Mass Spectra of Baryons, Mesons and Leptons

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In accordance with Sakurai's proposal, it is examined how the Yang-Mills gauge fields associated with the conservation of baryon number, strangeness, and isospin break the hypothetical mass degeneracy. Use is made of Nambu's self-consistent method in order to obtain the mass formulas. It is shown that reasonable sizes of coupling constants and of cutoff momenta lead to the observed mass spectra of the stable or metastable baryons and mesons \(N, A, \Sigma, \Xi; \eta, K\). The determination of the resonant state levels seems to be beyond the scope of the present treatment.

A similar attempt is made to explain the lepton mass spectrum by introducing a gauge field associated with the conservation of leptonic strangeness or muon number.

§ 1. Introduction

The observed mass difference of baryons and that of mesons presumably indicate the existence of some symmetry violating strong interactions. Needless to say, if \(\pi\) and/or \(K\) coupling constants are assumed to be suitably unequal from baryon to baryon, the observed baryon mass splitting may easily be understood. Even when assuming equal coupling constants, it has been shown\(^1\) that if the relative parity of \(N\) and \(\Xi\) is odd, the charge independent \(K\)-baryon interaction with a common coupling constant gives the approximately correct interval ratio of the mass splitting, \(\frac{m_\Xi - m_{\Sigma, \Lambda}}{m_{\Sigma, \Lambda} - m_N} = 1\), in the lowest order perturbation, irrespectively of cutoff momentum.

In the theory of unitary symmetry, Okubo\(^2\) has recently obtained the generalized Gell-Mann mass formula in the lowest order perturbation for a moderately strong \(U_3\)-violating interaction but in any orders for \(U_3\)-conserving very strong interactions. Moreover Katayama\(^3\) has made an interesting proposal of the empirical mass formulas for the particles and resonances in connection with the Sakata model. These proposals seem quite attractive in view of accommodating some of the recently discovered resonances, but the presence of a mysterious symmetry violating strong interaction is still obscure.\(^4\)

In the present report, apart from any special type of the symmetry model,

\(^{1}\) M. Baker and S. L. Glashow (Phys. Rev. 128 (1962), 2462) have found that without the introduction of any symmetry-violating interaction in the higher symmetry model, there exist solutions with the lower symmetries of isospin and hypercharge.
it is examined the possibility that the Yang and Mills gauge fields break the mass degeneracy dynamically. According to Sakurai's idea, three kinds of the Yang-Mills fields, $A_{B}$, $A_{S}$, and $A_{I}$, are introduced, in order that the conservation laws of baryon number $B$, strangeness $S$, and isospin $I$ be consistent with the concept of localized fields. The "strangeness" is here chosen instead of "hypercharge" in Sakurai's work, since the choice of the latter brings some complication for explaining the baryon mass splitting. In § 2, starting with a vanishing bare mass and making use of Nambu's self-consistent procedure, we obtain the baryon and meson mass formulas which give the observed mass splitting correctly in terms of reasonable magnitudes of coupling constants and of cutoff momenta.

It is shown in § 3 that the resonant states cannot be discussed on an equal footing with the metastable particles in our model, probably because the resonant state levels would be determined essentially by the strong interactions between the constituent particles.

A similar attempt is made for the lepton mass splitting in § 4. Introducing a gauge field associated with the conservation of leptonic strangeness or muon number, we may interpret the electron mass as an electromagnetic effect and most part of the muon mass as an effect of the leptonic strangeness gauge fields.

