A Theory of Weak Interaction Based on the Possible Existence of Strong Octet Gauge Bosons and Weak Intermediate Bosons

Mitsuru HAMA and Sho TANAKA

Department of Physics, Kyoto University, Kyoto

(Received October 25, 1963)

Possible models of the weak interaction are proposed, which are based on the Sakata model and able to explain the characteristic feature of the weak interaction including the $|\Delta I|=1/2$ rule and the occurrence of both $\Delta S/\Delta Q=\pm 1$ phenomena in the nonleptonic and leptonic decays of the strange particles. In this theory an octet local gauge invariance combined with the $SU(3)$ symmetry for the basic baryons of the Sakata model is assumed, which necessarily introduces the octet vector gauge bosons. The weak currents of the baryon-boson system are formed in a certain relation with the octet conserved vector currents, which are constituted of baryons and octet gauge bosons. One finds an essential difference between the structures of the $\Delta S/\Delta Q=1$ currents and the $\Delta S/\Delta Q=-1$ currents, that the latter currents ($I=3/2$, $S=1$) must be constituted of only the octet gauge bosons. This situation seems to have an intimate connection with the selection rule at the leptonic decay of the kaon ($K_{1d}$) reported recently.

§ 1. Introduction

The present status of the weak interaction of elementary particles seems to be characterized by the manifest violation of several conservation laws which hold in the strong and the electromagnetic interactions. Accumulated experimental data are making the detailed structure of this violation clearer, for instance, the $V-A$ law in the parity violation, the nonrenormalizability of the vector coupling of the beta-decay, the approximate $|\Delta I|=1/2$ rule in the nonleptonic decay of the strange particles, the prohibition of the nonleptonic processes with $|\Delta S|\geq 2$, and so on.

Among these, recent experiments, showing possible existences of the processes, $\Sigma^+\rightarrow n+\mu^++\nu$ and $K^0\rightarrow \pi^+e^-+\nu$, both of which obey $\Delta S/\Delta Q=-1$, are quite noticeable in making the mechanism of the $\Delta S/\Delta Q=\pm 1$ rule clear, together with further reports on the kaon-decay into two pions and leptons ($K_{1d}$). As is well known, those phenomena with $\Delta S/\Delta Q=-1$, can be explained only by the induced 6-spinor interaction $f_i A_{\mu \nu \rho} p^\rho \bar{\nu}_\nu$, from the standpoint of the Sakata model (which takes $p$, $n$ and $A$ as the basic baryon fields) as early pointed out by Taketani and Tati, or by increasing the number of basic baryon fields as in the Takeda or Lee models. Nevertheless, it seems difficult to comprehend the complex feature of the weak interactions, as seen in the violation of conservation laws or especially in the necessity of the 6-spinor interaction, in terms of the structureless point-interaction such as the Fermi interaction. In this connection, one of us (S.T.) proposed earlier an attempt
on the weak interaction based on the possible existence of the intermediate bosons and on the Sakata model, trying to ascribe the above complexity of the weak interaction exclusively to the attribute of the intermediate bosons as much as possible. We considered there that the induced 6-spinor interaction is only of a phenomenological character and it should be rather understood as the one caused by the more primary interaction, for instance, by three-vertex interaction among the intermediate bosons. If that is true, the form factor in the 6-spinor interaction appeared in the Taketani-Tati model is understood as the propagation function of the intermediate bosons. This understanding of the induced 6-spinor interaction may find its precedent in the electrodynamics, where the induction phenomena of the electromagnetism which has a character of action at a distance at the first glance was understood essentially in terms of the electromagnetic fields. However, we have to encounter now the situation that the coupling strength of this three-vertex interaction among the intermediate bosons must be extremely strong in order to give rise to the induced 6-spinor interaction with a correct order of strength when we restrict all the intermediate bosons participated in this coupling only to the conventional weak bosons.

This implausible consequences can be avoided if we succeed in taking one or two among three bosons in this interaction to be bosons strongly interacting with baryons (hereafter, called strong bosons). From our present intention, such strong bosons, of course, should be fundamental in the same level as the basic baryons $p, n$ and $A$ in the Sakata model. As the most suitable candidate for them, we could take the hypothetical strong octet gauge bosons, which will be introduced as a generalized Yang-Mills field combined with the invariance requirement under $SU(3)$ symmetry and considered just to afford a binding force for the composite particles in the Sakata model.

