  Journal of Hydroinformatics | 07.1 | 2005

Development of a discharge equation for side weirs using artificial neural networks

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ABSTRACT

Flow over a side weir is one of the more complex flows to simulate in one-dimensional unsteady flow analysis. Various experiments have been applied, but no agreement is apparent in the literature about the best method. In this study, an Artificial Neural Network model has been used to extract a discharge equation for side weirs which accurately estimates overflow discharges. The proposed methodology gives the advantage of accounting for both the geometric and hydraulic characteristics of the overflow structure. The developed model is calibrated and validated using experimental data. Model calibration is achieved by using a Multi-Layer Perceptron (MLP), trained with the back-propagation algorithm. In order to highlight the advantage of the developed model over an existing model widely in use, the model's performance is evaluated according to three comparison criteria. The provided results clearly reflect the ability of the developed model to overcome the weakness of conventional models.

Key words | artificial neural network, non-linear model, overflow discharge, side weir overflow

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INTRODUCTION

The hydraulic performance of overflow structures and particularly of side weir overflows is quantitatively and qualitatively difficult to model. These structures are often the site of complex hydraulic phenomena, and compared to frontal weirs their study is relatively recent. The first difficulty is to represent the surface profile along a side weir. The second is to determine the discharge coefficient. This coefficient depends not only on the head on the weir and the overflow structure geometry but also on flow conditions and pipe slopes upstream and downstream of the side weir.

The earliest studies of the hydraulic characteristics of side-channel weirs were concerned primarily with the analytical prediction of the effects of the weirs on the longitudinal water surface profile in the channel for the idealized case of a rectangular channel with a vertical weir plate and a constant discharge coefficient (De Marchi 1934; Frazer 1957; Henderson 1966). Some other studies on the evaluation of the discharge over the side channel weirs are those of Subramanya & Awasthy (1972), El-Khashab & Smith (1976), James & Mitri (1981) and Hager (1987). The general character of flows over side weirs is then understood. However, quantitative values for key coefficients are not well known. In addition, most of the work reported on flow over side weirs is for sharp-crested weirs, whereas most prototype systems use broad crested weirs. Very little information is available concerning the effect of the weir crest thickness on the side weir hydraulics. The report by Tynes (1989) contains measurements for flow over a broad-crested side weir; however, his analysis was limited and most of his results are specific to the particular configuration tested. Moreover, Krajewsky *et al.* (2000) shows that the discharge coefficient is a major source of uncertainty in estimating discharge, a fact reflecting both the lack of standardization and the great diversity of overflow structures. Various experiments have been applied, but no agreement is apparent in the literature about the best method. Besides, most of the previous work has considered only forward flows, and no detail was given on the