§ 2. Baryon and meson mass formulas

Strong interactions of the baryons with the three gauge fields can be written as

$$\mathcal{L}_{B} = -if_{B} \bar{\psi} \gamma_{\mu} B \psi A_{B\mu}$$

$$= -if_{B}(\bar{N} \gamma_{\mu} N + \bar{A} \gamma_{\mu} A + \bar{\Sigma} \cdot \gamma_{\mu} \Sigma + \bar{\Xi} \gamma_{\mu} \Xi) A_{B\mu}, \quad (2\cdot1)$$

$$\mathcal{L}_{S} = -if_{S} \bar{\psi} \gamma_{\mu} S \psi A_{S\mu}$$

$$= -if_{S}(\bar{A} \gamma_{\mu} A - \bar{\Sigma} \cdot \gamma_{\mu} \Sigma - 2 \bar{\Xi} \gamma_{\mu} \Xi) A_{S\mu}, \quad (2\cdot2)$$

$$\mathcal{L}_{I} = -if_{I} \bar{\psi} \gamma_{\mu} I \psi A_{I\mu}$$

$$= -if_{I}(\bar{N} \gamma_{\mu} \frac{\tau}{2} N + i\bar{\Sigma} \times \gamma_{\mu} \Sigma + \bar{\Xi} \gamma_{\mu} \frac{\tau}{2} \Xi) A_{I\mu}. \quad (2\cdot3)$$

Assuming a vanishing bare mass for the hypothetical baryon in the event of the gauge field interactions being switched off, we follow Nambu's self-consistent procedure and obtain the nontrivial solution of the self-energy equation which determines the observed baryon mass $m$ in terms of coupling constants $f^{*}$s, cutoff momenta $\Lambda$'s, and rest masses of the gauge particles $\mu$'s.

The interactions $\mathcal{L}_{B} + \mathcal{L}_{S} + \mathcal{L}_{I}$ lead to the self-consistent equation for the baryon mass in the lowest order,
The proper self-energy part of the baryon is given by

\[ \Sigma(p, m, f, A, \mu) = m \cdot \int \frac{d^4k}{4\pi^3} \frac{1}{k^2 + \mu^2} F(k, A), \]

where

\[ F(k, A) \]

and \( F(k, A) \) is a cutoff factor. Now we have the baryon mass formula as the nontrivial solution \((m \neq 0)\) of the self-consistent equation,

\[ 1 = \left| \left( \frac{A}{m}, \frac{\mu}{m} \right) \right| + \left( \frac{A}{m}, \frac{\mu}{m} \right) S^2 + \left( \frac{A}{m}, \frac{\mu}{m} \right) I(I+1). \]

To evaluate \( L \) is straightforward, but the expression is rather lengthy, and so the result only for the special case \( \mu = 0 \) is shown here. If we use a straight cutoff at \(|k| = A\), we get

\[ L \left( \frac{A}{m}, 0 \right) = \frac{1}{2\pi} \left[ 3 \ln \left( \frac{A}{m} + \sqrt{1 + \frac{A^2}{m^2}} \right) - \frac{A}{m} \left( \sqrt{1 + \frac{A^2}{m^2}} - A \right) \right]. \]

If we adopt an invariant cutoff at \( k^2 = -A^2 \) after the change of path, \( k_i \to ik_i \), we obtain

\[ L \left( \frac{A}{m}, 0 \right) = \frac{1}{2\pi} \left[ -\frac{3}{2} \ln (1 + \frac{A^2}{m^2}) + \frac{A}{m} \tan^{-1} \left( \frac{A}{m} \right) \right]. \]

Next we shall consider the mesons. Strong interactions with the gauge fields can now be written as

\[ L_S = -if \left[ \frac{\partial \psi}{\partial x_\mu} S \psi - \psi^* S \frac{\partial \psi}{\partial x_\mu} \right] A_{\mu} + f_s \psi^* S^\dagger \psi A_{\mu}. \]

\[ = -if \left[ \frac{\partial K}{\partial x_\mu} K - \psi^* \frac{\partial K}{\partial x_\mu} \right] A_{\mu} + f_s \psi^* K K A_{\mu}, \]

Following the original work of Yang and Mills, we assume that the bare rest mass of the isotopic gauge particle is zero, and that a finite rest mass \( \mu_1 \), if any, may be produced as a result of its self-interactions. Here we do not use the self-consistent mass determination procedure for the gauge particles so as to avoid complications.

