On accretion flow penetration of magnetospheres

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ABSTRACT

We address the problem of plasma penetration of astrophysical magnetospheres, an important issue in a wide variety of contexts, ranging from accretion in cataclysmic variables to flows in protostellar systems. We point out that under well-defined conditions, penetration can occur without any turbulent mixing (driven, for example, by Rayleigh–Taylor or Kelvin–Helmholtz instabilities) caused by charge polarization effects, if the inflowing plasma is bounded in the direction transverse to both the flow velocity and the magnetic field. Depolarization effects limit the penetration depth, which nevertheless can, under specific circumstances, be comparable to the size of the magnetosphere. We discuss the effect of ambient medium on plasma propagation across the stellar magnetic field and determine the criteria for deep magnetosphere penetration. We show that, under conditions appropriate to magnetized white dwarfs in AM Her type cataclysmic variables, charge polarization effects can lead to deep penetration of the magnetosphere.

Key words: accretion, accretion discs – magnetic fields – polarization – stars: magnetic fields – novae, cataclysmic variables.

1 INTRODUCTION

Magnetic fields are thought to play an important role in regulation of accretion in a wide variety of astrophysical systems, ranging from protostellar systems, cataclysmic variables (CV) and neutron stars to active galactic nuclei. In many of these systems, the central accreting object is surrounded by a magnetosphere; and a key problem in understanding the accretion process in such systems is the physics of magnetosphere penetration by the accreting material, which is typically highly ionized and weakly collisional. At large distances from the central object, the plasma motion is unimpeded by the weak magnetic field: surface plasma currents screen the external magnetic field \( B \) from the plasma interior and plasma propagates ballistically (Chapman 1923; Ferraro 1931). In this ‘diamagnetic’ mode of propagation, the plasma flow velocity of the infalling matter, respectively). Penetration is possible even at low magnetic field. Depolarizing effects result in drag force that decelerates the plasma flow (similarly to the effect of the Alfvén wave emission considered by Drell, Foley & Ruderman 1965); nevertheless, the penetration length can be significant. In fact, as we shall demonstrate, the penetration length can be comparable to or larger than the size of the magnetosphere in the particular case of a white dwarf magnetosphere.

This mechanism of magnetic field penetration by plasma was originally discussed in the context of geomagnetic storms by Chapman (1923), who concluded that an ionized stream impinging upon the Earth’s magnetic field would be deflected by it only very slightly; a closely analogous mechanism is
responsible for the instability of a plasma supported by a magnetic field against gravity (Kruskal & Schwarzschild 1954; Rosenbluth & Longmire 1957). Of particular importance for verifying the validity of this process were the theoretical investigations of Schmidt (1960) and the experiments of Baker & Hammel (1965). Schmidt showed that charge separation can explain the observations (Bostick 1956; Wetstone, Ehrlich, & Finkelstein 1960) that sufficiently dense plasma injected into a curved magnetic field propagates unaffected by it; Baker & Hammel corroborated this explanation by observing that injected plasma penetrates the magnetic field, causing only a small disturbance of the vacuum field, and that polarization charge on the plasma boundary is responsible for the instability of a plasma supported by a magnetic field. For the sake of simplicity, we restrict the analysis. When it is not, finite-temperature effects can be readily included, albeit at the price of increased complexity of the analysis.

With the mirroring force neglected in the cold plasma approximation, the parallel flow velocity varies along the trajectory according to

$$\frac{du}{dt} = \vec{u} \cdot \nabla \vec{V},$$

where $d/du = d/dt + \vec{u} \cdot \nabla$ is the convective derivative.

To obtain the zeroth-order transverse equation of motion requires the determination of the electric field. The latter results from the charge separation that arises as a consequence of the first-order drifts:

$$u^{(1)}_s = \frac{1}{\Omega_s} \left[ q_s - u_s^2 \hat{b} - \frac{d}{dt} E \right] \times \hat{b},$$

where $\Omega_s = q_s B/m_s c$ is the cyclotron frequency of species $s$ (with charge $q_s$ and mass $m_s$), $\hat{b}$ is the gravitational acceleration vector, $\hat{b} = B/B$ is the unit vector in the direction of the magnetic field and $\hat{b} \cdot \nabla \hat{b}$ is the magnetic field curvature. The first term on the right-hand side is the gravitational drift, the second is the curvature drift, and the last is the polarization drift. We have omitted here the grad-$B$ drift, in accord with our assumptions.