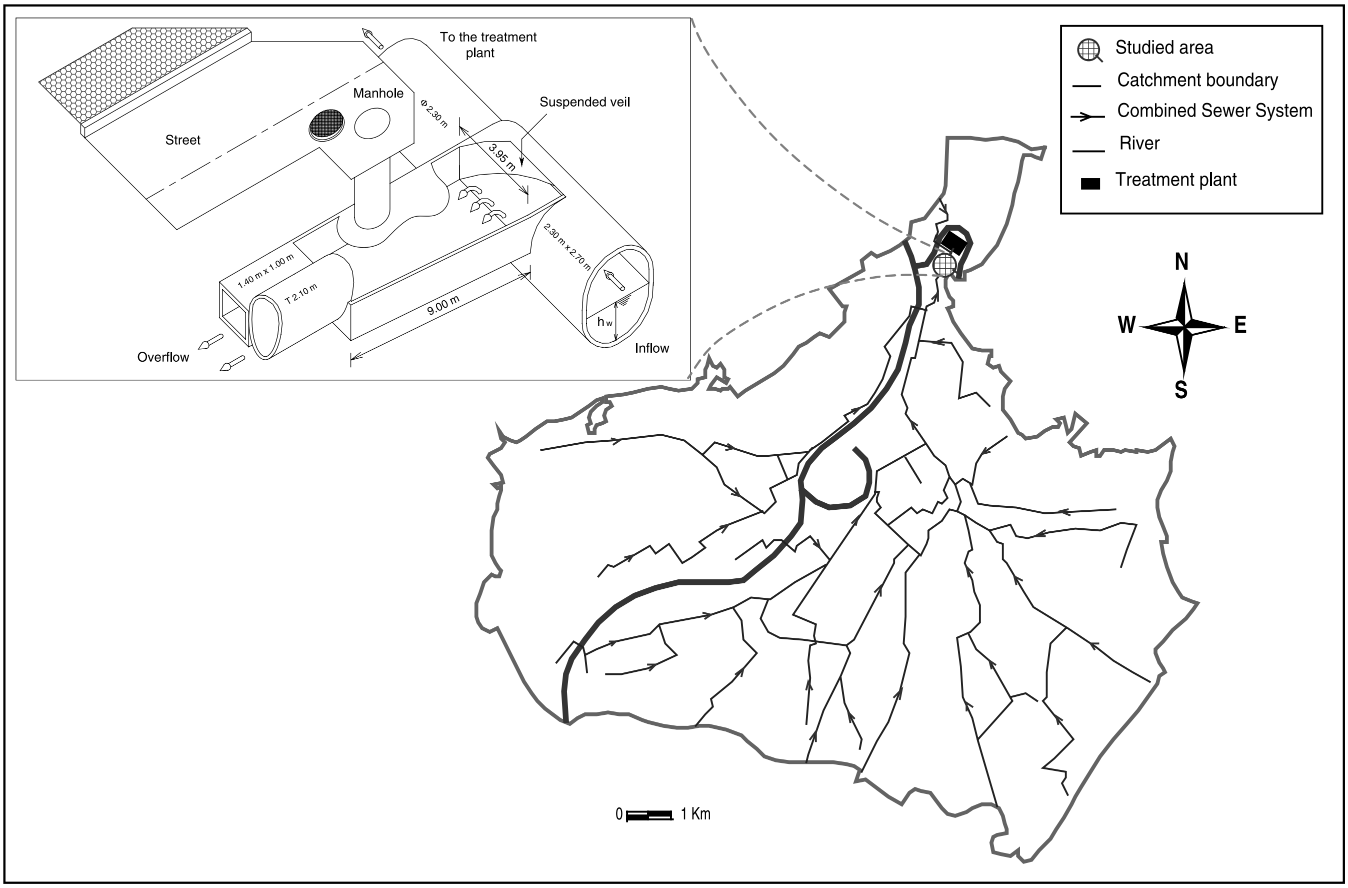


Figure 1 | Studied area location and side weir overflow description.

hydraulic behavior of side weirs in cases of surcharging and backwater effects.

Taking into account these considerations, this study focuses on the development of a non-linear model of side weir overflows using the Artificial Neural Network (ANN) approach. Referred to as ‘‘black box’’ models, the main advantages of neural network models, compared to conventional simulation models, include their computational speed and ability to learn the relationship between sets of inputs and outputs, without a prior knowledge of the underlying physical process that connects them (Price *et al.* 1998; Hall & Minns 1998).

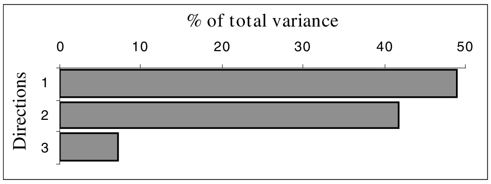
The main goal of this study is to design an easy-to-run and fast, but still accurate, model of side weir overflows using neural networks. In this paper, experimental data including sewer flow characteristics near the studied side weir overflow are at first analyzed. As a next step, the resources of ANNs are explored for the approach implementation. Results of the comparison between the developed model and the De Marchi model as well as some concluding remarks on the potential of the developed approach are included in the final section.

EXPERIMENTAL SITE

Description

The side weir overflow under study is located at the outfall of the Lille urban catchment (North France). The catchment area is approximately 155 km^2, mainly drained by combined sewer systems. As illustrated in Figure 1, the side weir overflow is fitted out along the side of the

	Eigenvalues	% variance	Plurality %
F1	1.96	49.12	49.12
F2	1.67	41.86	90.98
F3	0.29	7.48	98.46



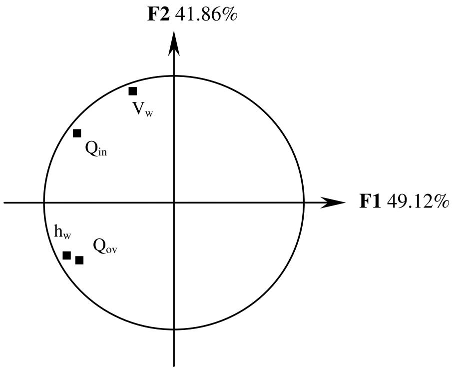


Figure 2 | Results from Principal Component Analysis.

main collector. The weir crest elevation is 1.67 m and the weir length is 3.95 m. The analysis of the sewer system hydraulics upstream of the treatment plant confirmed the fact that the studied side weir overflow always operates under backwater effects. This is due to the operation mode of the pumping station located downstream of the side weir.

