Peculiar velocities and the mean density parameter

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ABSTRACT
We study the peculiar velocity field inferred from the Mark III spirals using a new method of analysis. We estimate optimal values of Tully–Fisher scatter and zero-point offset, and we derive the three-dimensional rms peculiar velocity ($\sigma_v$) of the galaxies in the samples analysed. We check our statistical analysis using mock catalogues derived from numerical simulations of cold dark matter (CDM) models considering measurement uncertainties and sampling variations. Our best determination for the observations is $\sigma_v = (660 \pm 50)\, \text{km s}^{-1}$. We use the linear theory relation between $\sigma_v$, the density parameter $\Omega$, and the galaxy correlation function $\xi(r)$ to infer the quantity $\beta = \Omega^{0.6}/b = 0.60^{+0.13}_{-0.11}$, where $b$ is the linear bias parameter of optical galaxies and the uncertainties correspond to bootstrap resampling and an estimated cosmic variance added in quadrature. Our findings are consistent with the results of cluster abundances and redshift-space distortion of the two-point correlation function. These statistical measurements suggest a low value of the density parameter $\Omega \sim 0.4$ if optical galaxies are not strongly biased tracers of mass.

Key words: galaxies: general – cosmology: miscellaneous – distance scale.

1 INTRODUCTION
Recent developments on extragalactic distance indicators (Djorgovski and Davis 1987; Dressler et al. 1987) allow us to study the peculiar galaxy velocity field in the local Universe up to $\sim 50 – 100 \, h^{-1}\, \text{Mpc}$ (see Giovanelli 1997, or Strauss and Willick 1995, for a review). These measurements of peculiar velocities provide direct probes of the mass distribution in the Universe and set constraints on models of large-scale structure formation. The density parameter $\Omega$ can be estimated by a comparison of the mass distribution implied by the velocity field with the observed distribution of galaxies. Nevertheless, $\Omega$ may only be determined within the uncertainty of the bias parameter $b$ ($b = 1/\sigma_8$ is the inverse of the root-mean-square (rms) mass fluctuations in spheres of radius $8 \, \text{Mpc} \, h^{-1}$) through the factor $\beta = \Omega^{0.6}/b$.

Bertschinger & Dekel (1989) developed the POTENT method whereby the mass distribution may be reconstructed by using the analogue of the Bernoulli equation for irrotational flows. This method was used to analyse the peculiar velocity field out to $60 \, h^{-1}\, \text{Mpc}$ (Bertschinger et al. 1989). Dekel et al. (1993) compared the previously determined velocity field with the observed distribution of galaxies concluding that $\Omega^{0.6}/b = 1$ provides the best-fitting to the data. Sigad et al. (1998) compared the galaxy density field extracted from the IRAS 1.2 Jy redshift survey to the mass density field reconstructed by the POTENT method from the Mark III catalogue. They measure $\beta_1 = \Omega^{0.6}/b_1 = 0.89 \pm 0.12$ within a volume of effective radius $40 \, h^{-1}\, \text{Mpc}$. This value is consistent with $\Omega = 1$ and $b_1 = 1$. Another comparison between IRAS and POTENT density fields was performed by Willick & Strauss (1998) using a rigorous maximum likelihood method called VELMOD. Implementing this method, they find $\beta_1 = 0.5 \pm 0.04$ (1$\sigma$ error) at $300 \, \text{km s}^{-1}$ smoothing of the IRAS-predicted velocity field.

Analysis of the velocity tensor (Gorski 1988; Groth, Juszkiewicz & Ostriker 1989) also provide useful insights on the velocity field. Zaroubi et al. (1997) reconstructed the large-scale power spectrum from the velocity tensor of the Mark III data and found it consistent with a cold dark matter (CDM) model with $\sigma_8 \Omega^{0.6} = 0.8$ although a different result, $\sigma_8 \Omega^{0.6} = 0.35$, is found by Kashlinsky (1997) in a similar analysis.

The action variational principle has been used to find non-linear solutions for the orbits of mass tracer galaxies in the nearby Universe (Shaya, Peebles & Tully 1995). This analysis yields $\Omega = 0.17 \pm 0.10$ at one standard deviation, in strong disagreement with a critical density universe.