The zeroth-order motion is the same for electrons and ions and will be identified with the plasma flow velocity: $\vec{V} = \vec{u}^{(0)}$. First-order drifts are different for electrons and ions and they give rise to the charge polarization. The resulting electric field leads to the transverse plasma drift.

The inflowing plasma will be modelled as a narrow rectangular slab, with poloidal width $2w$ and toroidal width $2h \ll 2w \ll 2R$ (see Fig. 1). In the chosen inflow geometry, first-order drifts are primarily toroidal, resulting in the predominately toroidal electrostatic field $E = -\vec{V}_0 = -E \hat{\Phi}$, which gives rise to the transverse motion of the slab in the poloidal plane.

Introducing local cylindrical coordinates $(r, \theta, z)$ such that $\hat{r} = -\hat{b}/\kappa$, $\hat{\theta} = \hat{b}$ and $\hat{z} = \hat{\Phi}$, the $z$-component of equation (4) can be

$$\frac{d\tilde{V}}{dt} = -E \hat{\Phi}$$

where $d/du = d/dt + \tilde{u}^{(0)} \cdot \nabla$ is the convective derivative.

In the following quantitative discussion we adopt the approach of Schmidt (1960) and employ the guiding-centre approximation to describe the motion of plasma particles. In this approximation, the zeroth-order motion $\tilde{u}^{(0)}$ consists of the electric drift $u_\perp = e E \times B/B^2$ and parallel flow $u_\parallel$ along the magnetic field line:

$$\tilde{u}^{(0)} = u_\perp + u_\parallel.$$

In order to simplify to essentials, we shall assume that the inflowing plasma is cold, i.e. that its sound speed is much smaller than the inflow velocity. This assumption allows us to ignore the grad-$B$ drift, thermal expansion of the plasma stream along the field direction during infall and mirroring effects of the dipole field. The cold plasma approximation is justified in many physical situations of interest; when it is not, finite-temperature effects can be readily included, albeit at the price of increased complexity of the analysis.
expressed in the form
\[ u^{(1)}_x = \frac{1}{\Omega} \left[ g_x + \frac{u_x^2}{r} - \frac{d}{dt} \left( \frac{E}{B} \right) \right]. \] (5)

In the same coordinate system as above, the \( r \)-component of the zeroth-order velocity is given by
\[ u^{(0)}_r = \frac{E}{B} c, \] (6)
while
\[ u^{(0)}_\theta = u_\theta. \] (7)

First-order drifts give rise to the current in the \( z \)-direction
\[ J_z = \sum_s q_s n_s u^{(1)}_z, \] (8)
where \( n_s \) is the number density of the species \( s \). This current leads to an accumulation of charges in regions where \( \delta J_z \neq 0 \).

Assuming that the plasma density is uniform in the \( z \)-direction except in narrow boundary layers, the charge accumulation occurs in these layers: positive charges on one side of the slab and negative charges on the other side. The electric field is uniform in the central part of the slab, but non-uniform in the boundary layer, and it vanishes outside the plasma slab (whether the outside medium is vacuum or lower-density plasma) apart from a small stray field caused by the finite slab size, which gives rise to magnetic field-aligned currents outside the slab (see later). Thus the \( E \times B \) drift is confined to the slab, as is required for self-consistency of the model.

The self-consistent electric field is determined from charge conservation and the Poisson law. Assuming that the charge layer is narrow compared with the slab width \( h \), we can approximate it by a surface charge with density \( \sigma \). This charge is found by integrating the charge continuity equation, which yields
\[ \frac{\partial \sigma}{\partial t} + V \nabla \sigma = J_z - \frac{\sigma}{\tau_L}, \] (9)
where \( \tau_L \) is the time-scale of charge loss from the boundary layer (resulting from currents through the ambient medium, etc.). As, in the slab approximation, \( E = 4\pi\sigma \), equation (9) implies
\[ \frac{dE}{dt} = 4\pi \sum_s q_s n_s u^{(1)}_z - \frac{E}{\tau_L}. \] (10)