Available measurements

The data used in this study are collected from two measuring points in the studied area. The first located in the main collector, upstream of the side weir, provides water depth (h_w), water velocity (V_w) and inflow (Q_{in}) measurements. The second in the discharge collector allows overflow discharge (Q_{ov}) measurements for the same periods. All the data which are available from the Meteorological Section of Lille Urban Community, corresponding to 25 rain events, are collected at 5-minute intervals.

METHODOLOGY

Statistical analysis

In order to analyze the linear correlation between the different flow characteristics measured near the side weir overflow, a Principal Component Analysis (PCA) was undertaken. The studied variables are the upstream water depth (h_w), water velocity (V_w), inflow (Q_{in}) and the overflow discharge (Q_{ov}). The total number of observations is 1460. The basic goal in PCA is to reduce the dimensionality of the data. This can be achieved by finding p principal directions $F_1, F_2, \ldots, F_p$ such that the first direction explains most of the variance of the data. Having found this direction, the second direction is constructed such that this direction explains most of the remaining variance. This ''iteration'' is repeated to find the third and the fourth principal directions. Mathematically it turns out that these principal directions are formed by the eigenvectors of the covariance matrix of the data. The projection into a two-dimensional space, as illustrated in Figure 2, is useful for data visualization. It appears that the first two principal directions (F_1 and F_2) can be considered as prevalent directions, since they explain more than 90% of the total variance.

The correlation coefficients generated by PCA are those exposed in Table 1. In general, the optimal

Table 1 | Correlation matrix from PCA

Variables	h_w	V_w	Q_{in}	Q_{ov}
h_w	1.00	−0.19	0.39	0.72
V_w	−0.19	1.00	0.76	−0.18
Q_{in}	0.39	0.76	1.00	0.26
Q_{ov}	0.72	−0.18	0.26	1.00

correlation value is unity and a value smaller than 0.70 is assumed to be problematic. Taking into account the weak correlations between either the inflow (Q_{in}) and the overflow discharge (Q_{ov}) or the upstream water depth (h_w) and the overflow discharge (Q_{ov}), the studied variables can be considered as linearly independent. Such a result confirms the fact that a linear relationship does not allow an accurate estimation of overflow discharges, at least in this case study where the side weir always behaves under backwater effects.

Conceptual approach

Proposed model

Considering the results from PCA, it was therefore decided to see whether a non-linear ANN model would perform any better on the overflow discharge estimation. The developed model is based on the study of the following relationship:

$$Q_{ov} = aH^b \tag{1}$$

where Q_{ov} is the overflow discharge, H is the upstream head on the side weir, and a and b are the calibration parameters.

The choice of such an approach over an inflow–overflow discharge relationship lies in the fact that this latter does not make it possible to take into account backwater effects. As mentioned previously, this configuration is likely to occur when pumping station capacities, at the downstream of the side weir, are exceeded.

Currently, ANNs are being used for modeling, especially when no accurate physically based model can be built either because the processes are not known or are too complex to reproduce. In the context of sewer system hydraulics, neural networks have been successfully used for sewer flow and water quality simulations (Proano *et al.* 1998; Price *et al.* 1998; Morshed & Kaluarachchi 1998). Since a neural network can only learn from examples, it cannot learn more from the information than is presented to it in a defined training set. Particular attention has therefore been given to collecting a representative data set. The events to be used are selected taking into account that they must be representative for all sewer flow

