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Water distribution system model calibration under uncertainty environments

S. Takahashi, J. G. Saldarriaga, M. C. Vega and F. Hernández

ABSTRACT

The calibration process of water distribution system models allows for accurate and reliable hydraulic analysis results. Thus, calibration is of utmost importance if adequate operation and maintenance model-based procedures are sought. However, in emerging economies, there is a series of factors that make it more difficult to construct accurate models, including very poor information management, unusually high leakages and the presence of a large number of illegal connections. While some of the model variables are assumed to be known under normal circumstances, these factors make it necessary to consider them for calibration as well. This paper presents a calibration methodology flexible enough to address such problems allowing the calibration of pipe diameter, roughness and minor losses, and nodal demands and leakages. A genetic algorithm was implemented as well as a constraint programming algorithm that makes direct use of hydraulic criteria to advance in the solution space. The methodology was tested on a real system in Colombia with a satisfactory outcome. The use of these techniques results in major reduction of calculation time and similar or superior results in comparison to manual methods.

Key words | constraint programming, genetic algorithms, water distribution system calibration

S. Takahashi
Departamento de Ingeniería de Sistemas y Computación,
Universidad de los Andes,
Carrera 1 Este No. 19A-40,
Bogotá,
Colombia
E-mail: *stakahas@uniandes.edu.co*

J. G. Saldarriaga
M. C. Vega
F. Hernández
Centro de Investigaciones en Acueductos y Alcantarillados (CIACUA),
Universidad de los Andes,
Carrera 1 Este No. 19A-40,
Bogotá,
Colombia
E-mail: *jsaldarr@uniandes.edu.co*;
m.vega222@egresados.uniandes.edu.co;
feli-her@uniandes.edu.co

INTRODUCTION

Water distribution system (WDS) models have become everyday tools for planners, designers, maintainers and operators. Due to accurate hydraulic analysis calculation methods such as the Gradient Method (Todini & Pilati 1987), WDS models have found applicability in design, performance evaluation, rehabilitation, enlargement, risk management, operation and failure recovery procedures, among others. Moreover, these models are increasingly becoming, not only planning tools, but also real-time decision-making tools.

However, WDS models are often dissimilar from the actual systems in terms of the different elements that constitute them. These differences have a direct impact on the model's hydraulic performance and the reliability of any simulation results. This problem is far more complicated in developing countries where water utilities have very poor customer and asset information management programs, as well as limited resources for adequate management policies. As a result, input information for the construction of models is mostly deficient, proper instrumentation is lacking and there are important operation problems that make the model parameter estimation even more complex.

Thus, the models are generally built from the original project drawings and other documentation if any is available. Even under the assumption that these correctly represent the original system's topology, there are different factors that progressively modify the various elements making the documents no longer accurate. Rapidly, the available documentation becomes outdated if no significant updating efforts are made. Some of the factors that alter the system's topology, amongst other critical variables for modelling, are listed below.

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Unreported interventions

Installed new pipes or replaced old ones that are not listed in the modeller's documents. The model would not include new pipes and nodes or would not use appropriate diameter, roughness and minor loss coefficient values.

Pipe deterioration

Through years of operation, pipes may change their diameter, roughness and minor loss coefficient values as a consequence of corrosion, abrasion, oxidation, deposition and incrustation processes.

Operation procedures

Valve, pump or hydrant operation can drastically affect the system's hydraulic response. Operation procedures can be applied frequently, causing even topological changes and thus making a static model useless under alternative operation schemes.

Inaccurate measurement of domestic consumption

Flow meters may be damaged or intentionally manipulated. In most cases, precise demand values cannot really be expected anywhere.

Defective customer data bases and illegal connections

Some users could have gained access to the water supply service without the water utilities' knowledge or control. Poor asset and information management policies make this a major problem in many cities around the world.

Water losses

Failures in pipe walls or joints cause leakages that add to the nodal demands. In developing countries, water losses can account for up to 50% of the total water introduced into the systems (Kingdom *et al.* 2006). Such percentage is unacceptable by any standard in developed countries.