Using equation (4), one obtains
\[ (1 + \chi) \frac{du_x}{dt} + u_x \frac{d}{dt} \ln B = \chi \left( g_x + \frac{u_x^2}{r} \right) - \frac{u_E}{\tau_L}, \] (11)
where \( \chi = 4\pi ne^2/B^2 \) is the low-frequency (transverse) electric susceptibility and \( \rho \) is the mass density. If the plasma density is sufficiently high, so that \( \chi \gg 1 \), the second term on the left-hand side of equation (11) can be neglected so that (with equations 6 and 7 and recalling that \( V = u^{(0)}_r \))
\[ \frac{dV_r}{dt} = g_r + \frac{V_r^2}{r} - \nu V_r, \] (12)
where \( \nu = 1/\chi \tau_L \). Equations (3) and (12) can thus be expressed in a coordinate-independent form as
\[ \frac{dV}{dt} = g - \nu V, \] (13)
where the subscript \( \perp \) denotes the component perpendicular to \( B \).

In the absence of charge losses from the boundary (\( \nu = 0 \)), the above result implies that the plasma slab is accelerated by the gravitational field as if the magnetic field were absent. This is the reason why, in particular, the growth rate of the Kruskal†Schauss, instability of a plasma supported by a magnetic field against gravity is exactly the same as that of the Rayleigh-Taylor instability of a heavy neutral fluid supported against gravity by a lighter fluid (Kruskal & Schwarzschild 1954; Rosenbluth & Longmire 1957). Charge losses give rise to a drag force that decelerates the plasma, a phenomenon akin to the stabilizing effect of line-tying (Kunkel & Guillory 1966; more recently, Berk & Kotelnikov 1993 and references therein).

Observe that the \( E \times B \) drift occurs on equipotential surfaces, because \( u_E \nabla U = 0 \). Consequently, the potential difference \( 2U_0 \) between toroidal boundaries of the slab remains constant during its motion.

We note that while the propagation mechanism discussed here requires deviation from strict charge neutrality, this deviation is exceedingly small for typical astrophysical conditions. To see this, consider the toroidal density profile near the slab edge \( (\zeta \in [h, h + a]) \) given by \( n = n_0 [1 - (\zeta - h)/a]\), and the relative displacement of electron and ion slabs \( \delta \). Then \( n_e = n_1 - n_2 = (\delta/a)n_0 \approx n_0 \delta/a \ll 1 \). As \( V = c/E \) and \( E = 4\pi n_e a \), it is easy to see that \( \delta = \rho R_L \), where \( R_L \) is the Larmor radius corresponding to the inflow velocity \( V \). As \( \rho R_L \ll a \) in astrophysical situations of interest, \( n_e \ll n_0 \) for \( \chi > 1 \).

### 3 Permittivity of the Accretion Stream

For the case of a purely radial infall on to an object of mass \( M \), the second term on the left-hand side of equation (13) can be neglected if \( \nu \tau_L \ll 1 \) where \( \tau_L = \sqrt{2R^3/GM} \). Thus a sufficient condition for deep magnetospheric penetration by an infalling plasma is \( \chi \gg 1 \) and \( \nu \tau_L \leq 1 \) in a large fraction of the magnetosphere. It should be noted that \( \chi \gg 1 \) at least at the entry into the magnetosphere: \( R = R_0 \sim R_A \). As \( \chi \) can be expressed in terms of the inflow velocity \( V_0 \) into the magnetosphere, i.e. \( \chi = \beta c^2/V_0^2 \), it is immediately apparent that for non-relativistic inflows \( \chi \gg 1 \) for \( \beta \sim 1 \), i.e. \( R \sim R_A \).