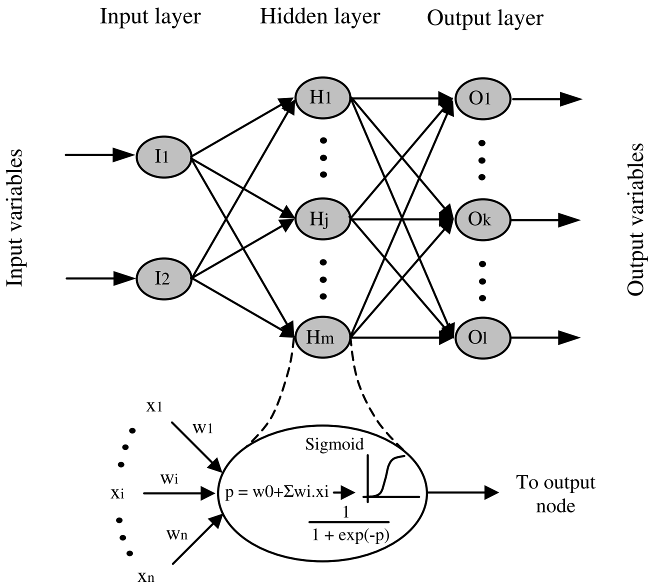


Figure 3 | Typical neural network architecture and neuron activation principle.

conditions including surcharging and backwater effects. This process of selection gives a complete well-mixed set of events to be used for both the training and validation sets as well as for the testing set.

Neural network software used in this study was the *Neural Connection* developed by SPSS Inc. and Recognition Systems Inc. Of the many ANN architectures available within *Neural Connection*, the "standard" multi-layer perceptron (MLP), trained with the back-propagation algorithm, was chosen. A typical architecture of the neural network model is given in Figure 3.

As indicated above, the ANN model is used here for the development of a non-linear model of side weir overflows by identifying the relationship between the upstream head on the side weir (H) and the overflow discharge (Q_{ov}). Subsequently, the input to the neural network model is the upstream head on the weir (H) and the output is the overflow discharge (Q_{ov}).

Model calibration

Among the 25 rain events which were available, 19 events (1100 observations) were used for the neural network model calibration. This phase groups together both training and validation sets. The 6 remaining events (360 observations), referred to as the testing data set, were used for model performance evaluation.

An important aspect in the development of an ANN is to ensure the network extracts the necessary features from the data. This is referred to as ''training'' the ANN. Training involves calibrating the parameters of an ANN using part of the available data set, referred to as the ''training data set''. During training, the data are presented respectively to the network until the parameter values are determined such that the network output adequately reproduces the training data. The ''validation data set'' is used to check the progress of the network to define when training should be stopped; that is, to choose the best solution. To do this, after every training iteration the validation data set is passed through the network and the error over the data set is calculated. The best set of weights are defined as those that produce the lowest error over the validation data set.

According to the selected neural network architecture, the activation function is of a sigmoid type in the hidden layer but a linear activation function was found necessary for the output layer. The use of a sigmoid function was to enable non-linearity of the network. For the output layer, the choice was made on a linear function because a sigmoid function forces an output to be in the range of 0–1. This condition is undesirable for this study since the maximum overflow discharge value usually is unknown. The difference between the actual and the desired output values is measured and the connection weights are changed so that the outputs produced by the neural network become closer to the desired outputs. This is achieved by a backward pass during which connection changes are propagated back through the network, starting with the connections to the output layer and ending with those to the input layer. The best set of weights are defined as those that produce the lowest error over the validation data set. Note that 80% from the available data for model calibration (880 observations) are used for the training set and 20% (220 observations) for the validation set. Once the best solution was found, the weights are fixed, and no further training takes place. The optimum values of the connection weights are illustrated in Figure 4.

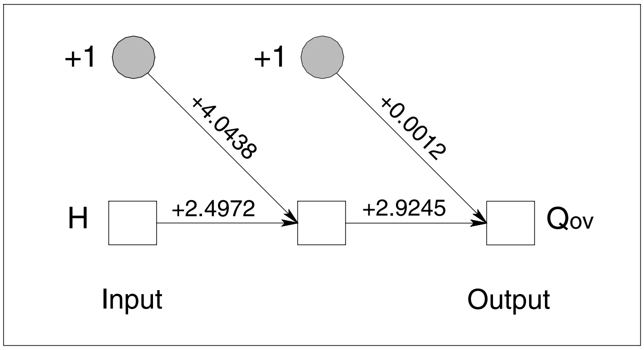


Figure 4 | Configuration of the connection weights.

In order to adapt neural network model output to the basic form of the discharge equation (7), the model input was transformed by the logarithmic function as follows:

$$x = \ln(H + c) \quad (2)$$

where c was set to 10^{-4} just to avoid the entry of zero head on the side weir in the ln function.