In the next section, we find as a necessary consequence of the Yang-Mills scheme that strong conserved octet currents, which couple with the corresponding octet gauge bosons, involve the part constituted only of the octet gauge bosons in addition to the ordinary baryon part. On the other hand, the well-known nonrenormalizability of the vector coupling constant of the beta-decay strongly suggests that at least the vector part of the source current of weak bosons are just the same as the above conserved strong currents, that involve the octet boson part. Thus three- (and four-) vertex interactions among the strong and the weak bosons mentioned above turn out to be the reasonable consequence from our formulation. The latter interactions themselves, however, are not yet able to explain the $\Delta S/\Delta Q = -1$ reactions. But we can easily form such a weak current from the octet gauge bosons that has the properties $I = 3/2$ and $S = 1$ and makes the above reactions possible, by taking into consideration the above three-vertex interaction as the prototype. In §4, concrete models of the weak interaction are proposed along the above line of thought which explain at least qualitatively the main feature of the weak interactions.
§ 2. Strong and weak currents

First let us assume that the system composed of \( p, n \) and \( A \) is invariant under the following octet local gauge transformation:

\[
\psi(x) \rightarrow \psi'(x) = \exp[i \epsilon \lambda_i A^i(x)] \cdot \psi(x), \quad i = 1, 2, \ldots, 8
\]

(2.1)*

where \( \psi = \begin{pmatrix} p \\ n \\ A \end{pmatrix} \) and \( \lambda_i \)'s are the \( 3 \times 3 \) matrices satisfying

\[
[\lambda_i, \lambda_j] = 2if_{ijk} \lambda_k
\]

(2.2)
in conformity with the Gell-Mann's notation. As is well known, this requirement necessitates the octet vector gauge bosons \( B_{\mu}^i \)'s. And the Lagrangian of the whole system satisfying this invariance is given as follows:

\[
\mathcal{L} = -\frac{1}{4} B_{\mu \nu}^{ij} - \bar{\psi} \gamma_{\mu} (\partial_{\mu} - i \lambda_i B_{\mu}^i) \psi - m \bar{\psi} \psi,
\]

(2.3)

where

\[
B_{\mu \nu}^{ij} = \partial_\mu B_{\nu}^i - \partial_\nu B_{\mu}^i - 2if_{ijk} B_{\mu}^j B_{\nu}^k.
\]

(2.4)

Taking into account the field equations derived from the above Lagrangian, we can find the following octet conserved currents \( J_{\mu}^i \)'s:

\[
J_{\mu}^i = \bar{\psi} \gamma_{\mu} \lambda_i \psi - 2if_{ijk} B_{\mu}^j B_{\nu}^k, \quad \partial_{\mu} J_{\mu}^i = 0.
\]

(2.5)

Instead of \( B_{\mu}^i \)'s and \( J_{\mu}^i \)'s we prefer to use, in what follows, more intuitive quantities with a definite isospin, strangeness and charge:

\[
\begin{align*}
I = 1 & \quad U^+ = (B^+ + iB^0) / \sqrt{2} & J_{\mu 1}^+ = (J_{\mu 1}^+ + iJ_{\mu 2}^+ ) / 2 \\
S = 0 & \quad U^0 = B^0, & J_{\mu 0}^0 = J_{\mu}^0 / \sqrt{2}, \\
I = 1 / 2 & \quad V^+ = (B^+ - iB^0) / \sqrt{2} & J_{\mu 1}^+ = (J_{\mu 4}^+ - iJ_{\mu 5}^+ ) / 2 \\
S = 1 & \quad V^0 = (B^0 - iB^0) / \sqrt{2}, & J_{\mu 1}^0 = (J_{\mu 6}^0 - iJ_{\mu 7}^0 ) / 2, \\
I = 0 & \quad S^0 = B^0, & J_{\mu 0}^0 = J_{\mu}^0.
\end{align*}
\]

(2.7)

(2.8)