All of the mentioned issues reduce the accuracy of a model which, from the starting point, could have had significant discrepancies from the system it intends to represent. This highlights the importance of calibration processes in order to make modelling results trustworthy. This way, the process of calibrating a WDS model should consist in the search of a model whose hydraulic behaviour differs as little as possible from that of the actual system through the adjustment of all unknown or uncertain variables. Nevertheless, this should be done in accordance to the modelling objectives and the available budget.

WDS model calibration has evolved from manual to automatically optimised methods. Traditional manual calibration methods repeatedly apply changes to some of the system's variables and, by means of hydraulic simulation, compare the model's results with the prototype's measured series. Both the modeller's judgement and experience plays a major role in the method's precision and efficiency. Still, the solution space is extensive: There are numerous possibilities for each of the variable values and exponential combinations of these values for all the system's elements. This makes calibration a NP-Hard problem (Saldarriaga 2007; Vega 2007). Intuitively and manually created calibration scenarios fall short in terms of the extent of the exploration efforts. Therefore, the use of alternative techniques and algorithms becomes imperative.

Current research focuses on proposing methods that deal with still incompletely resolved issues such as inclusion of calibration variables, measurement accuracy, model accuracy, variable uncertainty, friction models and algorithmic efficiency. Haestad *et al.* (2003) summarize publications dealing with WDS model calibration between 1980 and 2002. The most common approaches make use of genetic algorithms. This technique was introduced to WDS model calibration by Savic & Walters (1995). Genetic algorithms can be found, for instance, in the commercial package Darwin Calibrator for WaterCAD (Walski 2007). Darwin Calibrator uses pipe roughness, demands and, recently, leakages (Wu 2008) as the variables to be calibrated. Other authors proposing genetic algorithms-driven methods are Vítkovský *et al.* (2000), who calibrate pipe roughness and leakages complementing the genetic algorithm with the inverse transient method and Jamasb *et al.* (2008), who calibrate pipe diameters, Hazen-Williams coefficients and demands.

Other approaches include the minimum night flow method (García *et al.* 2006) for estimating leakages through extrapolation of measured data and application of mass conservation. In an implementation of the Levenberg-Marquardt algorithm, a roughness function dependent on the age of the pipe is calibrated rather than the roughness itself (Koppel & Vassiljev 2009). Notwithstanding, several adjustments are needed for the Levenberg-Marquardt algorithm to work correctly when dealing with large systems. Within these calibration techniques, it is a common practice to group calibration elements in order to reduce computation time and memory requirements (Mallick *et al.* 2002).

The mentioned approaches focus on the variables that are commonly the most uncertain. Other variables are assumed to be known with an acceptable accuracy and are not considered for calibration. However, this assumption may be too strong for WDSs in emerging economies, following the factors described above and the poor quality of the information available. Thus, in these cases, some of the variables that are traditionally thought to be known must also be subject to a calibration process. This article describes the implementation of a calibration methodology for pipe diameter, roughness and minor losses, and nodal demands and leakages.

The rest of the paper is organized as follows. In the calibration methodology section, the whole process is described highlighting the most important aspects. The two search methods used are described: a constraint programming algorithm and a genetic algorithm. In the results and discussion section, the results from testing the methodology on the WDS in the town of Candelaria, Colombia are presented. Finally, conclusions are drawn from the results and their implications and some future work guidelines are stated.

CALIBRATION METHODOLOGY

This section describes the proposed calibration methodology. Four critical aspects are treated: measurement at prototype system, selection of variables to be calibrated, determination of hydraulic resemblance between model and prototype and search method.

Prototype system instrumentation

An exact calibration method implies the measurement of flow and pressure series by means of the instrumentation of every pipe and node in the system. This is not an economically or technically viable approach due to the cost of instrument acquisition, installation and operation. Therefore, an instrumentation scheme must be sought where few measurement points are required, giving enough information to capture the system's hydraulic behaviour. The hydraulic variables that best describe system performance are flow on pipes and pressure on nodes. Thus, the instruments selected should directly or indirectly measure them.