Outside the magnetosphere, at \( R > R_A \), the plasma motion is ballistic. If the accretion is caused by the Roche lobe overflow of the secondary, the gas motion can be described in the manner of Lubov & Shu (1975): the accretion occurs in the form of a stream with the cross-sectional area \( A \) and density \( \rho = M/V_A \), where \( M \) is the mass accretion rate. If the mass of the secondary is smaller than that of the primary, \( A \approx c_s^2/a^2 \), where \( c_s \) is the isothermal sound velocity, \( \omega = 2\pi/P \) and \( P \) is the orbital period (Lubov & Shu 1975). For such a steady plasma stream inflowing in the vicinity of the magnetic equator, the Alfven radius is given by
\[ R_A = \left( \frac{c_s^2 B_0^2}{16\pi n_0 M V_0} \right)^{2/11}, \] (14)
where \( V_0 \) is the escape velocity from the stellar surface. For example, for characteristic parameters of AM Herculis cataclysmic variables (\( M = 1 M_\odot \), \( B_0 = 25 \mathrm{MG} \), \( R_0 = 5 \times 10^8 \mathrm{cm} \), \( M = 4 \times 10^5 \mathrm{g} \mathrm{s}^{-1} \), \( P = 2 \mathrm{h} \) and plasma temperature \( T = 10^7 \mathrm{K} \); see, e.g., Shapiro & Teukolsky 1983; Liebert & Stockman 1985 and Lamb 1988), one finds that \( R_A = 16R_0 \); it then follows that \( \chi(R_A) \approx 3 \times 10^4 \).

As the flow penetrates deeper into the magnetosphere, both the
magnetic field and the plasma stream density increase. The latter
dependence can be deduced from mass conservation, which for a
steady-state inflow implies that $\rho V A = \text{constant}$ where $A = 4\pi hw$
is the slab cross-section and $h$ and $w$ are the toroidal and poloidal
half-widths, respectively. For a radial inflow in the poloidal plane,$w/R = \text{constant}$. The radial dependence of the toroidal width $h$
can be determined from the fact that the potential difference
between the slab boundaries remains constant during the inflow
(see earlier) so that $hVB = \text{constant}$. Thus $\rho V B = \text{constant}$,
which implies that $\chi \sim R^2$. From this it follows that for the above-quoted parameters of cataclysmic variables, $\chi(R_c) \approx 100$. Thus,
for these parameters, the plasma stream will penetrate to the
stellar surface, as if the magnetic field were absent, provided the
drag force resulting from charge losses is small.$^1$ As we shall see
in the next section, the effect of the latter can be significant and
can modify the above conclusion.

4 DEPOLARIZATION EFFECTS ON STREAM
PENETRATION

The principal question concerning the validity of the mechanism,
proposed in the present paper is whether sufficient polarization
can arise on the free-fall time-scale. Within the framework of the
guiding-centre theory, the initial polarization is set up by the
gravitational drift. By comparing the current $J_g = eng/\Omega_i$ due to
the gravitational drift with the required charge density $\sigma = VB/4\pi c$
it is easy to see that the polarization occurs on the time-
scale $\tau_p = \tau_d/\chi$. Thus for $\chi \gg 1$, the polarization occurs on a
time-scale much shorter than the free-fall time.

The next question is under what circumstances the drag force,
resulting from depletion of the charge layers, is small enough to
allow significant penetration. We address this question in the
remainder of the present section.

Several mechanisms can cause charge loss, either by eroding
the charged boundary layers or by closing the circuit between the
electron and ion layers. If the charge depletion is sufficiently
large, it will limit the penetration distance (see Borovsky 1987).
We shall limit our discussion to processes that are relevant to the
steady-state accretion problem, discussed in the previous section.

In the case of a plasma stream with a finite extent along the
magnetic field, the fastest charge-loss mechanism is caused by
currents flowing parallel to the magnetic field. If the total current
per unit length in the flow direction (i.e. the surface current
density) flowing out of each charge layer is $J_l$, the time-scale for
charge loss from the boundary layers is given by

$$\tau_c = \frac{w R}{J_l},$$

(15)

from which the drag coefficient can be deduced. Before, however,
calculating the magnitude of the drag on the accretion stream
caused by parallel currents through the ambient plasma, let us
address the question of how large a magnetic field modification
these currents can produce in the vicinity of the accretion stream.