Obviously, \( U^{+0}, V^{+0}, \) and \( S^0 \) have, at least, in their quantum number, a correspondence to the existing vector bosons \( \rho, K^* \) and \( \omega \) (possibly \( \phi \)) respectively. The concrete forms of the above currents are shown as follows,

\[
\begin{align*}
J_{\mu 1}^+ &= \bar{n} \gamma_{\mu} p - \sqrt{2}(U^0, U^+)_\mu + (\bar{V}^0, V^+)_\mu, \\
J_{\mu 4}^+ &= (\bar{p} \gamma_{\mu} p - \bar{n} \gamma_{\mu} n) / \sqrt{2} - \sqrt{2}(U^+, U^-)_\mu + \{(\bar{V}^-, V^-)_\mu - (\bar{V}^0, V^0)_\mu \} / \sqrt{2}, \\
J_{\mu 6}^+ &= \bar{p} \gamma_{\mu} n + \sqrt{2}(U^0, U^-)_\mu + (\bar{V}^+, V^0)_\mu,
\end{align*}
\]

(2.9)

Repeated indices indicate the usual summation convention.
M. Hama and S. Tanaka

\[
\begin{align*}
J_{\rho}^{1/2} & = \tilde{A}_{\rho} \rho + (V^0, U^+)_{\rho} + (V^+, U^0)_{\rho} / \sqrt{2} + V^{3/2}(V^+, S^+), \\
J_{\rho}^{0} & = \tilde{A}_{\rho} n - (V^0, U^0)_{\rho} / \sqrt{2} + (V^+, U^-)_{\rho} + V^{3/2}(V^0, S^0), \\
J_{\rho}^{0} & = (\bar{\rho} \gamma_{\rho} \rho + \bar{n} \gamma_{n} n - 2\bar{A}_{\rho} A) / \sqrt{6} + V^3(V^+, V^+) + V^3(V^0, V^0),
\end{align*}
\]

(2·10)

by making use of the abbreviated notation,
\[
(A, B)_{\rho} = A_{\rho} B_{\rho} - B_{\rho} A_{\rho}.
\]

(2·12)

Now let us turn our attention to the source currents of the weak intermediate bosons W's (the weak currents). The well-known non-renormalizability of the vector coupling constant of the beta-decay seems to suggest that at least the vector parts of the weak currents are nothing but the above strong conserved vector currents J_{\rho}^{0}'s. At the same time, it must be also recognized that the weak currents can never be fully confined within the strong currents, but the characteristic currents of the weak interaction are needed in addition, at least by the following two reasons.

a) One of them is the well-known parity violation or the V-A law in the beta-decay and the muon-decay which enforces the weak currents to be a mixture of the vector and axial vector currents. Here, it seems quite important for understanding the mechanism of the weak interaction to make it clear whether the parity violation has its origin exclusively in the baryon currents (the first standpoint) or in both of the baryon and boson currents (the second standpoint). At present we have no definite answer to this question, but it turns out that this question has an intimate connection with the selection rule in the kaon decay into two pions and leptons, as will be referred in §§ 3 and 4. For the moment, we take the weak currents according to the first standpoint, replacing \( \gamma_\rho \) by \( \gamma_\rho (1 + \gamma_5) \) in the baryon parts of the strong conserved currents, while otherwise we must further mix the axial vector current of the boson fields such as

\[
[A, B]_{\rho} = \epsilon_{\mu \nu \rho \sigma} (A_{\mu} B_{\nu, \sigma} - B_{\mu} A_{\nu, \sigma}),
\]

(2·13)

with the preceding vector currents of the type, \((A, B)_{\rho}, (2·12)\).

b) As the second reason it is pointed out that so far as we restrict all the weak currents within the octet currents with the above prescribed isospin, strangeness and charge in (2·7), it is impossible to get a scheme which explains the \( A S / A Q = -1 \) phenomena or possibly the approximate \( |\Delta I| = 1/2 \) rule without neutral currents (the so-called veton scheme\(^{10,16}\)). In fact, in the Lee model\(^b\) the current with \( I = 3/2 \) and \( S = 1 \), which is composed of \( \Sigma^{\pm, 0} \) and \( N(n, \rho) \), was introduced to make the \( A S / A Q = -1 \) reactions possible, and in the d'Espagnat\(^b\) and the Bludman\(^b\) models (the veton scheme) further current with \( I = 0 \) and \( S = 2 \), which is composed of \( \Sigma^{3-} \) and \( N \), was necessary to get the \( |\Delta I| = 1/2 \) rule without use of neutral currents.