Ferreira et al. (1999) developed a method to estimate directly the pairwise velocity dispersion from the peculiar velocity data. This method can provide an estimate of $\Omega^{0.6} \sigma_8^2$ for a range of $\sigma_8$ according to their analysis in numerical simulations.

Relations between the rms mass fluctuation on a given scale and the density parameter may also be obtained in studies of cluster abundances in different cosmological models. These analyses place constraints, of the form $\Omega^{\alpha}/b = 0.4 – 0.6$ with $\alpha = 0.4 – 0.6$, on a variety of cold and mixed dark matter models (see Eke, Cole & Frenk 1998; Gross et al. 1998). Similarly, studies of redshift-space distortions of the galaxy two-point correlation function also provide a useful restriction to the parameter $\beta$, as, for instance, in Ratcliffe et al. (1997) who find $\beta = 0.5$. 

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We have taken into account Malmquist effects in our analysis of Mark III data to study the peculiar velocity field. The relevance of these corrections may be appreciated in Landy & Szalay (1992). Based on the distribution of estimated distances, the authors eliminate the need for assumptions concerning the true underlying distribution of galaxies and find that part of the signal of the Great Attractor is spurious.

The organization of this paper is as follows. Data characteristics are presented in Section 2. Section 3 provides an outline of the statistical procedure. In Section 4 we analyse the Mark III spirals, and in Section 5 we test our procedure using mock catalogues according to different observers in fully non-linear numerical simulations. In Section 6 we determine the parameter $\beta$ and finally in Section 7 we provide a discussion of the results.

## 2 DATA

We use samples of spiral galaxies taken from the Mark III catalogue (Willick et al. 1995, 1996, 1997) to analyse the peculiar velocity flow. This catalogue lists Tully–Fisher and $D_0 - \sigma$ distances and radial velocities for spiral, irregular, and elliptical galaxies. For spiral galaxies, the velocity parameter $\eta = \log \Delta V - 2.5$ is determined either from H$\alpha$ profiles or from optical H$\alpha$ rotation curves. The Tully–Fisher (TF) relations and their corresponding fractional distance scatters for the different samples of spiral galaxies are given by Willick et al. (1997) and are shown in Table 1, where the absolute magnitude $M$ satisfies $M = m - 5 \log (\text{cz})$. The galaxy apparent magnitudes $m$ of the Tully–Fisher distances are corrected for Galactic extinction, inclination and redshift (see Willick et al. 1997 for details).

The selection bias in the calibration of the forward TF relation can be corrected once the selection function is known. But then the TF inferred distances and the mean peculiar velocities are subject to Malmquist bias. Suitable procedures for considering these biases, induced both by inhomogeneities and selection function, have been discussed (see for instance Freudling et al. 1995, and references therein) where the spatial distribution, selection effects and observational uncertainties are realistically modeled through Monte-Carlo simulations. We have adopted in our analysis inverse TF distances with reference to the cosmic microwave background frame (Willick et al. 1995, 1996, 1997). Inverse TF distances overcome distance-dependent selection bias (see for instance Teerikorpi et al. 1998), nevertheless we have also tested the results using forward TF distances, fully corrected for inhomogeneous Malmquist bias by Willick et al. (1997).

## 3 OUTLINE OF THE ANALYSIS

The different methods applied to infer the distance to a galaxy are subject to uncertainties due to observational errors as well as scatter and systematics of the galaxy parameters. Distances are derived from linear relations between absolute magnitudes $M$ and physical properties independent of distance as for instance the circular velocity $V_c$ in the Tully–Fisher relation, or the central velocity dispersion in the $D_0 - \sigma$ relation. Both rms scatter and possible shifts in the zero point of the distance relation should be taken into account in studies of the peculiar velocity field, considering their strong influence on the results (Padilla, Merchán & Lambas 1998).