On the contrary if we start with a nonvanishing rest mass of the isotopic gauge particle, we get a quadratically divergent contribution with negative sign to the baryon self-energy, due to the additional term \( \mu_1^2 k_\mu k_\nu (k^2 + \mu_1^2)^{-1} \) in the propagator.
The self-consistent equation for the meson mass is similarly obtained,
\[ m^2 = \Sigma(q, m, f, \mu, A, \mu) |S^2 + \Sigma(q, m, f, \mu, A, \mu)| I(I+1). \]  
(2.12)

The proper self-energy part of the boson is given by
\[ \Sigma(q, m, f, A, \mu) = m^2 \frac{f_s^2}{4\pi} Q \left( \frac{A}{m}, \frac{\mu}{m} \right), \]  
(2.13)

where
\[ Q \left( \frac{A}{m}, \frac{\mu}{m} \right) = -\frac{1}{4i\pi^3 m^3} \int d^4k \left[ (2q-k)_\mu (q-k)^2 + m^2 (2q-k)_\mu - \delta_{\mu\nu} \right] \frac{1}{k^2 + \mu^2} F(k, A'). \]  
(2.14)*

Thus we have the meson mass formula as the solution of the self-consistent equation,
\[ 1 = \frac{f_s^2}{4\pi} Q \left( \frac{A_s'}{m}, \frac{\mu_s}{m} \right) S^2 + \frac{f_t^2}{4\pi} Q \left( \frac{A_t'}{m}, \frac{\mu_t}{m} \right) I(I+1). \]  
(2.15)

The expression of \( Q \) for the case \( \mu = 0 \) is shown as follows. A straight cutoff gives
\[ Q \left( \frac{A}{m}, 0 \right) = \frac{1}{2\pi} \left[ \frac{A}{m} \left( \sqrt{1 + \frac{A^2}{m^2} + 2 \frac{A}{m}} \right) + 3 \ln \left( \frac{A}{m} + \sqrt{1 + \frac{A^2}{m^2}} \right) \right]. \]  
(2.16)

An invariant cutoff leads to
\[ Q \left( \frac{A}{m}, 0 \right) = \frac{1}{2\pi} \left[ \frac{3}{2} \frac{A^2}{m^2} + \frac{3}{2} \ln \left( 1 + \frac{A^2}{m^2} \right) \right]. \]  
(2.17)

The fact that both \( L(A/m, \mu/m) \) and \( Q(A/m, \mu/m) \) are positive clearly indicates that the mass formulas (2.7) and (2.15) can give the correct direction of the observed mass splitting, so far as the stable baryons and mesons are concerned. Further we shall show that these mass formulas can determine the observed masses with the correct level intervals in terms of reasonable magnitudes of coupling constants, cutoff momenta and gauge particle masses.

* See the footnote on page 644. In the case of mesons, however, even if we start with a finite mass of the isotopic gauge particle, we have no additional contribution to the meson self-energy.
From (2.7) and (2.15) we have a system of six coupled equations for the stable baryons, \(N(939\text{ MeV}, S=0, I=1/2), \(A(1115\text{ MeV}, S=-1, I=0), \(\Sigma(1192\text{ MeV}, S=-1, I=1)\) and \(\Xi(1319\text{ MeV}, S=-2, I=1/2)\), and for the stable mesons, \(\pi(137\text{ MeV}, S=0, I=1)\) and \(K(496\text{ MeV}, S=1, I=1/2)\). There are eleven unknown parameters, \(f_B, f_S, f_I; A_B, A_S, A_I, A'_B, A'_S, A'_I; \mu_B, \mu_S, \mu_I\). We now reduce the number of parameters to six. If the gauge particle mass \(\mu\) is smaller than the baryon mass, \(L\) and \(Q\) are not very sensitive to \(\mu_i\), so we shall assume some suitable values for every \(\mu_i\), say \(\mu_i=0\), or a few times the pion mass. In addition, we should note that \(Q\) depends quadratically on the cutoff \(A\), while \(L\) depends logarithmically, that is, \(L\) does not so strongly depend on the cutoff as \(Q\) does. We may then assume a common cutoff parameter for the baryons, \(A_B=A_S=A_I=A\).