As the charge layer width is small compared with the plasma
stream width, which in turn is small compared with both the field
line length and the distance to the star, we need to compute only
the magnetic field close to the two narrow current ‘sheets’ flowing
parallel to the magnetic field out of charged boundary layers. The
magnetic field induced in the vicinity of current sheets with surface
current density $J_l$ is $B = 4\pi J_l/c = BV/\pi R^2$ (cf. equation 15), where $B$
is the unperturbed magnetic field. Expressing $\tau_l$ through the friction coefficient $\nu = B^2/4\pi p c^2\tau_l$,
then, one finds that $B/B = \nu R_0/2R$, where $\beta = 4\pi p v^2/B^2$.
As $v \ll R$ and $\beta < 1$, it follows that $\nu R_0/2R \ll 1$. Thus in the range of parameters for which deep
magnetospheric penetration takes place, the magnetic field can
be approximated by the stellar dipole field.

As the magnetic field lines are attached to a highly conducting
medium (the stellar ‘surface’ or photosphere), the electrostatic
potential difference between the accretion stream boundaries
results in the potential drop along the magnetic field. For potential
differences characteristic for astrophysical systems, discussed in
the previous section, the collisional mean free path of particles
accelerated by parallel electric fields would exceed the length of
the field line. Thus the parallel current would not be collisionally
supported; instead it would be determined by space-charge
effects.

4.1 Penetration in ‘vacuum’

Let us first consider the case when effects of the ambient plasma
are not important (we shall presently make this statement more
precise). We shall estimate the magnitude of the steady-state
space-charge-limited current in the one-dimensional (1D) approxi-
mation, from the Child–Langmuir law (Child 1911; Langmuir
1913). As in astrophysical situations of interest $\alpha = eU_0/m_p c^2 > 1$
($m_p$ is the proton mass) for even relatively narrow plasma streams
(in the example in the last section, this occurs for $h > 2$ km at
$R = R_e$), the space-charge limited flow of both electrons and ions
is ultrarelativistic. In such a case the 1D space-charge limited
current density is given by (Litwin & Rosner 1998)

$$J_{sc} = \frac{\alpha c U_0}{\pi d^2},$$

(16)

where $d$ is the distance between the electrodes; we shall identify it
with the distance between the accretion stream and the stellar
surface along the magnetic field line: $d \approx 1.4R$, for the geometry
specified in Fig. 1. The above current is much larger than the
space-charge limited current when only one charge-particle
species is involved (which is formally given by equation (16)
with $\alpha = 1/2$, cf. Jory & Trivelpiece (1969)); this is caused by the
space-charge neutralizing effect of the second species.

From equations (15) and (16), with $J_l = a J_{sc}$ ($a \ll h$ is the
charge layer width) and recalculating that $\sigma = VB/4\pi c$, the friction
coefficient $\nu = (1/\chi\tau_c)$ can be readily found, yielding

$$\nu = \frac{4ac ah}{\chi w d^2}.$$ 

(17)

Let us again consider a purely radial inflow in the vicinity of the
magnetic equator. Recalling that $hVB = \text{constant}$ and $\rho V B = \text{constant}$, it follows that $\nu = \lambda x R_e/\psi V A$, where $\lambda$
$= 4\mu R_0 \Omega_b h^2/\psi V A$, $\mu = a_{sc}/w A$, $\Omega_b = eB A/m_p c$, $x = R/R_e$, $\psi = V/V A$ and the subscript ‘A’ denotes the value at $R = R_e$.
Here and in the following we assume that $a/c = \text{constant}$, i.e. that
the shape of the density profile remains constant. The equation of

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$^1$The underlying assumption of the above discussion is that the thermal
effects on the plasma motion are negligible. This is justified as long as the
ion sound speed $c_s < V$. In the example discussed above, thermal effects
can be neglected along the whole inflow trajectory as long as temperature
$T < 6 \times 10^5 K$ at the magnetosphere boundary, a much higher temperature
than generally expected, even if the plasma compression during the inflow
were adiabatic (i.e. $T \sim R^{-8/3}$).
motion (13) then becomes
\[
\frac{d\psi}{dx} = -\frac{1}{2\xi^2} + \frac{\lambda x}{\psi},
\]  
(18)

We solve the above equation in the region 0 < x ≈ 1 for various values of parameter \(\lambda\), with the initial condition \(\psi(1) = 1\), corresponding to the free-fall flow velocity at \(R = R_A\). We find that for \(\lambda \approx \lambda_{crit} \approx 1.66\) the flow is initially decelerated and subsequently reaccelerated by the gravitational field; for \(\lambda > \lambda_{crit}\) the flow is continuously decelerated until it stops at a distance \(R_x > R_{crit} \approx 0.51R_A\) without reaching the stellar surface. In Fig. 2 we show examples of the two classes of solutions and compare them with the free-fall solution.