Since peculiar velocities of galaxies are inferred from redshifts and independently estimated distances, the effect of a zero-point shift would be observed as a systematic motion of a shell of galaxies proportional to distance. The effects induced in the velocity field by the scatter in the distance relations ($\sigma_{DR}$) depend on the catalogue radial gradient which is affected by distance uncertainties.

We correct the observed radial gradient by considering a Gaussian distribution of distance uncertainties. Therefore the resulting distribution of distance measurements of galaxies restricted to the same true distance bin $d_a$ is approximately Gaussian centred at $d_a$, with scatter $= d_a \sigma_{DR}$. Here $d_a$ is given in units of km s$^{-1}$ and $\sigma_{DR}$ corresponds to a distance fraction. Galaxies from other distances $d_a (n_{ab})$ will also contribute to the measured number of galaxies at $d_a$. The corresponding contribution from objects at $d_a$ can be expressed as:

$$n_{ab} = T(d_b - d_a, \sigma)n_{ab}, \quad (1)$$

$$T(d_b - d_a, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(\frac{-x^2}{2\sigma^2}\right) dx,$$

where $\sigma = d_b \sigma_{DR}$ and $\Delta = 300$ km s$^{-1}$ is the adopted binning of galaxy distances corresponding to shells. Then, the number of galaxies measured at distance $d_a (n_0^a)$ takes into account contributions from all other distances:

$$n_0^a = \sum_{d_a=0}^{d_{max}} n_{ab}, \quad (2)$$

where $d_{max}$ is the limiting distance imposed by the catalogue.

In order to solve equation (2), we define the vectors $N' = n_0^a$ and $N = n_b$ and the matrix $T = n_{ab}$. Then equation (2) can be rewritten as

$$N' = TN.$$

$T$ can be inverted to obtain the true number count of galaxies, $N$, unaffected by the distance estimator scatter taken into account in the matrix $T$. However, a direct inversion of the matrix (using the Gauss method for instance) produces diverging solutions for the last components of $N$. This divergence is produced by accumulation of large errors through the calculation over matrix rows. To avoid this problem, we used an iterative method in which we apply $T$ to $N'$ and obtain $N''$. Since $T \sim I$, the vector

<table>
<thead>
<tr>
<th>Subsample</th>
<th>Number of Gx.</th>
<th>TF relation</th>
<th>Fractional distance $\sigma_{TF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaronson et al. Field (1982)</td>
<td>359</td>
<td>$M_H = -5.95 + 10.29\eta$</td>
<td>0.235</td>
</tr>
<tr>
<td>Mathewson et al. (1992)</td>
<td>1355</td>
<td>$M_I = -5.79 + 6.8\eta$</td>
<td>0.215</td>
</tr>
<tr>
<td>Willick, Perseus Pisces (1991)</td>
<td>383</td>
<td>$M_I = -4.28 + 7.12\eta$</td>
<td>0.190</td>
</tr>
<tr>
<td>Willick, Cluster Galaxy (1991)</td>
<td>156</td>
<td>$M_I = -4.18 + 7.37\eta$</td>
<td>0.190</td>
</tr>
<tr>
<td>Courteau-Faber (1993)</td>
<td>326</td>
<td>$M_I = -4.22 + 7.37\eta$</td>
<td>0.190</td>
</tr>
<tr>
<td>Han-Mould et al., Cl. Gx. (1992)</td>
<td>433</td>
<td>$M_I = -5.48 + 7.87\eta$</td>
<td>0.200</td>
</tr>
</tbody>
</table>
$$N_1 = N^l + (N^l - N^r),$$ will be a better approximation to $N$ than $N^r$. After $k$ iterations, we obtain

$$N_{k+1} = N_k + (N^l - N_k^l)$$

(4)

where we impose the condition $N_k \geq 0$.

This method also accumulates errors in the last components of $N_k$, but the solutions start to diverge only when more than three iterations are applied. The optimal number of iterations was found to be $k = 2$ or 3. The results of the iterative method have also been checked with the mock catalogues analysed in Section 5, and were found to be accurate.