Thus we have a system of six coupled equations for six unknowns, \(f_B, f_S, f_I; A, A'_S, A'_I\). In order to solve the equations, the computations have been carried out on the KDC-I at Kyoto University. Table I shows that the reasonable coupling constants are obtained for the cutoff parameters in the proximity of \(m_N\). These coupling constants satisfy the inequality

\[
f_B^2/4\pi \gg f_S^2/4\pi \gg f_I^2/4\pi,
\]

(2.18)

<table>
<thead>
<tr>
<th>Cutoff</th>
<th>Mass of gauge particles</th>
<th>(f_B^2/4\pi)</th>
<th>(f_S^2/4\pi)</th>
<th>(f_I^2/4\pi)</th>
<th>(A/m_N)</th>
<th>(A'_S/m_N)</th>
<th>(A'_I/m_N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>(m_N/4)</td>
<td>7.0</td>
<td>0.40</td>
<td>0.023</td>
<td>0.47</td>
<td>1.2</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>(m_N/2)</td>
<td>7.2</td>
<td>0.65</td>
<td>0.022</td>
<td>0.69</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Invariant</td>
<td>(m_N/4)</td>
<td>6.3</td>
<td>0.56</td>
<td>0.046</td>
<td>0.61</td>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>(m_N/2)</td>
<td>7.6</td>
<td>0.79</td>
<td>0.044</td>
<td>0.83</td>
<td>1.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

\[A_0, A_S, A_I, \ldots \]

Fig. 1.

\[A_0, A_S, A_I, \ldots \]

Fig. 2.
as was proposed by Sakurai, though we take $f_s$ instead of his $f_y$, and our results of $f'$s are somewhat smaller than his. Also it should be noted that these results for $f'$s and $A'$s are insensitive not only to the masses of gauge particles but also to the method of cutoff.

Although the present calculation is not based on the perturbative procedure but on Nambu's method, the coupling constant relation (2.18) may allow us to interpret the mass splitting scheme as the successive steps shown in Figs. 1 and 2.

§ 3. Other possible particles and resonances

Besides the presently observed stable baryons and mesons, the Nishijima and Gell-Mann rule suggests the possible existence of other spin 1/2 baryons and spin 0 mesons for $|S| ≤ 3$.

Baryons: $Z^+(S=1, I=0)$ and $\Xi^-(S=-3, I=0)$.

Mesons: $\pi^{09}(S=0, I=0)$ and $\Delta^+(S=2, I=0)$.

If the self-consistent mass formulas in the preceding section can be applied to these particles, their masses are easily evaluated as

$$m_{Z^+} = m_{\Delta},$$

$$m_{\Xi^-} = 1617 \text{ MeV} \text{ (straight cutoff)},$$

$$= 1775 \text{ MeV} \text{ (invariant cutoff)},$$

$$m_{\pi^{09}} = 0,$$

$$m_{\Delta^+} = 2238 \text{ MeV} \text{ (straight cutoff)},$$

$$= 1610 \text{ MeV} \text{ (invariant cutoff)}.$$

Here the gauge particle masses have been assumed to be zero. The choice of the finite mass smaller than the nucleon mass does not change the above results greatly. In view of the use of crude cutoff techniques, these numerical values should not be taken very seriously. The $\gamma$ cannot be identified with the $\pi^{09}$ having a zero mass, and then none of the resonances observed so far corresponds to these possible particles.

It will be further shown that the application of the self-consistent formulas to the observed resonant states fails to explain their mass levels. Let us first consider the spin 1/2 resonant state $N_{11}^+(1690 \text{ MeV}, I=1/2, S=0)$. Since the quantum numbers $B$, $S$, and $I$ of this state are the same as those of the nucleon, the preceding mass formula gives the mass of $N_{11}^+$ being equal to $m_N$. In addition if the $Z_1^+(1620 \text{ MeV}, I=1/2, S=0)$ and the $Y_{13}^+(1385 \text{ MeV}, I=1, S=-1)$ are also the spin 1/2 states, their masses are to be $m_N$ and $m_\pi$ re-

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*3 According to M. Roos's review article, the working names and subscripts for the resonances are used in what follows.
spectively in contrast with the observed data. In order to explain the observed masses we have to take the cutoff parameters being 1.5 to 1.8 times as large as the values listed in Table I.