Relationship induced by the ANN

The output from the ANN model can be expressed mathematically as:

$$Q_{ov} = 0.0012 + 2.9245 \left(\frac{1}{1 + \exp\{-[4.0438 + 2.4972\ln(H + 10^{-4})]\}} \right) \quad (3)$$

which can be reduced to:

$$Q_{ov} = \frac{2.925}{H^{2.5} + 0.0175} H^{2.5} \quad (4)$$

The calibration parameters (a and b) of the developed model defined in Eq. (1) will then have the following expressions:

$$a = \frac{2.925}{H^{2.5} + 0.0175} \quad (5)$$

$$b = 2.5 \tag{6}$$

Referring to the basic form of the discharge equation,

$$Q_{\text{ov}} = C_w L\sqrt{2g}H^{1.5} \tag{7}$$

where Q_{ov} is the overflow discharge, C_{w} is the discharge coefficient, L is the length of the weir, g is the gravitational acceleration and H is the head above the weir, the relationship induced by the ANN model can be written as:

$$Q_{\text{ov}} = \frac{0.167H}{H^{2.5} + 0.0175} L\sqrt{2g}H^{1.5} \tag{8}$$

The discharge coefficient will therefore have the following expression:

$$C_{\text{w}} = \frac{0.167H}{H^{2.5} + 0.0175} \tag{9}$$

It is quite obvious that the discharge relationship induced by the ANN model is non-linear and the discharge coefficient is non-constant. The criteria used to evaluate the performances of the developed model are discussed in the following subsection.

Model performance criteria

Model performances are evaluated using the remaining part of the available data corresponding to the 6 rain events (360 observations) which have not been introduced in the calibration phase. To achieve this task, three criteria are used. As the accurate estimation of peak flows is of particular interest for the sewer system hydraulics, the weighted quadratic deviation (WQD) (Petit *et al.* 1998) is selected. This criterion can be specified as:

$$\text{WQD} = \frac{\left(\sum_{i=1}^{n_t} Q_i\hat{Q}_i\left(Q_i - \hat{Q}_i\right)^2\right)^{1/2}}{\sum_{i=1}^{n_t} Q_i\hat{Q}_i} \tag{10}$$

where Q_i and $\hat{Q}_i$ are respectively the measured and simulated overflow discharges at time step i and n_t is the total number of time steps for each hydrograph. The WQD criterion has the following advantages:

- it is dimensionless;
- it grants a larger weight to the significant discharges thanks to the product $(Q_i\hat{Q}_i)$;
- it gives the same deviation if we invert the two hydrographs.

As it shows the global goodness of the fit, the WQD criterion is difficult to interpret independently. Therefore, an easy to interpret and dimensionless criterion has been included. This is the peak discharge criterion (PDC) which can be written as:

$$\text{PDC} = \frac{Q_p - \hat{Q}_p}{Q_p} \tag{11}$$

where Q_p and $\hat{Q}_p$ are respectively the measured and the simulated peak discharge. For both the WQD and the PDC, a value equal to zero shows a perfect model. To estimate the efficiency of the fit, the R^2 criterion is also considered. The optimum R^2 value is unity and an R^2 smaller than 0.7 corresponds to a very poor fit.

In order to highlight the improvements given by the developed approach, the results provided by the developed model of side weir overflow are compared to those given by an existing model (the De Marchi model) which uses the standard weir equation:

$$Q_{\text{ov}} = \frac{2}{3}C_{\text{dm}}L\sqrt{2g}H^{1.5} \tag{12}$$

where C_{dm} represents the De Marchi coefficient.

The De Marchi coefficient is empirically defined according to the Froude number (*Fr*) at the upstream of the side weir (El Khashab & Smith 1976). The Froude number is specified as:

$$Fr = \sqrt{\frac{Q_{\text{in}}^2 B}{gS^3}} \tag{13}$$

where Q_{in}, B and S are respectively the inflow, the top width of the flow cross section and the wetted cross

Table 2 | Empirical formulation of the De Marchi coefficient according to the Froude number (El Khashab & Smith, 1976)

Froude number value	De Marchi coefficient value
$Fr < 0.6$	$C_{\mathrm{dm}} = 0.611\sqrt{1 - \dfrac{3Fr^2}{Fr^2 + 2}}$
$0.6 < Fr < 1$	$C_{\mathrm{dm}} = 0.45 - 0.06(Fr - 0.6)$
$1 < Fr < 1.8$	$C_{\mathrm{dm}} = 0.95\sqrt{2 - \dfrac{3Fr^2}{Fr^2 + 2}}$
$1.8 < Fr$	$C_{\mathrm{dm}} = 0.632 - 0.018(Fr - 1.8)$