In order to study the effects induced by the TF scatter in the velocity field, we calculate the effect of $\sigma_{\text{DR}}$ on the mean radial velocity of galaxies ($v_a$) at distances $d \in [d_a - \Delta, d_a + \Delta]$ under the assumption that the shells do not expand or contract. We may write the apparent mean velocity of the shell $v'_a$ at distance $d_a$ as:

$$v'_a = \frac{1}{n_a} \sum_{i=1}^{n_a} v'_i,$$

(5)

where $v'_i$ is the individual velocity of the galaxy $i$. We recall the fact that at distance $d_a$ there are contributions from other distances. If a galaxy $j$ at real distance $d_b$ is measured to be at distance $d_a$, the inferred velocity will be $v'_j = v_j - (d_b - d_a)$, where $v_j$ is the real peculiar velocity of the galaxy $j$. Finally, if we sort by distance single galaxies accidentally in the shell at distance $d_a$, we can rewrite the last expression as:

$$v'_a = \frac{1}{n_a} \sum_{i=1}^{n_a} \sum_{j=1}^{n_b} [v_i - (d_b - d_a)].$$

(6)

If we consider our assumption $v_b = 0$ and add the possible presence of a zero-point shift $P_0$, we find the final expression:

$$v'_a = \frac{1}{n_a} \sum_{i=1}^{n_a} \sum_{j=0}^{n_b} [v_i - d_b + P_0(d_b)].$$

(7)

The inputs of this equation are the number count of galaxies as a function of distance, the scatter $\sigma_{\text{DR}}$, and the zero-point shift, $P_0$. By comparing the measured values $v'_a$ from a catalogue with the calculated values given by equation (7), we may infer the uncertainties that affect the measured distances.

A similar deduction can be applied to obtain the apparent rms velocities $\sigma'_v$ corresponding to galaxies in a given shell at distance $d_a$. The assumption made here is that the true rms velocity of a shell is independent of distance.

$$\sigma'_v = \sigma_{\text{idim}}.$$

This quantity can be calculated from the following equation:

$$\sigma_{\text{idim}}^2 = \frac{\sum_{i=1}^{n_a} (n'_i \sigma'^2_i - \sum_{i=0}^{n_a} n_{ab}[d_b - d_a + P_0(d_b)])^2}{\sum_{i=0}^{n_a} n'_a}. $$

(8)

The inputs of equation (8) are the observed $\sigma'_v$, $n'_v$, the distance relation scatter $\sigma_{\text{DR}}$, and its zero-point offset, $P_0$. The one-dimensional velocity dispersion, $\sigma_{\text{idim}}$, is thus directly obtained from radial velocity data. The three-dimensional velocity dispersion $\sigma_v$ is simply $\sigma_v = \sqrt{3} \sigma_{\text{idim}}$, assuming isotropy.

4 APPLICATION TO THE MARK III CATALOGUE

The scatter of the Mark III spiral TF relation (hereafter $\sigma_{\text{TF}}$) has been extensively studied (Willick 1991; Mathewson, Ford & Buchhorn 1992; Mo, Mao & White 1997), and similarly uncertainties of the TF zero point, $P_0$ (Willick 1991; Shanks 1997). The value of the mean fractional distance scatter is $\sigma_{\text{TF}} = 0.2$ for the samples of spiral galaxies of the Mark III catalogue with a null zero-point offset with mean deviation $\pm 0.035$ (Willick 1991).

Since the sample of Mark III spirals has a reasonably smooth sky coverage the analysis outlined in the previous section is suitable for statistical purposes. We applied equation (7) to the Mark III spirals restricted to distances $d < 6000\text{ km s}^{-1}$ since beyond this distance the uncertainties make peculiar velocities unreliable. We have calculated a $\chi^2$ deviation between predicted and observed mean peculiar velocity of shells $v'_a$ as a function of $\sigma_{\text{TF}}$ and $P_0$. The best fitting was obtained for $\sigma_{\text{TF}} = 0.205$, $P_0 = -0.025$.