Next we shall consider the baryon and meson resonances with higher spins. The self-consistent equations can also be written in the same forms as (2·4) and (2·12). For the spin 3/2 baryons, the proper self-energy part is written by a quadratically divergent integral with positive sign.9 Therefore the mass formula gives the relation, \( m(N_{3/2}^*) < m(N_{3/2}^*) < m(Y_{13}^*) < m(Y_{13}^{**}) < m(\Xi_1^*) \), which is evidently in contradiction to the observed data in Table II. Furthermore if the \( Z_1^*, Z_5^*, Y_{15}^*, \) and \( Y_{05}^* \) are also the spin 3/2 states, the formula provides us with \( m(Z_1^*) = m(N_{13}^*), m(Z_5^*) = m(N_{35}^*), m(Y_{15}^*) = m(Y_{15}^{**}), \) and \( m(Y_{05}^*) = m(Y_{05}^{**}) \) against the data.

### Table II. The observed spin 3/2 resonances.7) The state shown in parentheses means that its spin is not affirmative.

<table>
<thead>
<tr>
<th></th>
<th>( N_{3/2}^* )</th>
<th>( Y_{13}^* )</th>
<th>( N_{35}^* )</th>
<th>( Y_{05}^* )</th>
<th>( \Xi_1^* )</th>
<th>( Z_1^* )</th>
<th>( Y_{13}^{**} )</th>
<th>( Y_{05}^{**} )</th>
<th>( Z_5^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>3/2</td>
<td>1</td>
<td>1/2</td>
<td>0</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>3/2</td>
<td>0</td>
</tr>
<tr>
<td>( S )</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-2</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Mass (MeV)</td>
<td>1237</td>
<td>1385</td>
<td>1517</td>
<td>1520</td>
<td>1533</td>
<td>1650</td>
<td>1660</td>
<td>1815</td>
<td>1920</td>
</tr>
</tbody>
</table>

### Table III. The observed spin 1 resonances.7,8) The state shown in parentheses means that its spin is not affirmative.

<table>
<thead>
<tr>
<th></th>
<th>( K_1^{*+} )</th>
<th>( \rho )</th>
<th>( \omega )</th>
<th>( K_1^{**} )</th>
<th>( \varphi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>1/2</td>
<td>1</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>( S )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Mass (MeV)</td>
<td>730</td>
<td>757</td>
<td>781</td>
<td>888</td>
<td>1019</td>
</tr>
</tbody>
</table>

Also for the spin 1 bosons, the proper self-energy part becomes a quadratically divergent integral with positive sign.10 In this case the mass formula leads to \( m(\omega) = m(\varphi) = m(\rho) = m(K_1^{*+}) = m(K_1^{**}) \) in the event of the \( K_1^{**} \) being also the spin 1 state, without fitting the observations.

The above arguments indicate that the self-consistent mass formulas are unsuccessful to explain the masses of the resonances. This may be a matter of course, however, because the mass levels of the resonances are to be dominantly governed by the nature of strong interactions between the constituents, even though the constituents, for example the nucleon and pion in the case of \( N_{3/2}^* \), are stable particles whose masses may be determined by the gauge fields. Then the determination of the resonant levels is beyond the scope of the present treatment, and requires a further development such as a "bootstrap" method.11}
§ 4. Lepton mass splitting

Katayama and Taketani\(^{(12)}\) have shown the possibility of explaining both of the observed electron and muon masses in terms of only the electromagnetic self-energy by assuming a singular charge distribution for the muon. Several authors\(^{(13),(13)}\) have also proposed other possibilities to explain the muon-electron mass splitting from various viewpoints.