\(^a\) \( \tilde{B} \) denotes the hermite conjugate field of \( B \).
In our present proposition, those currents which are characteristic of weak interactions, can be easily formed only from the octet gauge bosons, as a reasonable extension of such a type of currents already introduced in the octet currents (2.9)–(2.11) as \((U^\alpha_5, U^\alpha_8)\), or \((V^\alpha_5, U^\alpha_8)\), the occurrence of which is a necessary consequence from the conserved vector current and gives rise to three- (and four-) vertex interaction among the strong and the weak bosons, as emphasized in the introduction. In fact, they are given as follows:

\[
\begin{align*}
J^{\pm}_{\mu} &= - (V^+, U^\pm)_{\mu}, \\
J^0_{\mu} &= \pm \sqrt{3} + \sqrt{2/3} (V^+, U^0)_{\mu}, \\
J^0_{\mu} &= \pm \sqrt{2/3} (V^0, U^+ - U^-)_{\mu}.
\end{align*}
\]

Thus we get first the current \(J_{\mu3/2}^+ = (V^0, U^-)_{\mu}\) that has the desired property, \(\Delta S/\Delta Q = -1\).

Here again we encounter the question of the two standpoints concerning the parity violation mentioned above. If we take the first standpoint, the currents of (2.14) and (2.15) themselves just come to describe the correct expression for the weak currents, because the baryon components are not concerned in this current from the beginning.

In the next section we consider the concrete models of the weak interaction based on the above baryon-boson currents in connection with the following lepton currents:

\[
j_\mu = \bar{e}\gamma_\mu (1 + \gamma_5)\nu_e + \bar{\mu}\gamma_\mu (1 + \gamma_5)\nu_\mu,
\]

and possibly

\[
j_\mu' = \bar{e}\gamma_\mu (1 - \gamma_5)\nu_e + \bar{\mu}\gamma_\mu (1 - \gamma_5)\nu_\mu,
\]

where \(\nu_e\) and \(\nu_\mu\) are the two kinds of neutrinos which are usually considered to accompany with \(e\) and \(\mu\) in the beta-decay and the pion-decay.

### § 3. Possible schemes of the weak interaction

**Model I.** In this model we assume the existence of three complex weak bosons \((W^+, W^0, W^-)\), and their conjugate bosons \((\bar{W}^-, \bar{W}^0, \bar{W}^+)\), both of which form different sets with isospin 1. The characteristic feature of this model is that the strict \(|\Delta I| = 1/2\) rule and the occurrence of \(\Delta S/\Delta Q = -1\) reactions are guaranteed by the full use of the \(J_{\mu3/2}^+ (I = 3/2, S = 1)\) currents (2.14) (the Lee scheme\(^a\)).

\(^a\) Here we take only the currents which has the property antisymmetric in the bilinear boson fields, after the prototype expression in the conserved currents, though, of course, other currents which have a symmetric property are also conceivable.
The interaction Hamiltonian is taken as follows:

\[ H_I = g (j_\rho W_\rho^- + j_\rho^* W_\rho^-) + \text{h.c.}, \]  
\[ H_{1/2} = f_{3/2} \left( \mathcal{G}^{+} W_\rho^+ + \sqrt{2} \mathcal{G}^{0} W_\rho^0 - \sqrt{3} \mathcal{G}^{-} W_\rho^- \right) + \text{h.c.}, \]  
\[ H_0 = g (\mathcal{G}^{+} W_\rho^+ + \mathcal{G}^{0} W_\rho^0 + \sqrt{2} \mathcal{G}^{-} W_\rho^-) + \text{h.c.}, \]

where \( \mathcal{G}_\rho \)'s stand for the already introduced baryon-boson vector currents \( J_\rho \)'s plus the axial vector currents, the form of which depends upon our choice of the two standpoints concerning the parity violation referred in the previous section. \( H_{1/2} \) and \( H_0 \) of (3.2) and (3.3) have, as a whole, the transformation property of \( I = 1/2 \) and \( I = 0 \), respectively. Of course, this model is able to explain the \( |\Delta I| = 1/2 \) rule and the occurrence of both \( \Delta S/\Delta Q = 1 \) and \( -1 \) reactions just in the same way as the Lee model. The only difference from the latter model will be pointed out in the last of this section, which concerns the detailed structure of the \( \Delta S/\Delta Q = \pm 1 \) rule.