Notice that these values of $\sigma_{\text{TF}}$ and $P_0$ are consistent with those quoted in Willick et al. (1997). Figs 1(a) and (b) show the observed mean velocities and the results of equation (7) for the values of $\sigma_{\text{TF}}$ and $P_0$ obtained using inverse TF and forward corrected TF respectively. In Fig. 1(b) the predicted $v'_a$ with no zero-point shift $P_0$ and $\sigma_{\text{TF}} = 0.2$ is also shown. In order to measure the accuracy of the determination of the distance uncertainties, we apply a $\chi^2$ test to obtain $\sigma_{\text{TF}}$ and $P_0$ for a large set of catalogues obtained through bootstrap resampling. The number of realizations with resulting $\sigma_{\text{TF}}$ and $P_0$ are shown in Figs 2(a) and (b) for inverse and forward corrected TF distances respectively.

These results obtained for the inverse TF distances may be compared with the homogeneous calibration of the Tully–Fisher relation corresponding to different samples of spirals of the Mark III catalogue given by Willick et al. (1995, 1996, 1997), which corresponds to values of fractional distance scatter $\sigma_{\text{TF}}$ in the range $0.19$–$0.235$ (see Table 1). We have obtained the galaxy rms velocity by application of equation 8 to the sample $d < 6000\text{ km s}^{-1}$. The value obtained is $\sigma_v = (660 \pm 50)\text{ km s}^{-1}$, where the error was obtained from 1000 bootstrap resamplings (Barrow, Bhavsar & Sonoda 1984). The distribution of $\sigma_v$ derived from the different bootstrap resamplings is shown in Fig. 3.

5 TESTING THE METHOD WITH MOCK CATALOGUES

We test the results of our analysis using mock catalogues derived from the numerical simulation. We adopted a $\Omega = 0.5$, $\Omega_\Lambda = 0$ COBE normalized CDM model which reasonably reproduces several statistical tests of large-scale structure such as cluster abundances, correlation functions, etc. This particular model requires no strong bias, so each particle of the simulation corresponds to a galaxy.

We have considered 1000 random observers in the numerical simulation by defining cones with different positions and orientations in our computational volume. We have included the observed strong radial gradient which corresponds approximately to a selection bias due to a magnitude limit cut-off in the data to
Figure 1. (a) Mean radial velocities of shells of galaxies from Mark III using inverse TF distances (solid lines) with $\sigma_{TF} = 0.205$ and $P_0 = 0.025$. The dashed line corresponds to the model results with the same values of $\sigma_{TF}$ and $P_0$. (b) Mean radial velocities using forward TF distances corrected for inhomogeneous Malmquist bias (solid lines). The dashed line corresponds to the model results with parameter $\sigma_{TF} = 0.15$ and $P_0 = -0.065$, and the dotted line to a model with $\sigma_{TF} = 0.2$ and $P_0 = 0$.

Figure 2. Occurrence of values $P_0$ and $\sigma_{TF}$ by random resamplings of the sample. (a) Using inverse TF distances, and (b) using forward corrected TF distances.
Figure 3. Distribution of $\sigma_v$ for the observational sample corresponding to bootstrap resampling.

Figure 4. Mean radial velocities of shells according to models (solid line) and mock catalogues (dashed lines). Error bars correspond to fluctuations arising from different observers in the simulations.
the mock catalogues. This can be seen from the observed distribution of absolute magnitudes which is nearly Gaussian with mean \( M^* \) (the knee of the luminosity function) and \( \sigma = 1.5 \) mag. Nevertheless, for our statistical purposes it is not necessary to adopt a Monte Carlo model using the galaxy luminosity function in the simulations. It suffices to reproduce the observed radial gradient in the numerical models through a Monte Carlo rejection algorithm. Furthermore, we restrict the resulting number of particles of the mock catalogues to be equal to the number of galaxies in the observational sample.

We have considered a Gaussian distribution of relative distance uncertainties according to the TF relation so that for each galaxy \( \Delta d/d = \sigma_{\text{TF}} \). Namely, we assign to each particle in the mock catalogue a new distance \( d_{\text{new}} = d(1 + s) \), where \( s \) is taken from a

Figure 5. Occurrence of values \( P_0 \) and \( \sigma_{\text{TF}} \) in random resamplings of a mock catalogue with \( \sigma_{\text{TF}} = 0.205 \) and \( P_0 = -0.025 \).