In this section we shall examine whether the mass splitting of leptons can be treated in like manner as made for the metastable baryons and mesons in § 2. Let us prescribe that the charge doublets \((e^-, \nu_e)\) and \((\mu^-, \nu_\mu)\) have the opposite lepton number \(L\)\(^{(10)}\) and that the leptonic strangeness \(S_l\) or muon number \(S_{l/2}\) discriminates between the two groups. Although many attempts\(^{(15)}\) to assign the quantum numbers for the leptons have been presented in accordance with the existence of two kinds of neutrino, we shall here choose the assignment shown in Table IV. This is not the only possibility, of course, but we take it for convenience’ sake. Now the analog of the Nishijima and Gell-Mann relation holds:

\[
\frac{Q}{e} = I_3 + \frac{S_l - L}{2} \tag{4.1}
\]

In addition, with this assignment, unwanted processes, \(\mu \rightarrow e + \gamma, \mu \rightarrow 3e\), and \(\mu + p \rightarrow e + p\), can be forbidden by the conservation law of \(S_l\) as well as by that of \(L\)\(^{(14)}\)

<table>
<thead>
<tr>
<th>Table IV.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>(e^-)</td>
</tr>
<tr>
<td>(\nu_e)</td>
</tr>
<tr>
<td>(\mu^-)</td>
</tr>
</tbody>
</table>
| \(\nu_\mu\) | \(
\frac{1}{2}\) | -1| -2|

The fact that the observed mass of the neutrino \(\nu_e\) is almost zero (<0.00025 MeV) and that of \(\nu_\mu\) is smaller than 2.5 MeV seems to suggest that the gauge field for lepton number has no appreciable contribution to the lepton masses, even if its effect exists some few. So we shall neglect the lepton number gauge field henceforth. The gauge fields to be taken into account are the electromagnetic field \(A_e\), which is supposed to be responsible for the major part of the electron mass, and the leptonic strangeness gauge field \(A_{s_{l/2}}\), which may explain the sufficient amount of the muon mass.

It has been pointed out by Katayama and others\(^{(19),(19)}\) that in order to interpret the observed electron mass (0.51 MeV) as an electromagnetic effect, an additional tensor type interaction is needed, even if use is made of Nambu’s
self-consistent procedure. Okabayashi has also stated the necessity of the tensor type interaction from a different viewpoint by his even-odd argument. Then we assume the electromagnetic interaction of leptons as

$$\mathcal{L}_{EM} = -ie\bar{\psi}_{\nu} \gamma_{\mu} \left( I_3 + \frac{S_1 - L}{2} \right) \psi A_{\mu} + \frac{e}{4m_e} \partial_{\nu} \sigma_{\mu\nu} (I_3 + aS_1 + bL) \psi F_{\mu\nu},$$  \tag{4.2}

where \(m_e\) is the observed electron mass, which has been introduced here as a sort of universal constant, \(\delta\) stands for the deviation of the magnetic moment, and \(a\) and \(b\) mean certain numerical constants. The experimental data of the electron and muon magnetic moments\(^{(17)}\) give the restrictions: \(|\delta(1/2 + b)| < 10^{-8}, |\delta(-1/2 - 2a - b)| < 10^{-6}|. In addition the astrophysical consideration in conjunction with the \(\nu_{\mu}\) scattering and \(\nu_{\mu}\)-pion production provides the upper limits of the neutrino magnetic moments\(^{(18)}\), i.e. \(|\delta(1/2 + b)| < 10^{-10}\) to \(10^{-9}\), \(|\delta(1/2 - 2a - b)| < 10^{-10}\) to \(10^{-8}\). Putting together these inequalities, we may roughly determine the values,

$$a = (1/2), \quad b = -(1/2), \quad \delta \leq 10^{-6}. \tag{4.3}$$

Similarly the interaction of the leptonic strangeness gauge field is assumed to be written as

$$\mathcal{L}_{S} = -if_{\nu} \bar{\psi}_{\nu} S_{\mu} \psi A_{\mu} + \frac{f_{\nu} f_{\mu}}{4m_e} \bar{\psi}_{\nu} \sigma_{\mu\nu} (I_3 + a'S_1 + b'L) \psi F_{\mu\nu} \tag{4.4},$$