Measurement points should be placed at key locations where information is most significant. Flow meters should be placed at every system or hydraulic sector entry point in order to control the complete mass entering and conforming metered district areas. Pressure meters are cheaper than flow meters and should be placed alongside them. Additionally, pressure meters should be placed downstream into the system balancing two opposing criteria: First, they should be placed on main routes where pipe diameter and flow values are larger to measure mean pressure rather than variations in a daily cycle; second, they should be placed so as to maximise total afferent area. The afferent area of a measuring point is the set of system elements whose variables affect the values being measured at that point. As these are mainly upstream elements, this criterion favours segments in the far ends of the WDS where pipes are smaller, more specific elements are targeted and pressure variations are the largest throughout the day.

While the first criterion optimises calibration for the most important elements, the second allows for the calibration of as many elements as possible. An optimal measuring layout maximises total afferent area while minimising measurement point count, making sure the most important elements are being taken into account. There have been several research projects on system instrumentation optimisation including that of Zhang & Huang (2009).

Calibration variables

The model variables to be calibrated are selected given their uncertainty as a result of the factors described in the introduction. None of the previous works allow for the calibration of all of them. The variables are the following.

Nodal base demands

These are the base value of constant output flow at the system's nodes. Demand curves are built from base demands and demand patterns which are series of multiplier values. Base demands should account for known and unknown (e. g. illegal) output flows.

Leakages

These are modelled in a WDS model through emitters. Emitters are pressure-driven output flows at nodes. Emitter flow (Q_e) is calculated by using Equation (1), where h is the nodal pressure, C is the emitter coefficient and x_e is the emitter exponent. x_e usually takes the value of 0.5 for circular orifices. Thus, the only variable to be calibrated is C, the emitter coefficient.

$$Q_e = C \cdot h^{x_e} \tag{1}$$

Diameter

The real inner diameter of pipes. Along with the deterioration processes described before, the deficient quality of data bases and documents in poorly managed water utilities add to the uncertainty of this variable.

Roughness

The mean thickness of pipe wall irregularities (k_s) used in the Colebrook-White friction factor equation. These irregularities may be inherent to the pipe material or caused by external agents as described in the introduction.

Minor loss coefficient

Head-loss due to accessories in a pipe (h_m) is calibrated through the minor loss coefficient (k_m). This coefficient includes the effect of partially closed valves, situation that is not uncommon as are the cases where this is unknown by the WDS operators.

Base demands and leakages are classified as mass variables within the presented methodology. Diameter, roughness and minor loss coefficient are classified as energy variables.

Hydraulic resemblance

It is important to measure how well a WDS model represents the hydraulic performance of the prototype. Hydraulic performance is best described by flow values on pipes and pressure values at nodes. Consequently, hydraulic resemblance should be measured comparing model and prototype flow and pressure series. This can only be done where prototype series are available. Model series are obtained by running an extended period hydraulic analysis. The mean relative error of a series (E_s) can be calculated using Equation (2).

$$E_s = \frac{\sum_{i=1}^{NT}\left(\frac{|x_{p,i}-x_{m,i}|}{x_{p,i}}\right)}{NT} \tag{2}$$

$x_{p,i}$ is the prototype measured value at time step i, $x_{m,i}$ is the modelled value at time step i and NT is the number of time steps or periods of the series. x can be either flow or pressure. The model error can be calculated from individual series errors. A weighting factor p_j can be used for series j. The mean weighted relative error of the model (E_m) can be calculated using Equation (3).

$$E_m = \frac{\sum_{j=1}^{NS} p_j \cdot E_{s,j}}{\sum_{j=1}^{NS} p_j} \tag{3}$$

$E_{s,j}$ is the error for series j and NS is the number of series available. E_m can be used to measure hydraulic resemblance between the model and the prototype, allowing for the comparison between different candidate calibrated models.

