Remarks on Broken $G_2$ Symmetry

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The $G_2$ symmetry has been proposed by many authors as an alternative of the $SU(3)$ in the strong interactions as well as in the weak interactions. We show in this note that the broken $G_2$ symmetry scheme contradicts to the notion of the so-called spontaneous breakdown or the self-consistent deviation.

As is well known, the seven baryons ($\bar{p}$, $n$, $\Sigma^+$, $\Sigma^0$, $\Sigma^-$, $\Xi^0$, $\Xi^-$), the seven pseudoscalar mesons ($K^+$, $K^0$, $\bar{p}^+$, $\bar{p}^0$, $\bar{p}^-$, $K^0$, $K^-$) and the seven vector mesons ($K^{*+}$, $K^{*0}$, $\rho^+$, $\rho^0$, $\rho^-$, $K^{*0}$, $K^{*-}$) are assigned to the $7$ dimensional representation. According to the formalism of spontaneous breakdown, the following self-consistency equations are obtained,

$$
\delta \mu_{(i)}^{(G)} = T^{(i)} \times \begin{pmatrix}
\delta m_{(i)}^{(G)} \\
\delta \mu_{(i)}^{(G)} \\
\delta \mu_{(i)}^{(G)} \\
\delta \mu_{(i)}^{(G)} \\
\end{pmatrix},
$$

where $\delta m_{(i)}^{(G)}$, $\delta \mu_{(i)}^{(G)}$ and $\delta \mu_{(i)}^{(G)}$ are bilinear irreducible tensors constructed from the field operators of the baryon and the antibaryon, the $p$-s mesons and the vector mesons, respectively; while $\delta G_{(i)}^{(G)}$ is a trilinear tensor constructed from those of the baryon, the antibaryon and the boson, if it is associated with the Yukawa type coupling. $T^{(i)}$ is a $4 \times 4$ matrix. $i$ runs over all the irreducible representations resulting from the direct product $7 \times 7$, namely $1(\pm)$, $7(-)$, $14(-)$ and $27(\pm)$, among which the first and the fourth are associated with symmetric tensors, while the second and the third with antisymmetric ones. $\mu_{(i)}^{(n)}$ is identically zero for $i=7(-)$ and $14(-)$ because of Bose statistics.

The spontaneous splitting says that

$$\det(T^{(i)}-1)=0 \quad (2)$$

for one of the above-mentioned representations, and that the symmetry breaks down according to that representation. If Eq. (2) holds for only one $i$, we are immediately led to a contradiction. Indeed, $i$ must be $27$ in order to realize the $K\pi$ and the $K^*\rho$ splittings. Since the baryons are subjected to the same violation, the $\Sigma N$ splitting never occurs although $\Sigma N$ splitting may be surely realized. Conversely, if one requires the $\Sigma N$ splitting (7 dimensional spurion, as is suggested by Behrends and Landovitz) the $K\pi$ and the $K^*\rho$ splittings are forbidden at least to the lowest order approximation.

One of the apologies will be that the 27 dimensional spurion in the meson families is induced by the 7 dimensional splitting of the baryons as the second order effect. It does not, however, look so plausible, since the $K\pi$ mass splitting is too large.

Another way is to require that Eq. (2) holds for two different representations. It will be more accidental than the usual spontaneous splitting. In that case an additional restriction is put on the strong interaction parameters. This version involves another unpleasant feature. It has been shown in the previous work that the spontaneous splitting predicts the corresponding light bosons. When it is applied to this exceptional case, we have 27-fold scalar mesons as well as 7-fold ones which are responsible for the $\Sigma N$ splitting. They are 34 scalar mesons in total. They are too copious in the light of the present experimental data. Thus, this looks hopeless.

We do not succeed in obtaining reliable mass formulas in the $G_2$ symmetry scheme. If we abandon the concept of the spontaneous splitting, we will have no plausible basis of choosing an irreducible tensor for the effective symmetry-violating interaction.
It is specific to the $SU_3$ symmetry that the mass splittings of the baryons and bosons can be simultaneously interpreted in spite of the difference between the statistics and their mass spectra. It is because the two 8 dimensional representations ($8_4$ and $8_5$) exist in the $SU_3$ and a transition is allowed between them.

In conclusion, the $G_2$ symmetry is very inferior to the $SU_3$ in the strong interactions at least from the viewpoint of broken symmetry.

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