Figure 6. Probability distribution of \( \sigma_v \) corresponding to mock catalogues with \( \sigma_{\text{TF}} = 0.15 \) and \( P_0 = 0 \) (solid lines), \( \sigma_{\text{TF}} = 0.205 \) and \( P_0 = -0.025 \) (dashed lines), and \( \sigma_{\text{TF}} = 0.2 \) and \( P_0 = -0.1 \) (dotted lines).
Gaussian distribution with dispersion corresponding to the TF distance uncertainty. Then, as the particle peculiar velocities are inferred from the galaxy redshift and distance, \( v_{\text{new}} = v - ds \).

The \( N \)-body numerical simulation was performed using the adaptive particle–particle–mesh (AP3M) code developed by Couchman (1991). The initial condition was generated using the Zeldovich approximation and corresponds to the adiabatic CDM power spectrum with \( \Omega = 0.5 \) and \( \Omega_0 = 0 \). We have adopted the analytic fit to the CDM power spectrum given by Sugiyama (1995):

\[
P(k) \propto \frac{k}{A} \left[ \ln(1 + 2.34q) \right]^2, \tag{9}
\]

where \( A = [+3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4]^{1/2} \), \( q = (k/0.5) \text{Mpc} \), \( \Gamma = \Omega \theta^2 / [h \exp(-\Omega_0 / \sqrt{h/0.5 \Omega_0 / \Omega})] \), \( \theta \) is the microwave background radiation temperature in units of 2.7 K, and \( \Omega_0 = 0.0125 \ h^{-2} \) is the value of the baryon density parameter given by nucleosynthesis. The normalization of the CDM power spectrum is imposed by COBE measurements using the value of \( \sigma_8 \) and \( h \) (the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\)) chosen from table 1 of Gorski et al. (1995) corresponding to an age of the Universe \( t_0 = 12 \) Gyr. The computational volume is a periodic cube of side length 300 Mpc. We have followed the evolution of \( N = 5 \times 10^5 \) particles with a \( 64^3 \) grid and a maximum level of refinements of 4. The resulting mass per particle is 2.05 \( \times 10^{12} h^{-1} \text{M}_\odot \). The initial condition corresponds to redshift \( z = 10 \) and the evolution was followed using 1000 steps. At the final step (\( z = 0 \)) the linear extrapolated value of \( \sigma_8 \) is compatible with the normalization imposed by observed temperature fluctuations in the cosmic background.

As a test of the ability of equation (7) to reproduce the distorted values of mean velocity, we plot in Fig. 4 the resulting \( v'_{\text{obs}} \) for the average of 1000 mock catalogues, each with distance uncertainties included. The error bars correspond to the rms deviation of \( v' \) from the different observers in the numerical simulation. We adopt the values of \( \sigma_{\text{TF}} \) and \( P_0 \) used in the construction of the mock catalogues to perform the calculation of \( v'_{\text{obs}} \) through equation (7). The excellent agreement between the results of the calculation and the mock catalogues can be seen in the figure. We have applied a \( \chi^2 \) method to derive \( \sigma_{\text{TF}} \) and \( P_0 \). We find that our method gives accurate estimates of the actual \( \sigma_{\text{TF}} \) and \( P_0 \) when applied to the mock catalogues. In Fig. 5 we show the frequency of \( \sigma_{\text{TF}} \) and \( P_0 \) obtained from bootstrap resampling objects corresponding to a mock catalogue with an imposed scatter \( \sigma_{\text{TF}} = 0.2 \) and \( P_0 = -0.025 \) as derived from the observations.