A deep penetration thus requires \(\lambda \leq 1.6\). This is equivalent to
\[
h_A^2 < h_{crit}^2 \approx 0.4 \frac{c X_A}{\mu R_A \Omega_A} d_R^2.
\]
Thus a deep penetration occurs if the accretion stream is sufficiently narrow; the maximum stream width \(h_{crit}\) depends, in particular, on the charge layer aspect ratio \(\mu (= a_h/w_A \ll 1)\) at \(R = R_A\). As \(a_h \ll R_A \ll w_A\), we assume \(\mu\) to be in the range \(10^{-2} - 10^{-1}\). For the case of accretion in a cataclysmic variable, discussed in the previous section, equation (19) implies that the charge-loss-induced drag force can be neglected if the stream cross-field width at the magnetospheric boundary \(2h_A < 4 \times 10^{5}\) km (for \(\mu = 0.05\)); the latter value is in particular larger than the estimated stream width \(\sim c_s / 2\omega = 5 \times 10^4\) km.

4.2 Penetration in ambient plasma

The situation can be different if ambient plasma of sufficient high density \(n_A\) is present, specifically if \(n_A\) is so high that the Bohm current \(J_B = e\eta c_s\) (Bohm 1949) exceeds \(J_{sec}(U_0, d)\). In this case, an electrostatic sheath is established along the magnetic field, with the width \(l < d\) such that \(J_{sec}(U_0, l) = 0.6J_B\) (Lieberman & Lichtenberg 1994).

The drag coefficient is obtained from equation (15), with \(j_L = 0.6\mu a_r\); one then finds, following reasoning similar to before, the equation of motion
\[
\frac{d\psi}{dx} = -\frac{1}{2\xi^2} + \frac{\xi x}{\psi} \frac{\lambda}{\psi},
\]
(20)

where \(\xi = 2.4\pi\mu R_A J_B / c B_A\). Assuming that \(J_B = \text{constant}\) (see however the later discussion), numerical solutions of equation (20) are found to be qualitatively similar to those shown in Fig. 2; one finds that deep magnetospheric penetration occurs if \(\xi < 2.4\), i.e. if
\[
n_A < n_{crit} = \frac{1}{\pi \mu e c B_A \Omega_a}.
\]
(21)

For the case of accretion on a cataclysmic variable that we discussed earlier, the above result implies that the plasma flow is unimpeded, e.g. (again assuming \(\mu = 0.05\)) if the ambient plasma temperature \(T_A = 10^4\) K (i.e. comparable to the temperature of the accreting stream material) and \(n_A < n_{crit} \approx 10^8\) cm\(^{-3}\).

A question thus arises: what is the ambient plasma density compared with \(n_{crit}\)? It is generally assumed in other theoretical works (e.g. see Joss, Katz & Rappaport 1979; Campbell 1983) that the region outside the accretion stream is essentially a vacuum, which a magnetosphere filled with plasma of density \(\sim n_{crit}\) is definitely not. Unfortunately, this question cannot be resolved, even qualitatively, on the basis of existing observations. One therefore has to resort to theoretical considerations. A fully reliable calculation – involving at minimum two-fluid magneto-hydrodynamics in three dimensions – is, however, beyond present capabilities, and we therefore have to rely on less certain model estimates.

First we note that if the ambient plasma were in hydrostatic equilibrium with the primary photosphere, its density in the vicinity of the Alfvén radius would be much smaller than \(n_{crit}\) because of its small scaleheight. If the companion star possessed a corona similar to the solar one, its scaleheight would be much larger; nevertheless, because of its low pressure and high ionization degree, this unbounded coronal plasma is not expected to penetrate the strong magnetic field (>1 kG) of the primary near the Alfvén radius.