Calibration methodology

The calibration methodology portrayed herein is further explained by Jurado (2007) and has been applied in various projects throughout Colombia by CIACUA[1]. The methodology is divided into five stages: baseline construction, sensitivity analysis, mass calibration, energy calibration and validation. These steps are explained below.

[1] CIACUA: Centro de Investigaciones en Acueductos y Alcantarillados (Water Distribution and Sewer Systems Research Centre).

Baseline construction

Flow and pressure series are analysed and prepared. The initial model is built upon available information. Values for the different variables are manually adjusted whenever reasonable. This includes the replacement of nominal with real inner diameter values and the assignment of minor loss coefficient values per unit length from valves and other accessories. The model obtained in this step is called Baseline 1.

Sensitivity analysis

Calibration variables are modified randomly in order to characterize the impact on flow and pressure curves. This is done to establish or validate variable priority and calibration order.

Mass calibration

The actual process of calibrating base demands and leakages. This can be done for all variables simultaneously or individually using search methods, which are explained in the following sections. Several iterations are needed until the results are satisfactory. The model obtained in this step is called Baseline 2.

Energy calibration

The actual process of calibrating pipe diameter, roughness and minor loss coefficient. The process is the same as with mass calibration. The model obtained in this step is the Unified Model.

Validation

In order to verify results, the calibrated model (Unified Model) is tested using flow and pressure series different from the ones used before.

Constraint programming

This programming paradigm is used to find solutions to problems from a set of constraints that the solution must meet. The solution is the assignment of values to the problem variables so that the constraints are met. Constraint programming has been found to be most successful for discrete variable domains (Abdennadher & Frühwirth 2003).

Solution algorithms must define a search heuristic where information available leads to decision-making. The information available is a set of input constraints and a set of deduced constraints. On every iteration, the search heuristic may infer new information, which adds to the deduced constraints. Ideally, this constraint propagation reduces variable domains to a single value.

In the problem of calibration, values must be found for the calibration variables mentioned above in order that hydraulic constraints are met. Constraints can be defined in terms of hydraulic resemblance, given that the solution values must constitute as accurate a model as possible. Calibration variables must be made discrete by defining a variation range around the initial value. A number of possible discrete values are defined within this range.

The search heuristic takes the error E_m from the model curves. In the case of energy curves, if the error suggests that the model has an average higher pressure value than the prototype, a variable must be changed in such a way that head-loss increases. On the other hand, if the error suggests that the model has an average lower pressure, a variable must be changed to decrease head-loss. In contrast with metaheuristic algorithms, hydraulic criteria are not only used for establishing the fitness of a candidate solution, but also as the direct driver of modifications themselves. The selection of the variable is done randomly. Variable changes are only made by single steps in the domain. If the change accounted for a smaller E_m, it is kept. Otherwise, it is reversed and a new search is performed. The algorithm stops when excessive iteration is reached without further improvements.

Genetic algorithm

Genetic Algorithms are bio-inspired algorithms that abstract concepts from Darwin's evolutionary theory in order to find satisfactory solutions for problems where solution space is extensive. They are commonly used in engineering problems such as WDS design. Implementation of a genetic algorithm for the problem of calibration requires the definition of individual genotypes, individual phenotypes, mutation, fitness measurement, selection and reproduction processes.

Since the goal is to find a WDS model whose hydraulic behaviour resembles the prototype's, different model configurations are the algorithm's individuals. Genotypes, the features subject to alteration through inheritance and mutation, are the calibration variables of each individual. Phenotypes, the features that determine fitness and are the manifestation of genotypes, are the simulated flow and pressure curves of each individual. An individual's fitness is measured through its hydraulic resemblance with the prototype. In other words, E_m is the objective function of the algorithm which needs to be minimised.