The values of \( \sigma_{\text{TF}} \) and \( P_0 \) inferred from the \( \chi^2 \) method can be used to calculate the value of \( \sigma_{\text{TF}} \) in the mock catalogue through equation (8). The distributions of inferred \( \sigma_{\text{TF}} \) found are shown in Fig. 6 where the stability of the method in estimating \( \sigma_{\text{TF}} \) under different zero-point offsets \( P_0 \) and \( \sigma_{\text{TF}} \) can be appreciated. The actual value of the rms peculiar velocity of the simulation is \( \sigma_{\text{TF}} = 780 \text{ km s}^{-1} \) which may be compared to the inferred values from equation (8) for different adopted values of \( \sigma_{\text{TF}} \) and \( P_0 \). For instance, we find \( \sigma_{\text{TF}} = (743 \pm 102) \text{ km s}^{-1} \), \( \sigma_{\text{TF}} = (770 \pm 117) \text{ km s}^{-1} \), and \( \sigma_{\text{TF}} = (772 \pm 145) \text{ km s}^{-1} \) for \( \sigma_{\text{TF}} = 0.15 \), \( P_0 = 0 \); \( \sigma_{\text{TF}} = 0.25 \), \( P_0 = 0 \); \( \sigma_{\text{TF}} = 0.30 \), \( P_0 = -0.025 \); and \( \sigma_{\text{TF}} = 0.2 \), \( P_0 = 0.1 \) respectively, indicating the ability of our procedure to derive the rms peculiar velocity irrespective of TF scatter and zero-point offset.

We find a zero-point shift \( P_0 = -0.025 \) in our analysis of the observations using inverse TF distances. It is of interest to test the probability of occurrence of such a value arising from our model assumptions such as \( v_0 = 0 \), etc. We use mock catalogues with imposed \( \sigma = 0.2 \) and \( P_0 = 0 \) in the TF calibration and we test the probability of finding different values of zero-point shifts. We apply a \( \chi^2 \) method to derive the pair of values \( \sigma_{\text{TF}} \) and \( P_0 \) that provides the best fitting of equation (7) to the actual values for each mock catalogue. We compute the frequency of occurrence of \( \sigma_{\text{TF}} \) and \( P_0 \) for the different observers, and we can estimate the probability of obtaining different values of zero-point shifts. We find that a random occurrence of the observed value \( P_0 = -0.025 \) is within a standard deviation, consistent with the estimate of Willick et al. (1997).

6 DETERMINATION OF THE \( \beta \) PARAMETER

The three-dimensional velocity dispersion \( \sigma_0 \) is directly related to the quantity \( \beta = \Omega^{0.6}/b \) through the relation (Peebles 1980)

\[
\sigma_0 = \left( \frac{H \beta}{b} \right) \int_0^{\beta_0} y(y) \, dy, \tag{10}
\]

where \( H \) is the Hubble constant, \( a \) is the expansion factor of the universe, \( y \) is expressed in Mpc, \( \xi(y) \) the galaxy spatial correlation function, \( f = \Omega^{0.6} \) the rate of growth of inhomogeneities, and \( b \) is the linear bias factor. We express \( y \) in units of Mpc \( h^{-1} \), therefore \( H = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \) and \( a = 1 \).

We estimate \( \Omega^{0.6}/b \) from the inferred value of the three-dimensional rms peculiar velocity. We adopt the power-law fit to

![Figure 7. Probability contours corresponding to the observational sample \( d < 6000 \text{ km s}^{-1} \). The dashed lines correspond to different values of \( \beta \). The plus sign indicates the best values of \( \sigma_{\text{TF}} \) and \( P_0 \).](https://academic.oup.com/mnras/article-abstract/310/1/21/972523)
the galaxy spatial correlation function estimated by Ratcliffe et al. (1998):
\
$$\xi(r) = \begin{cases} 
\left(\frac{r}{r_0}\right)^{-\gamma}, & \text{if } r \leq 50 \text{ Mpc}, \\
0, & \text{if } r > 50 \text{ Mpc}, 
\end{cases}$$
\
with $r_0 = 5.1$ Mpc and $\gamma = 1.6$.

We calculate $\beta$ from equation (10) using equation (8) to express $\sigma_v$ and therefore $\beta$ in terms of the parameters $\sigma_{TF}$ and $P_0$. In Fig. 7 we show equal $\beta$ contours in the $\sigma - P_0$ plane for our sample of Mark III spirals with distances $d \leq 6000$ km s$^{-1}$. Also shown in this figure are the 1$\sigma$ and 2$\sigma$ contour levels corresponding to the frequency of inferred $\sigma_{TF}$ and $P_0$ from bootstrap resamplings of the observational data set. The corresponding result is $\beta = 0.60^{+0.08}_{-0.05}$.