**Model II.** In this case, we try to avoid the neutral weak bosons, taking only two kinds of charged bosons: \( (W^+, W^-) \) and \( (W'^+, W'^-) \). Such a scheme is made possible by making use of \( J_{\rho s} \) \( (I=0, S=2) \) current (2.15) in order to explain the \( |\Delta I| = 1/2 \) rule (the von Schacky scheme\(^{15,16} \)) in addition to the \( J_{\rho s} \) \( (I=3/2, S=1 \) and \( \Delta S/\Delta Q = -1 \) current (2.14), which permits the \( \Delta S/\Delta Q = 1 \) reactions on one hand and give a slight deviation from the strict \( |\Delta I| = 1/2 \) rule on the other hand in contrast with Model I. The interaction Hamiltonian is given as follows:

\[ H = g (j_\rho + \mathcal{G}^{+} + \epsilon \mathcal{G}^{-}) W_\rho^- + \text{h.c.}, \]  
\[ H' = g' (\epsilon j_{\rho'} + \mathcal{G}^{+}_{1/2} + \mathcal{G}^{0}_{1/2}) W_\rho^- + \text{h.c.}. \]

The characteristic consequences of this scheme are as follows: i) The phenomenological Fermi coupling for the muon-decay becomes larger than that for the beta-decay\(^{17} \) since the former coupling is caused through both \( H \) and \( H' \). ii) The approximate \( |\Delta I| = 1/2 \) rule\(^{18} \) in the nonleptonic decay of strange particles are guaranteed by the main part due to the crossing term of the second and the third interactions in \( H' \), which is subject to the strict \( |\Delta I| = 1/2 \) rule, and by the correction part due to the crossing term of the second and the third interactions in \( H \) that is a superposition of \( I = 1/2, 3/2 \) and \( 5/2 \) currents with the respective weights \( 1/\sqrt{2}, -\sqrt{2}/5 \) and \( 1/\sqrt{10} \). iii) Both of \( \Delta S/\Delta Q = 1 \) and \( -1 \) reactions are made possible by the crossing term of the first and the second interactions in \( H' \) and of the first and the third interactions in \( H \), respectively.\(^{17,18} \) iv) The leptonic decay of the strange particles with \( |\Delta S| = 2 \) occurs, as in the

\(^{15} \) In the case of \( \Delta S/\Delta Q = 1 \), the neutrino flip phenomena will occur if we take the form of \( H' \), as it stands according to the original idea of Bludman.\(^{16} \)
A Theory of Weak Interaction

253

usual veton theory.

It is here of great interest to ask whether the qualitative consequences noted above can be quantitatively and consistently fit to the recent experimental results\(^{(1,17,18)}\) by suitably choosing the several constants \(g, g', \epsilon\) and \(\epsilon'\). However, we cannot draw any simple conclusion from it before concrete calculation of the matrix elements of the above various baryon-boson currents, since they have in general a complex structure especially in the case of the boson currents with the space-time derivatives.

Finally it should be remarked that as a common feature of the above two models there exists an essential difference between the structures of the \(\Delta S/\Delta Q=1\) and of the \(\Delta S/\Delta Q=-1\) currents. That is, as pointed out in the previous section, the latter is necessarily constituted only of the octet gauge bosons, though the former involves both baryon and boson parts. This means that the \(\Delta S/\Delta Q=-1\) current has a purely vector character with respect to the space-time transformation in contrast with the \(\Delta S/\Delta Q=1\) currents, if we take the first standpoint concerning the parity violation, namely, the parity violation stems exclusively from the baryon and lepton currents and the vector gauge bosons have no connection with it. On the other hand, the recent experiments\(^{(3)}\) concerning the kaon decay into two pions and leptons (\(K_{\mu}\)), which seem to show predominantly \(\Delta S/\Delta Q=1\) phenomena, support this standpoint, namely, the vector character of the \(\Delta S/\Delta Q=-1\) current.\(^{(3)}\) This problem will have a great importance in finding the structure of the weak interaction, as will be referred in the next section.

