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# Free Vibrations of Constrained Beams[1]

H. F. WEINBERGER.[2] The interesting computational method presented in this paper can be considered as a special case of the following results of the writer (an exception is the case of a continuous elastic foundation, which is not covered by these results):

Let problem A be a beam problem with boundary conditions such that its eigenvalues and eigenfunctions are explicitly known, for example, the authors' "simple beam." Associated with this problem is a "Green's function" or influence function $G(x, y;\ \Lambda)$. This function is the steady-state solution of the beam under an oscillating unit point force with frequency $\Lambda$ applied at $y$. In some cases, for instance for the uniform beam, the function $G$ can be found explicitly from its definition. Otherwise, $G$ can be evaluated from the formula

$$G(x, y;\ \Lambda) = \Sigma \frac{\phi_r(x)\ \phi_r(y)}{\Lambda^2 - p_r^2} \qquad [1]$$

which gives the connection of our results with those of the authors.

The problem whose eigenvalues (natural frequencies) are to be found is called problem B. It differs from problem A in that $n$ new conditions

$$J_i[Y] = 0 \qquad [2]$$

are imposed. At the same time, $n$ other conditions on the solution of problem A are relaxed. For example, if the beam is pinned at the point $h_1$, the condition $Y(h_1) = 0$ is imposed, while the continuity of the third derivative of $Y$ is no longer required at $h_1$.

If, now, the condition $J_i$ is imposed on $G(x, y;\ \Lambda)$ as a function of $x$, a function of $y$ alone is obtained. We impose condition $J_j$ on this function and write the result as $J_iJ_jG(\Lambda)$. This yields an $n$ x $n$ matrix whose determinant we call $D(\Lambda)$. This is precisely the determinant obtained by the authors. (To obtain their form in Equation [15], the fact that

$$\frac{\partial^3}{\partial x^3} G(x, y;\ \Lambda)$$

has a discontinuity of magnitude $1/EI(y)$ at $x = y$ must be used.) In terms of this determinant we can find not only the eigenvalues of problem B but also their proper multiplicities, i.e., the number of eigenfunctions associated with them. These are obtained in exactly the same way as the corresponding quantities are obtained from the so-called Weinstein determinant in the Weinstein method for plates; namely, if $M_A(\Lambda)$ and $M_B(\Lambda)$ are the multiplicities of $\Lambda$ as an eigenvalue of problems A and B, respectively, and if $z(\Lambda)$ is the order of $D(\Lambda)$, i.e., $(\Lambda - \Lambda_0)^{-z(\Lambda_0)} D(\Lambda)$ is regular and nonzero at $\Lambda_0$, for any fixed $\Lambda_0$; then

$$M_B(\Lambda) = M_A(\Lambda) + z(\Lambda) \qquad [3]$$

When the zeros of $D(\Lambda)$ are simple and $\Lambda$ is not an eigenvalue of problem A, this reduces to the results of the paper under discussion.

The occurrence of a Weinstein determinant leads one to inquire as to the relation of this method to the variational method of Weinstein. The latter consists of two parts: a variational idea and a computational method. The variational idea is based on the fact that if one eigenvalue problem is less constrained than another, it has lower eigenvalues. Thus it is possible to obtain a sequence of lower bounds for the eigenvalues of a given problem if another one which is infinitely less constrained has known eigenvalues and eigenfunctions. These lower bounds are the eigenvalues of the known problem with a finite number of added constraints. This variational idea does not concern us here.

However, the fact that makes the Weinstein method of practical interest is that the eigenvalues of a problem obtained by imposing a finite number of constraints on a known eigenvalue problem can be solved explicitly in terms of the Weinstein determinant. It is this part of the Weinstein method that is used in the paper under discussion.

The results given in this discussion hold not only when constraints are imposed, but also when they are removed or substituted, e.g., if the spring constant is changed. This simply means that the computational part of the Weinstein method holds not only for the imposition of constraints but for any finite change of constraints.

### AUTHORS' CLOSURE

The authors wish to thank Dr. Weinberger for his succinct and interesting discussion in which he so ably summarizes the existing situation.

The role played by the Green's function had been pointed out by one of us previously, reference (4), so that it was not repeated in the present paper.

The relation between the present method and that of Weinstein is particularly interesting. His procedure for calculating eigenvalues is to relax the outer boundary conditions in such a way as to get a problem which can be solved. Then the solution of the desired problem is approached by letting the boundary conditions approach the desired ones. In our case, however, the outer-boundary conditions remain the same but inner conditions are added which, as Dr. Weinberger points out, introduces discontinuities in the third derivative of the Green's function at the points where the constraints are added. These correspond to introducing reactions at the points of constraint.

[1] By W. F. Z. Lee and Edward Saibel, published in the December, 1952, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 74, pp. 471–477.

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# Deflection and Stresses in Beams Subjected to Bending and Creep[1]

FOLKE K. G. ODQVIST.[2] The writer appreciates the authors' new method of attack with a close approximation to the total creep over a considerable time. For the initial stages of creep in bending more accurate results will certainly be obtained than with the writer's formulas which correspond to the case $K = 0$ (authors' notation). Nevertheless, attention is drawn to the writer's work Hållfasthetslära, Stockholm, 1948, p. 805 (published in Swedish) where on pp. 737–741 some of authors' cases, including that of Fig. 3, are treated, as well as a few more. There is also to be found an estimation of the influence of shearing stress on deflection.

### AUTHORS' CLOSURE

The authors wish to thank Professor Odqvist for his interest in their paper and are glad that attention has been drawn to his fine work in this field. It should be clear, however, that in this particular paper, the authors were more interested in the development of theory rather than in the number of specific cases that could be solved.

[1] By Yoh-Han Pao and Joseph Marin, published in the December, 1952, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 74, pp. 478–484.

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