Scilight

NEWS | AUGUST 02 2019

New variational calculus equation calculates space-charge limited current for any geometry FREE

Jodi Ackerman Frank

Scilight 2019, 310005 (2019)
https://doi.org/10.1063/1.5121645

AIP Publishing

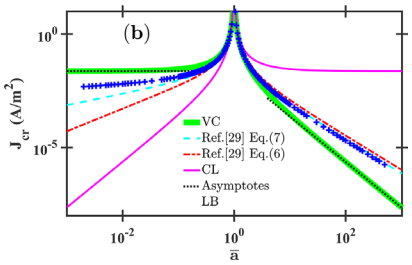
Scilight

30 July 2019

New variational calculus equation calculates space-charge limited current for any geometry

Jodi Ackerman Frank

Rather than directly solving Poisson's equation, researchers apply variational calculus to calculate space charge limited emission current density for cylindrical and spherical geometries and beyond.



Space-charge limited current (SCLC) is easy to calculate theoretically for one-dimensional (1D) planar geometry using Poisson's equation. But 1D cylindrical and spherical geometries require approximations because the curvature makes the differential equation nonlinear, which prohibits the derivation of a simple analytic equation.

To more accurately and efficiently determine SCLC in these curved geometries, Darr and Garner transformed Poisson's equation by turning to variational calculus (VC) to derive a differential equation for SCLC from first principles. Their new equation can calculate the SCLC not only for cylindrical and spherical shapes, but for *any* geometry.

Poisson's partial differential equation calculates electric potentials related to the charge density. The VC transformation eliminates the current density term by assuming that it is minimized, thus bypassing the unsolvable Poisson's equation for the analytically solvable Euler-Lagrange equation.

Darr and Garner found that VC provides a first-order method for assessing the behavior in more complicated geometries much faster than more detailed simulation techniques, such as particle-in-cell, thus enabling rapid screening before performing more fine-tuned simulations. After the discovery,

they did not want to limit their VC study of SCLC to numerical approaches as has been done in the past. Instead they focused on the feasibility of analytical methods.

"The immediate benefit of the analytical VC techniques we used is that the potential function was calculated exactly from first principles with no need to guess or provide artificial constraints upon its form," said author Allen Garner.

The researchers are now further developing the VC model to study magnetrons and expect to extend it to investigate other crossed-field devices in the future.

Source: "A coordinate system invariant formulation for space-charge limited current in vacuum," by Adam M. Darr and Allen L. Garner, *Applied Physics Letters* (2019). The article can be accessed at https://doi.org/10.1063/1.5115261.