [1993]).
from an analysis of Bell’s theorem, the relevant experiments, and the formalism of quantum mechanics’. Admittedly, as Shimony puts it, this thesis, ‘... when separated from the rest of Einstein’s thesis, leaves open the character of the real existence of physical things’ (Shimony [1988], in Shimony [1993], p. 179).

This compatibility claim appears to us very implausible. True, we argued previously that the assumption of separability for hidden variable theories is an unjustified restriction for such theories. However, hidden variable theories cannot be considered as a direct expression of an Einsteinian realism and separability is just one of the issues about which one can question the image of Einstein as ‘the most profound advocate of hidden variable theories’. The assumption of separability appears very well justified when Einstein’s philosophical view of physical reality is at stake. If we still want to define a proponent of a physical non-separable realism as ‘... a sympathetic but independent-minded follower of Einstein’ (Shimony [1988], in Shimony [1993], p. 179), we have to recognize that this proponent’s mind is so independent from Einstein’s one that it is very dubious whether we would be entitled to call him Einstein’s follower.

5 Quantifying separability?

In order to highlight the consequences of Bell’s theorem, D’Espagnat has proposed a quantitative formulation of a principle he called principle of separability (D’Espagnat [1984]). D’Espagnat’s proposal is grounded once again on the assumption that the notion of locality and the notion of separability amount to the same thing. We stress once again that this identification is implausible as a general philosophical point of view. In the present section we will analyse and criticize D’Espagnat’s view.

As a general introduction to the problem, D’Espagnat defines realism as ‘... the view that in some sense or other there should exist “something out there”, that depends neither on our knowledge nor on what we decide to measure although it influences the results of our measurements’ (D’Espagnat [1984], p. 204). Under this view, reality may be considered as consisting of distinct parts (that may interact) called physical systems, and the objective state of such systems is fully described by a set of parameters $\lambda$. D’Espagnat defines as existents the physical entities the $\lambda$ are supposed to refer to. A very general instance of a measurement process is then taken into account. Let $F$ be an apparatus that, in a bounded region, performs a measurement of an observable $A$ on a physical system $\alpha$ emitted from a source $S$ localized in some definite region. We then consider other macroscopic systems $\gamma$, and we denote by $G$ the compound systems
The possibility is considered that the source may emit some other system $\beta$, that can interact subsequently with some of the $\tau$. $G$ may even denote an apparatus set to measure some given observable on $\beta$. The notation $(X_1, X_2, \ldots, X_m \mid Y_1, Y_2, \ldots, Y_n)$ stands in general for the probability of particular values for the variables $X_1, X_2, \ldots, X_m$ given the particular values of the variables $Y_1, Y_2, \ldots, Y_n$. The principle of separability is then formulated by D'Espagnat as the conjunction of the following two assumptions.

**Assumption A:** Let $C_1, C_2, \ldots, C_n$ be distinct observable existents relative to $G$, $c_i (i = 1, \ldots, m)$ be the value of $C_i$ at time $t_i$. Let $a$ denote the result of the measurement of $A$ by $F$ and $A$ denote a set of parameters describing completely the objective state of the source $S$ at the time of emission. Then the distances between the source $S$, the apparatus $G$ and the $T$ can always be chosen large enough so that:

$$| (a \mid \lambda, c_1, c_2, \ldots, c_{m-1}, c_m) (a \mid \lambda, c_1, c_2, \ldots, c_{m-1})^{-1} - 1 | < 10^{-3},$$

$$| (a \mid \lambda, c_1, c_2, \ldots, c_{m-1}) (a \mid \lambda, c_1, c_2, \ldots, c_{m-2})^{-1} - 1 | < 10^{-3},$$

..........................................................................................................

$| (a \mid \lambda, c_1) (a \mid \lambda)^{-1} - 1 | < 10^{-3}$

**Assumption B:** Within an ensemble of similarly prepared sources $S$ the statistical distribution $\rho(\lambda)$ of the parameters $\lambda$ describing completely the objective states of these sources at the time of emission is independent of the $c_i$.