§ 4. Concluding remarks and further outlook

So far we have considered possible schemes of the weak interaction based on the hypothetical weak intermediate bosons and the Sakata model under the \(SU(3)\) symmetry. Here it is worth while to refer to the reason why we so stick to the Sakata model in spite of the fact that, otherwise, we can easily find possible models of weak interaction only by increasing the number of the basic baryons as in the Takeda\(^{(5)}\) or Lee\(^{(6)}\) models. We emphasize here only one point that this model seems to grasp the essential difference between the strong and the weak interactions in the fact that the former interaction, notwithstanding its great complexity and variety, is simply characterized according to this model by the strict conservation of the kind of the fundamental particles\(^{(3)}\) in marked contrast with the weak interaction.

As seen from the results obtained in the preceding sections, it seems possible to get the satisfactory interaction schemes of the intermediate bosons so as to fulfill at least qualitatively characteristic features of the weak interaction. In the course of establishing these interaction schemes, it was quite important to make the inner relation between the strong and the weak currents clear. For the strong current its structure was uniquely determined from the funda-
M. Hama and S. Tanaka

mental postulate on the strong bosons in association with the generalized local
gauge invariance, which is, of course, hypothetical at this stage, but seems to
have a somewhat persuasive theoretical ground. Contrary, with respect to the
weak bosons, we have not yet any theoretical principle to determine how they
couple with the baryons and the octet gauge bosons, other than purely empirical
facts such as the nonrenormalizability of the vector coupling constant of the
beta-decay, the parity mixing, the $|\Delta I|=\frac{1}{2}$ rule, the $\Delta S/\Delta Q=\pm 1$ rule and so
on. Therefore it is desirable to get a somewhat theoretical ground which could
afford the basis for the interaction scheme taken in the previous sections and
especially settle the question of the two possible standpoints concerning the
parity violation. In this connection, it seems promising that the weak vector
bosons are also generalized Yang-Mills type fields. Some authors$^{9,19}$ have sought
such a possibility. This approach, however, has to encounter the serious dif­
ficulties. In fact, in order that the weak boson is able to be a gauge boson,
its source currents must at least satisfy some sort of conservation law. But,
characteristic currents of the weak interaction such like the axial vector currents
or the $I=\frac{3}{2}$ and $S=1$ currents $J_{a/3}$, which were necessary for the parity
mixing or the $\Delta S/\Delta Q=\mp 1$ rule, respectively, can never be conserved without
taking further implausible assumptions so as to introduce additional fields or
to set the strong coupling constant and the baryon mass to be vanishing. There
is another difficulty in this approach, i.e. how to allow the finite mass for the
gauge bosons. With respect to the strong gauge bosons, Schwinger's conjecture$^{20}$
seems quite promising, which seeks the origin of the finite mass of the gauge
field in the magnitude of the coupling strength. Even if this is true, the cou­
pling of the electromagnetic interaction ought to be already lower than the limit
to allow a finite mass. Therefore this conjecture seems not to be applicable to
the weak vector boson because its coupling is considered to be certainly sma­
ller than the electromagnetic coupling except to the extraordinary case when the
weak boson gets an enormously large mass. Nevertheless, it seems further
necessary to qualify the structure of the weak interaction experimentally, before
we can get a definite answer to the above possibility.

Acknowledgement

Authors should like to express their sincere thanks to Prof. T. Inoue for
his continual encouragement and helpful discussion. They are also greatly
indebted to the colleagues of their laboratory for valuable discussions.

References

A Theory of Weak Interaction

See also S. Machida, Prog. Theor. Phys., to be published.
5) G. Takeda, Ann. of Phys. 18 (1962), 310.
7) S. Tanaka, Prog. Theor Phys. 29 (1963), 792.