Table 3 | Model WQD, PDC and R^2 statistics for the testing data set, De Marchi model (DM) and developed model (PM)

	WQD%		PDC%		R^2	
Event	DM	PM	DM	PM	DM	PM
1	10.19	0.51	39.01	0.01	0.96	0.99
2	30.49	5.95	53.55	− 12.51	0.95	0.97
3	30.79	4.44	41.45	− 2.11	0.92	0.93
4	21.67	5.26	56.99	− 5.00	0.92	0.94
5	15.97	1.47	55.77	− 1.79	0.95	0.95
6	19.87	12.47	55.14	− 6.20	0.95	0.95

section area, measured upstream of the side weir, and g is the gravitational acceleration.

According to the Froude number (Fr) value, the De Marchi coefficient can have four different expressions as shown in Table 2.

RESULTS AND DISCUSSION

As mentioned previously, the model's performances is evaluated on the 6 rain events which have not been introduced in the calibration phase. According to the selected criteria, the statistics summarized in Table 3 provide quantitative information on the performances of the two models: the De Marchi model (DM) and the developed model (PM) for the testing data set.

It is very interesting to observe that, in general, the developed model provides more accurate results than the De Marchi model for the observed events. As can be seen from Table 3, the WQD ranges between 0.5% and 6% for the proposed model, whereas the De Marchi model has WQD values ranging from 10% to 31%.

As discussed earlier, the PDC is a better indicator than the WQD to measure the model performance on the peak flows periods. Examining Table 3, the PDC statistics reveal that the De Marchi model significantly underestimates the peak flow discharges. Conversely, the developed model significantly improves the overall estimation accuracy. The performance of the developed model is clearly superior than the De Marchi model for the peak discharge estimation. Indeed, the developed model improves the peak discharge estimation accuracy on average by 91% by comparison to the De Marchi model.

In order to give some indications of the model's performance, a selective plot of overflow discharge simulations for one event taken from the testing data set (event 5), is given in Figure 5. This figure presents also the R^2 statistics for both the developed model and the De Marchi model. In general, an R^2 value greater than 0.9 indicates a very satisfactory model performance, while an R^2 value in the range 0.8–0.9 indicates a fairly good model, and values less than 0.8 indicate an unsatisfactory model.

It is very interesting to note that the results provided by the developed model appear to be more closely grouped around the line of equal values (the 45° line) showing sufficient model accuracy. Conversely, an important bias appears to be evident in the overflow discharge estimation from the De Marchi model. While the R^2 values are the same (0.95) for both the developed model and the De Marchi model, the previous results confirm that the latter is an unsatisfactory model according to the WQD and PDC statistics. It is therefore necessary to account for the "slope" of the regression line when interpreting the R^2