With regard to the Assumption A, to D'Espagnat it is essential that $\lambda$ be meant as a complete description of the source, namely that $\lambda$ be assumed as representing the maximal quantity of information available on the system. The assumptions A–B refer to the probability of a certain outcome conditional on certain values pertaining to observables of $G$: these last values can be actually seen as parameters relative to an apparatus that is set to measure some observable on a distant system. However, the assumptions A–B do not imply that to every $\lambda$ there always corresponds a pair of parameters describing completely the objective state of each system separately. What we mean by this remark is that also in the case of D'Espagnat's definition we are actually talking about a principle of locality and not of separability. Anything having to do with the description of physical systems as systems specified by distinct and separate states, although correlated, has to do with separability, whereas anything having to do with the invariance of the probability of a certain outcome, no matter what
the parameters of the apparatus set to measure some observable on a distant systems are, has to do with *locality*. If then the Bell theorem is held as usual to refute the conjunction of the Assumptions A—B, it is locality that is violated and not separability, as D’Espagnat claims. In the usual formulations of Bell’s theorem, as we previously recalled, the complete specification λ does not contemplate separate complete specifications for the two systems separately and it is locality that is put to the test. A significant remark by D’Espagnat shows that some ambiguities on this point could be avoided if only separability and locality were sharply distinguished at the beginning of the argument. D’Espagnat devoted a section of the cited 1984 article to show the validity of what he calls ‘two complementary assertions’. The two assertions are ‘... (a) that the principle of separability [i.e. Assumptions A—B] is not obeyed by the quantum state ... (b) that, nevertheless, this fact alone cannot be used for inferring that no separable conception of the world is compatible with quantum theory’ (D’Espagnat [1984], p. 208). That the quantum state violates a form of separability is easily shown by appealing to the well-known property of entanglement, but this last property is not at all equivalent to D’Espagnat’s principle of separability. If we define the property of entanglement as *Q*-nonseparability, it is clear then that the *Q*-separability does not establish by itself the incompatibility of Assumptions A—B (D’Espagnat’s principle of separability) with quantum theory. This is so precisely because D’Espagnat’s principle of separability actually amounts to a principle of locality. In fact, what the local hidden variable theories’ research programme attempted to achieve was to show the compatibility of more general local hidden variable theories that accepted a form of nonseparability for the description provided by λ (see again on this point the quotations of Bell, Howard, and Jarrett we previously discussed). A further remark by D’Espagnat shows that this equation between separability and locality is usually meant as a semantical convention, since in his view the act of equating separability and locality hardly needs to be justified. D’Espagnat explicitly states that

... the principle called here the ‘principle of separability’ is but a slight generalization of a principle on which the theories called ‘objective local theories’ by Clauser and Horne are based, and which is called ‘principle of locality’ by several authors ... For that reason the words *local*, *locality*, etc., are freely used below as synonymous to the words *separable*, *separability*, etc. ... ([1984], p. 208).  

5 See Healey’s remarks on D’Espagnat’s tendency to conflate nonseparability and action-at-a-distance (Healey [1994]). Healey’s paper is recommended for the analysis of (non)separability as applied to the wider notion of physical process in relativity theory and quantum theory.
With the discussion above, we would like to contribute, to the contrary, to the clarification of the distinction separability/locality: we argue that these two notions correspond to two metaphysically different properties, whose distinction is to be maintained.

6 Conclusions

The distinction between the principle of separability and the principle of locality, whose origins are to be found in Einstein’s incompleteness argument, turns out to be crucial in order to evaluate the consequences of Bell’s theorem for the foundations of quantum mechanics. With reference to Jarrett’s equivalence proof for stochastic hidden variable theories—according to which the factorizability condition is equivalent to the conjunction of two further conditions, the so-called parameter independence and outcome independence—Howard has proved that, under a certain condition, outcome independence is equivalent to separability. According to Bell’s theorem, if we want to retain parameter independence, we are forced to renounce outcome independence. Therefore, Howard’s proof supports his argument that the focus of Bell’s theorem is the violation of separability, rather than that of locality.

The aim of the present work has been to show that the latter argument is ill founded, since the condition that underlies Howard’s equivalence proof is untenable. Given the distinction of separability and locality, which is adequate and should be maintained, Howard’s condition of state separability has been shown to be an implausible requirement for the local hidden variable theories that, as is already clear from Bell’s original paper in 1964, were formulated as nonseparable theories. After a correct and—I think—natural reading of Bell’s result, we should be aware that nonlocality, rather than nonseparability, remains the price one must be willing to pay when searching for any hidden variable theory that might be able to reproduce the ‘strange’ quantum correlations.

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