The first generation can either be produced randomly or by means of mutation of the species' Adam: the initial model. Mutation occurs with a defined probability on each of the genotype's genes and by a random variation within a specified range. More specifically, a mutated individual resembles the original but has certain variables, like pipe roughness, modified randomly. At each generation, the individuals with better fitness (or smallest E_m) are selected for reproduction. The reproduction process can be done by several methods where the parent's genotypes are combined into new individuals. This process includes mutation to avoid stagnancy in local minimum.

As the algorithm spawns new generations of model configurations, these are expected to have a greater fitness than their predecessors. In the end, the individual that is most adaptable is chosen as the calibrated model. Consecutive breeding of new generations is done through selection and reproduction processes until a user-specified generation count is reached.

RESULTS AND DISCUSSION

The two search algorithms were implemented in the WDS hydraulic modeller software Redes, an application developed by the CIACUA. Redes includes static and dynamic hydraulic and water quality analysis as well as design, resilience, calibration, rehabilitation, skeletonization and transient modules.

The constraint programming algorithm was implemented for energy calibration only, given the broad range of possible demand and emitter coefficient values a node can take. Although it has a suitable heuristic search method, the algorithm cannot be considered purely a constraint programming one for two reasons. First, there are no practical constraints amongst variables to implement constraint propagation. In other words, variables are largely independent from each other. Second, it is practically impossible to reach a solution where there is no error so as to satisfy a constraint of perfect hydraulic resemblance. That is, only approximate solutions can be obtained.

The genetic algorithm was implemented allowing for the calibration of each variable independently. Several parameters can be adjusted by the user such as generation count, generation population, reproduction method, initial generation breeding method, mutation probability and allowable variation range. In both algorithms, calibration is executed using user-picked series on every run.

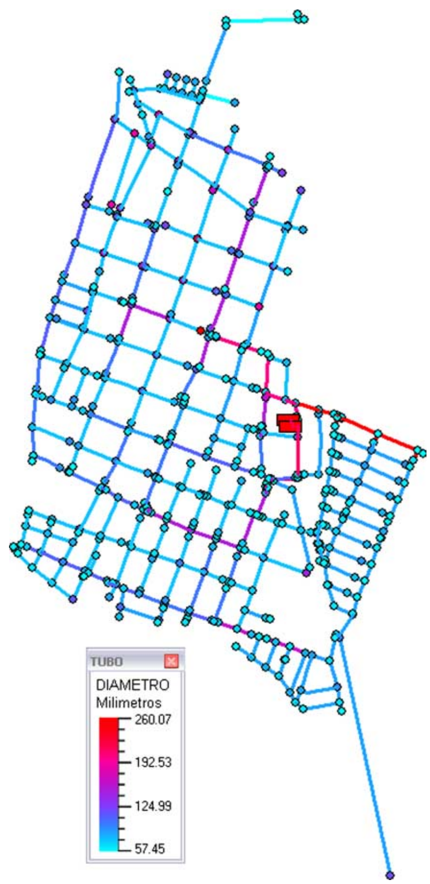

Figure 1 | Candelaria's WDS. Subscribers to the online version of Water Science and Technology: Water Supply can access the colour version of this figure from http://www.iwaponline.com/ws.

Table 1 | E_m and approximate time for resulting models

Model	E_m (%)	$E_m/E_{m,\text{BL1}}$ (%)	Approximate time (h)	App. time/App. time$_{\text{manual}}$ (%)
Baseline 1	20.068	100	–	–
Unified (manual)	6.298	31	192*	100
Unified (CP)	7.734	39	5:15	2.7
Unified (GA)	5.212	26	12:45	6.6

*The model was calibrated manually by a team of two engineers. Although it may seem a long time, two aspects must be taken into account. First, traditional approaches calibrate up to two variables whereas this process addressed five of them: any additional variable introduces the problem of having to find balance between modifications of different values producing similar hydraulic effects. Second, the model was calibrated with no previous training on such multivariable approach. Once trained, the same team could perform the calibration in less than half this time.