where \(a'\) and \(b'\) are also certain numerical constants. If the cutoff parameter is chosen to be \(A \gg m_N\), the self-energy parts of \(\epsilon^2\)-order and \(f_{\nu}^2\)-order both are logarithmically divergent and can be neglected in comparison with the other contributions which are quadratically divergent. Furthermore remembering that \(\delta(m/m_e) \gg \delta^2(m/m_e)\) even for the muon, we may safely omit the \(\delta^2\)-contribution. We shall make a similar assumption on the interaction \(\mathcal{L}_{S}\), namely \(f_{\nu} f_{\mu} (m/m_e) \gg f_{\nu}^2 (m/m_e)^2\) even for the muon. Thus the self-consistent equation for the lepton mass is represented by only the cross terms, and the nontrivial solution is given as follows,

$$1 = -\frac{e^2}{4\pi} \frac{\sigma}{m_e} Q_i \left( \frac{A}{m}, \frac{\mu}{m} \right) \left( I_3 + \frac{S_1 - L}{2} \right)^2$$

$$+ \frac{f_{\nu} f_{\mu}}{4\pi} \frac{m}{m_e} Q_i \left( \frac{A'}{m}, \frac{\mu}{m} \right) S_i (I_3 + a'S_1 + b'L), \tag{4.5}$$

where

$$Q_i \left( \frac{A}{m}, \frac{\mu}{m} \right) = -\frac{1}{4i\pi^3 m^2} \int d^4k \left[ \tilde{\tau}_{\nu} \cdot \frac{1}{i j} \cdot (p - k) + \frac{1}{m} \sigma_{\mu k} \tau_{\nu} - \frac{1}{i j} \cdot (p - k) + \frac{1}{m} \sigma_{\mu k} \tau_{\nu} \right]$$

$$\times \frac{1}{k^2 + \mu^2} F(k, A).$$

For the doublet \((e^-, \nu_e), (4.5)\) becomes
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\[ m_e = \sqrt{\frac{\varepsilon}{\pi}} s \frac{e^2}{4\pi} \delta A, \]  
\[ m_e = 0, \]  
(4.6)  
(4.7)

where \( \varepsilon \) denotes 1 for the straight cutoff and 1/2 for the invariant cutoff. If we take \( A = 10 \text{ BeV} \) and \( \delta = 10^{-5} \), we can get the observed electron mass.

For the doublet \( (\mu^-, \nu_\mu) \), in order to obtain the results, \( m_\mu = 106 \text{ MeV} \) and \( m_\nu_\mu \leq 2.5 \text{ MeV} \), the relation

\[ \frac{1}{2} - 2a' - b' \leq 1/40 \]  
(4.8)

must be satisfied. Then it follows from (4.5) that

\[ m_\mu = \varepsilon \frac{3}{\pi} \left[ \frac{e^2}{4\pi} A' + \frac{f_3}{4\pi} m_\mu - \frac{A''}{2} m_\mu \right] = \varepsilon \frac{6}{\pi} \frac{f_3}{4\pi} m_\mu, \]  
\[ m_\nu_\mu = \left( \frac{1}{2} - 2a' - b' \right) m_\mu, \]  
(4.9)  
(4.10)

where we have assumed that the cutoff \( A' \) is very large as compared with not only the muon mass but the gauge particle mass. If the same cutoff parameter \( A' = 10 \text{ BeV} \) is taken, the coupling constants \( f_3/4\pi = 10^{-5} \) and \( f_3/f_1 = 10^{-3} \) lead to the observed masses of the muon and muon-neutrino. The existence of the leptonic strangeness gauge field of such a strength would affect, for instance, the Möller scattering of two muons within the order of 1\% of the total cross section.