The true uncertainty of the global value $\beta = \Omega^{0.6}/b$ derived from a catalogue of peculiar velocities will be greater than that obtained from bootstrap resampling of the data due to cosmic variance. A suitable value of the uncertainty in $\beta$ can be estimated from the rms values of this parameter derived from the mock catalogues. According to our analysis $\Delta \beta = 0.1$ for a limiting distance $d_{\text{max}} \leq 6000$. Thus, adding in quadrature for both errors, we find $\beta = 0.60^{+0.11}_{-0.11}$ for $d \leq 6000$ km s$^{-1}$.

We have also considered another observational estimate of the galaxy spatial two point correlation function directly obtained from angular APM data. This correlation function provides an independent measurement and shows a more gentle decline to uniformity at scales $\leq 50 h^{-1}$ Mpc. Although this galaxy correlation function differs from the determination of Ratcliffe et al. (1998), we find in this case $\beta = 0.56$, entirely consistent with the previous result.

7 DISCUSSION

We have developed a method for the analysis of the peculiar velocity field inferred from peculiar velocity data and we apply this procedure to the spirals of the Mark III catalogue. We estimate optimal values of inverse Tully–Fisher scatter and zero-point offset for a sample of the catalogue with limiting distance $d_{\text{lim}} = 6000$ km s$^{-1}$. We derive the three-dimensional rms peculiar velocity of the galaxies $\sigma_v = (660 \pm 50)$ km s$^{-1}$ where the uncertainty has been obtained through bootstrap resampling of the data.

In our model, the shells do not have a net mean radial motion, and the mean square velocities of galaxies in different shells are described by a unique number $\sigma_v$. The comparison with mock catalogues derived from numerical simulations shows that these are reasonable hypotheses that allow one to obtain physical characterizations of the nearby Universe from observations. We have shown that corrected TF distances require a large fractional distance zero-point offset $P_0 = -0.075$ so that caution should be taken when they are used in statistical analysis. Landy & Szalay’s (1992) Malquist correction is also based on the distribution of estimated distances without the need for assumptions of the underlying distribution of galaxies.

Our approach uses mock catalogues derived from numerical simulations of CDM models considering measurement uncertainties and sampling variations to check our statistical analysis. We find a general good agreement between the results of the calculations and those measured in the mock catalogues. The spread of $\sigma_v$ measurements from different observers in the numerical simulations may be added in quadrature to the bootstrap resampling errors to provide a more reliable estimate of the uncertainty in $\sigma_v$. We infer $\sigma_v = 660 \pm 70$ km s$^{-1}$, and we conclude that $\beta = \Omega^{0.6}/b = 0.60^{+0.13}_{-0.11}$.

Estimates of the parameter $\beta$ from other analyses such as studies of redshift-space distortions of the galaxy two-point correlation function provide similar values of $\beta = 0.5$ (Ratcliffe et al. 1997). The comparison between IRAS and POTENT density fields (Willlick & Strauss 1998) using VELMOD also yields $\beta = 0.5 \pm 0.04$. Moreover, the confrontation of observed cluster abundances with the predictions of different cosmological models produce constraints of the form $\Omega^{0.6}/b = 0.4 - 0.6$, consistent with our determinations. Nevertheless, higher values of $\beta = 0.9$ are obtained by Sigad et al. (1998) comparing the IRAS 1.2 Jy galaxy density field to the mass density field reconstructed with POTENT from the Mark III catalogue; while the action variational principle (Shaya, Peebles & Tully 1995) gives a significantly lower value, $\Omega = 0.17 \pm 0.10$.

Our global approach is based on the determination of the mean velocity dispersion obtained from peculiar velocity data. Although sensitive to the adopted galaxy correlation function, our procedures provide a useful measure of $\beta$ in agreement with other independent estimates.

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REFERENCES


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Willick J., 1991, Doctoral thesis

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