Case study

The calibration methodology was tested on a real WDS in the town of Candelaria which is located in the southwest of Colombia. Its hydraulic model is composed of 567 pipes for a total of 23.3 km, 463 nodes, two reservoirs and counts with approximately 3,800 customers. According to flow measurements at the reservoirs and at consumption points, non-revenue water accounts for the 39.5% of the total mass entering the system in Candelaria. The model is shown in Figure 1.

The Candelaria's WDS was calibrated both manually and using the search methods described in this article. Results for E_m and approximate time are presented on Table 1. GA stands for genetic algorithm and CP for constraint programming. $E_{m,\text{BL1}}$ stands for the E_m of the Baseline 1 model. The time is measured in working hours of WDS modelling engineers.

The E_m for the Baseline 1 model shows the importance of the calibration process: model predictions using a model without calibration will have a mean error of around 20%. Note that the Baseline 1 model has already been subject to an initial manual depuration process and so the E_m for the initial model must be even bigger. All the methodologies reduce significantly the model inaccuracy as seen through its E_m. Simulation results obtained from the unified models can be trusted with a much smaller uncertainty.

A significant time reduction was accomplished through the use of the automatic search methods, especially with the constraint programming algorithm. Although the search itself is completely automated, the role of an expert is still critical in the sensitivity analysis and the unification steps. This follows the fact that each model usually needs a different approach in order to be calibrated properly. For example, in old WDSs with high pressures, leakages may be the most significant variables to be calibrated, while in industrial systems with a lot of accessories, minor losses could become even more important than roughness. Similarly, in cities where illegal connections are common and only shy controls are exerted, nodal demands are probably among the most uncertain variables. The proposed methodology grants the flexibility necessary to address these diverse scenarios unlike previous studies where some of the variables are assumed to be known.

Although the constraint programming algorithm showed a higher E_m than the genetic algorithm, it is still a sound candidate for calibration scenarios where a fast methodology is needed that produces acceptable results. This may be the case of the calibration of huge models, where the number of hydraulic simulations needed by the genetic algorithm may take a very long time. This is modest evidence of the strength of hydraulically-based approaches that may lead to good results while avoiding the sometimes cumbersome stochastic nature of metaheuristics and of other similar techniques.

CONCLUSIONS

A calibration methodology for water distribution system models was presented, taking into account the different factors that make the variables uncertain in several environments such as emerging economies. The methodology spanned measurement, calibration variable selection, calibration order and two Artificial Intelligence search methods: constraint programming and genetic algorithms. The constraint programming algorithm has a weaker stochastic nature than the genetic algorithm, compensated by the use of hydraulic criteria. Using the methodology, calibration is allowed for pipe diameter, roughness and minor losses, and for nodal demands and leakages.

The case study reiterates the importance of calibration showing that baseline models do not appropriately represent the hydraulic behaviour of the prototype systems. The mean weighted relative error of the model (E_m) was 20.068%. Results from calibration show a considerable time reduction in comparison to manual methods, especially when using the constraint programming algorithm. Moreover, the rapid learning curve of using the automated methods is reason enough to prefer them. Although the constraint algorithm spawned acceptable results (E_m of 7.734%), only the genetic algorithm (E_m of 5.212%) was able to produce a more accurate model than that obtained using manual calibration (E_m of 6.298%). However, constraint programming still could be used in certain cases where time is of the essence and computational resources are limited.

The constraint programming algorithm performance suggests that hydraulically-driven algorithms may overcome stochastic approaches in terms of computational efficiency. Thus, future work should expand this or other similar methods in order to reach better results maintaining time and memory efficiency. More particularly, the constraint programming algorithm should adopt the use of practical techniques such as the identification and manipulation of single hydraulic sectors in the system. Multi-variable calibration should also be addressed by automating the selection and unification steps in the methodology. However, this should be done without sacrificing the achieved flexibility or the capability of experts on taking major decisions throughout the whole process.

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