\section{5. Discussions}

With respect to the stable baryons and mesons, we have obtained the mass formulas (2.7) and (2.15) which explain the observed spectra. At a glance these formulas seem to have a resemblance to those obtained by other authors.\(^3\)\(^4\)

It is noted, however, that in our formulas the coefficients of \( B^3, S^2 \) and \( I(I+1) \) are not constant, but the mass is determined as a root of the somewhat complicated transcendental equation. In the limit of \( A \gg m, (2.7) \) and (2.15) become

\[ m \approx A \exp \left\{ -\frac{1}{2\pi} \left[ \frac{f_3^2}{4\pi} + \frac{f_1^2}{4\pi} S^2 \right] - \frac{f_1^2}{4\pi} I(I+1) \right\} \]  
for baryons

and

\[ m^2 \approx A^2 \varepsilon \frac{3}{\pi} \left[ \frac{f_3^2}{4\pi} S^2 + \frac{f_1^2}{4\pi} I(I+1) \right] \]  
for mesons

respectively, where \( \varepsilon \) denotes the same as in (4.6), though these limiting masses do not correspond to the observed ones.

Next we should remark that the assumption of a vanishing bare mass is
not indispensable to our argument. Instead if we start from a suitable finite
mass, we get a similar conclusion with somewhat different values of coupling
constants and cutoff parameters.

Further if we take into account the strong $\pi$ and $K$ interactions of baryons,
our mass formulas are naturally modified by additional contributions due to their
self-interactions. So far as the assumptions are made of all the stable baryons
having the same parity, the coupling without derivative, and the common unre­
normalized coupling constants, the additional terms to the right-hand side of
the baryon mass formula (2.7) are positive and logarithmically divergent. The
contribution turns out to be at most $0.03\alpha_s^2/4\pi$ for the cutoff $A=m$ and depends
directly on neither $S$ nor $I$. Hence this effect is effectively equivalent to modi­
yfying the value of $\alpha_s^2/4\pi$ by a factor somewhat smaller than 1. The right-hand
side of the meson mass formula (2.15), however, gets the contribution from
the vacuum polarization effect which is negative and quadratically divergent.
We are not in a position to give a definite answer as the case stands, though
this effect might be considered to be amalgamated with the vacuum. Anyway,
strictly speaking, we would have to solve much more complicated simultaneous
equations than those considered above, but we are hoping that at least qualitative
features of our conclusion would not be greatly changed.

The application of our mass formulas to the resonant states has been unsuc­
cessful. A proper treatment of the strong interactions between the constituents
would be needed.

With regard to the lepton mass splitting, it has merely been pointed out
that a similar explanation is possible, if the quantum numbers are suitably as­
signed and the tensor interaction term is added to both the electromagnetic
interaction and the leptonic strangeness gauge field interaction. In the case of
leptons we have taken a rather large cutoff parameter $A=10$ BeV, while for the
strongly interacting particles the cutoff $A=1$ BeV. The argument about this
matter has also been made by Solomon.\textsuperscript{13}

In the present report we have no mention of the electromagnetic mass shift
for the baryons and mesons. This shift may be regarded as a result of the
small perturbation of the electromagnetic interaction, but in order to calculate
the shift correctly the details of the electromagnetic form factors must be known.\textsuperscript{19}
In the light of the gauge fields, this problem will be discussed in a separate
report.

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Note added in proof: V. E. Barnes et al. (Phys. Rev. Letters 12 (1964), 204) have recently
reported the observation of an event which they believe to be an example of the production
and decay of Gell-Mann's $\Omega^-$ hyperon. The observed mass of $\Omega^-$, 1685±12 MeV, is rather close to our
predicted values, in view of the use of our crude cutoff. Note that our estimation has been carried
out assuming that the $\Omega^-$ belongs to the spin 1/2 family of metastable baryons, while in the unitary
symmetry scheme Gell-Mann and others have arranged a decuplet so that the $\Omega^-$ together with
the resonances, $N_{3/2}^*$, $Y_{13}^*$, and $\Sigma_{15}^{\ast}$, compose the spin 3/2 family, their observed masses being in
agreement with those predicted by the Gell-Mann and Okubo formula.