Another possibility is that the ambient plasma is not in hydrostatic equilibrium but is instead constantly fed by the mass loss from the accretion stream. We discuss this now.

4.3 Plasma filling of the magnetosphere

In the context of the model discussed in the present paper, it is possible to estimate the ambient plasma density on the basis of mass loss from the accretion stream. This mass loss is caused by the shearing-off of the boundary layers: because the electric field is non-uniform in the charge layer, the \(E \times B\) flow velocity is non-uniform as well. As discussed at the end of Section 3, for the density profile \(n(z) = n_0[1 - (z - h)\theta(z - h)/a]\) (where \(z \approx h + a\), \(a \ll h\) and \(\theta\) is Heaviside’s function), \(E = 4\pi e\delta n(z)\) (recall that \(\delta = V_0 / \Omega_b X_b\)) and therefore
\[
V(z) = \frac{4\pi e c \delta}{B} n(z) = \frac{n(z)}{n_0} V_0;
\]
(22)

here and in the following the subscript 0 denotes quantities in the plasma bulk. The portion of the stream within the sonic points \([V(z) \approx c_s]\) will penetrate the magnetosphere; however, plasma with a cross-field drift velocity much slower than the sound speed [say, with \(V(z) < c_s / 2\)] will primarily spread along the magnetic field. This occurs for \(z > z_s\), where \(z_s\) is determined from the condition \(V(z_s) = c_s / 2\) or
\[
n(z_s) = \frac{c_s}{2V_0 n_0}.
\]
(23)
and its flux density by \( \bar{n}_0 c_\perp \). The escaping plasma spreads along the magnetic field, reducing the gap between the accretion stream and the stellar surface; the current from the plasma to the star increases until it becomes so large that it erodes the plasma front: \( J_\perp \approx J_B \) where \( J_B \) is the Bohm current. As before, we find the density of the surface current flowing out of the charge layer, \( j_s \approx q_0 \bar{n}_0 c_\perp \Delta \); the corresponding charge-loss timescale is given by equation (15). The equation of motion then becomes

\[
\frac{d\phi}{dx} = -\frac{1}{2\pi} + \frac{\xi}{x\psi^2},
\]

where

\[
\xi = \frac{\mu R \Omega_A}{8 V_A} \left( \frac{c_s}{V_A} \right)^2 = \frac{\mu R \Omega_A}{16 V_e} \left( \frac{c_s}{V_e} \right)^3.
\]

As before, we solve the above equation numerically for various values of \( \xi \). We find that a deep penetration occurs for \( \xi < 0.72 \) or, alternatively, for

\[
T < T_{\text{crit}} \approx 8200 \left( \frac{\mu}{0.05} \right)^{-2/3} \left( \frac{M}{M_\odot} \right)^{4/3} \left( \frac{B_s}{25 \text{ MG}} \right)^{-2/3} \left( \frac{R_*}{10^8 \text{ cm}} \right)^{-2}.
\]

For significantly higher temperatures the flow is strongly decelerated in the vicinity of the Alfvén radius. This behaviour is illustrated in Fig. 3.

Thus, for typical parameters of AM Her cataclysmic variables, a deep penetration can occur if the plasma temperature is less than \( T_{\text{crit}} \sim 10^4 \text{ K} \). This temperature is comparable to the temperature near the Lagrange (L1) point (cf. Schmidt, Liebert & Stockman 1995) and thus can be expected to be the accreting gas temperature in the outer magnetosphere. In the closer proximity of the primary surface, the accreting plasma temperature may be higher, which would prevent further cross-field penetration. Thus, the ‘attachment’ of the accretion stream to the magnetic field occurs at a radius determined by the temperature profile rather than at the Alfvén radius; in the range of parameters of AM Her cataclysmic variables, both the attachment near the Alfvén radius and the penetration to the close vicinity of the stellar surface appear possible.