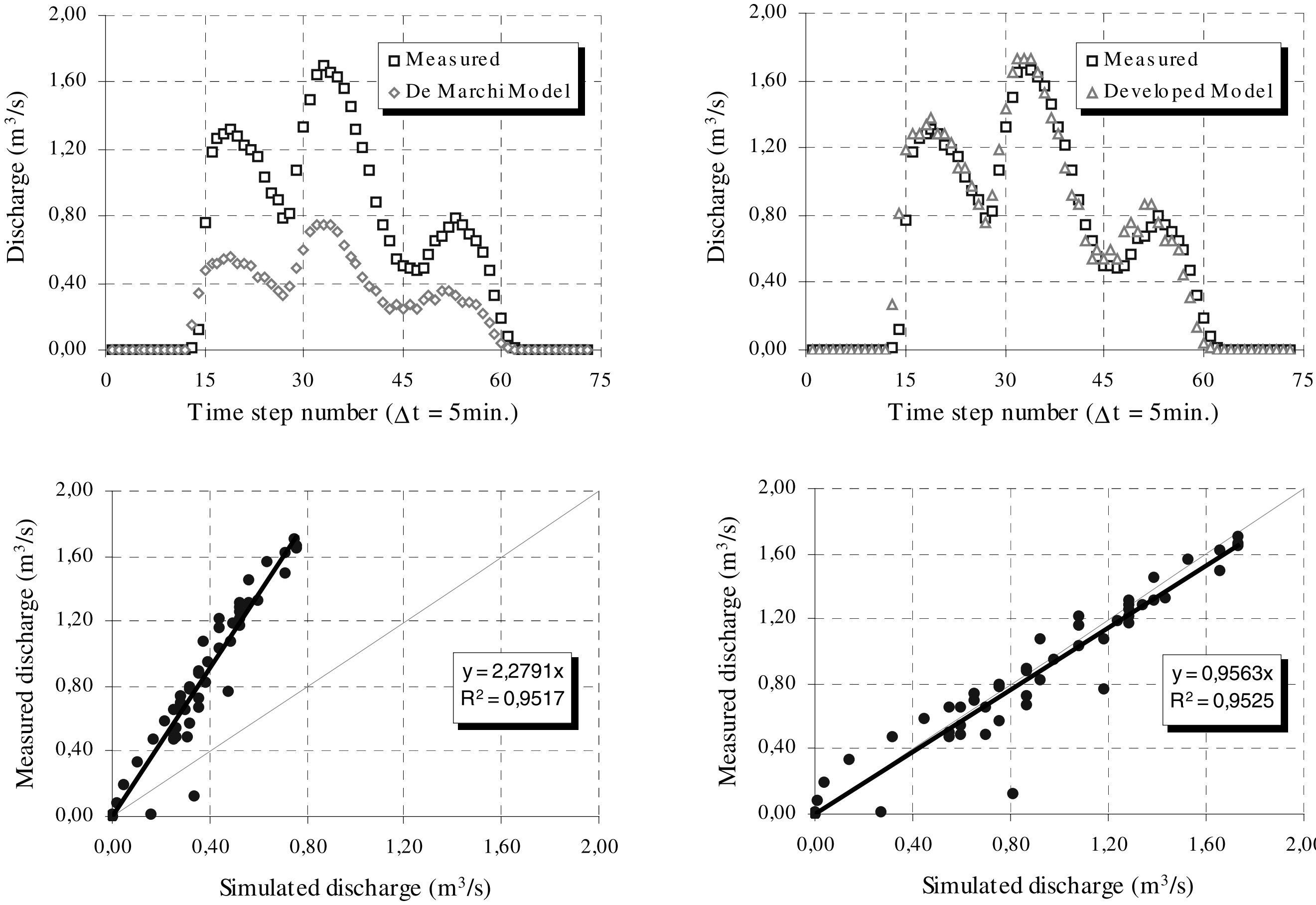


Figure 5 | Comparison between measured and simulated overflow discharges (event 5) for both the De Marchi model and the developed model.

statistics. As can seen from Figure 5, the regression line has slope values of 2.27 and 0.95 for the De Marchi model and the developed model, respectively, where a value equal to unity shows perfect model accuracy.

CONCLUSION

In this paper, a discharge equation for side weirs is developed using an Artificial Neural Network model. This original approach allows the extraction of the discharge coefficient from the weights of the trained neural network. The important advantage of the developed model over conventional models is that it allows us to account for both the geometric and hydraulic characteristics of the side weir, for forward flows as well as for surcharging and backwater effects. The comparison of the results provided by the developed model with those of the De Marchi model, according to the selected criteria, shows that the former has significantly better performance than the latter, in terms of accuracy and consistency.

While the calibration parameters of the developed model, as they have been defined, are only validated for the studied side weir overflow, the methodology can, however, easily be adapted to any other configuration. Such an approach has also the advantage of implicitly incorporating side weir overflow geometry, channel slope and roughness as well as upstream and downstream flow conditions in the developed discharge equation.

ACKNOWLEDGEMENTS

The authors are grateful to the Lille Urban Community for its financial support and for providing the experimental data.

NOTATION

a	= calibration parameter
B	= top width of the flow cross section
b	= calibration parameter
C_{dm}	= De Marchi coefficient
C_{w}	= discharge coefficient
exp( )	= exponential function
F_1	= first principal direction
F_2	= second principal direction
F_3	= third principal direction
Fr	= Froude number
g	= gravitational acceleration
H	= upstream head on the side weir
h_{w}	= upstream water depth
L	= length of the weir
ln	= natural logarithm
n_{t}	= number of time steps for a hydrograph
Q_{in}	= inflow
Q_{ov}	= overflow discharge
Q	= measured discharge
$\hat{Q}$	= simulated discharge
Q_p	= measured peak discharge
$\hat{Q}_p$	= simulated peak discharge
S	= wetted cross section area
V_{w}	= water velocity
W	= connection weight

Subscripts

i, p	= positive integer indices

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