5 DISCUSSION

In this paper we have argued that plasma accreting on to an object surrounded by a magnetosphere can, under well-defined circumstances, penetrate deeply into the magnetosphere, well past the Alfvén radius (and even to the close proximity of the stellar surface), if its permittivity is sufficiently high. The particular mechanism responsible for cross-field propagation is \( E \times B \) drift, caused by the electric field arising from polarization currents in the accretion stream. An essential aspect of such a penetrating accretion flow is its finite extent in the direction perpendicular to the stream velocity, i.e. the accretion flow must have the geometric characteristic of a slab or beam, with cross-field dimension smaller than the magnetosphere characteristic spatial scales. For this reason, our solution is not relevant to, for example, uniform spherical accretion, or to the problem of plasma penetration associated with magnetospheres embedded in an ambient wind (such as planetary magnetospheres); in both of these cases, the charge separation necessary to create the electric fields driving the cross-field motions cannot occur without the intercession of further instabilities. In contrast, examples of systems where such accretion via cross-field penetration might be of considerable interest include accretion on to the primary in AM Her cataclysmic binary systems, and other systems in which accretion occurs in the form of a finite cross-section stream.

A crucial requirement for the above mechanism to function is that the depolarizing effect of the surrounding medium (in particular that of the central object itself) should be sufficiently weak. We showed that these effects result in a drag on the plasma flow, similarly to the effect of Alfvén wave emission on the motion of a satellite in the ionosphere (Drell et al. 1965). Estimating the drag force resulting from magnetic field-aligned currents, we determined the criteria for deep magnetosphere penetration; in particular, we found that deep penetration results for sufficiently narrow streams (such as suggested by the analysis of Lubov & Shu 1975) and for sufficiently low ambient plasma densities and temperatures.

We have employed our model to determine the ambient plasma density on the basis of mass loss from the accretion stream. We then found that deep penetration will result if the accreting plasma temperature is lower that a certain critical temperature \( T_{\text{crit}} \). For the specific example of radial accretion on to a magnetized white dwarf, with a dipolar magnetic field and parameters characteristic of AM Herculis cataclysmic variables, \( T_{\text{crit}} \sim 10^4 \text{ K} \), which is similar to the temperature near the Lagrange (L1) point (Schmidt et al. 1995); thus it can be expected to be the accreting gas

\[\text{Figure 3. Solution of equation (26), normalized to the free-fall solution } (F = \psi_0/\Delta), \text{ for } \xi = 0.6 \text{ (long-dashed line), } 0.718 \text{ (solid line), } 0.719 \text{ (short-dashed line) and 0.9 (dotted line).}\]

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temperature, at least in the outer magnetosphere. In the closer proximity of the primary surface, the accreting plasma temperature may be higher, which would prevent further cross-field penetration. In such a situation, the accretion stream would become ‘attached’ to the magnetic field at a radius smaller than the Alfvén radius; the cross-field penetration can be either deep or shallow, depending on the temperature profile.

It is quite striking that, despite an admittedly crude model, the plasma temperature at which deep penetration is found to occur is, in the case of AM Her CVs, comparable to the temperature expected to be the accreting gas temperature in the outer magnetosphere. We therefore conclude that both deep penetration and ‘attachment’ near the Alfvén radius are plausible in the range of parameters characteristic for AM Her CVs. Indeed, observations indicate that while in some systems a deep penetration (even to the close vicinity of the stellar surface) takes place, in other systems the ‘attachment’ occurs far from the surface of the primary. The latter appears to be the case in HU Aquarrii (Schwope, Mantel & Horne 1997); the former might be the origin of the accretion ring in V1500 Cygni discussed by Schmidt & Stockman (1991).

We have focused our discussion on steady-state radial accretion on to a magnetized white dwarf with a dipolar magnetic field. In reality, radial accretion will not, in general, occur: the impact parameter of the inflow is finite, owing to the non-vanishing specific angular momentum of the accreting matter. Nevertheless, this example demonstrates that, for realistic parameters, the plasma stream can penetrate very deeply into the magnetosphere. The accreting flow can reach field lines that are never near the magnetopause boundary.

While the general formalism discussed here is valid for an arbitrary time-dependent flow, our specific examples were limited to the steady state. In a non-steady situation, other charge-loss mechanisms than those discussed here have to be taken into account. On the other hand, the inductive effects, which are known to impede discharging of charged boundary layers (e.g. Baker & Hammel 1965), are likely to be important. The analysis of these effects is relegated to future publications.

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