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Discussion

Nonideal Isentropic Gas Flow Through Converging-Diverging Nozzles[1]

J. C. Leung[2] and M. Epstein.[2] In their recent paper the authors gave a numerical method of treating nonideal gas flows through converging-diverging nozzles based on the Redlich-Kwong (R-K) equation of state (EOS). Though they present a case where nonideal gas effects are important, they did not provide the range of inlet conditions where this nonideality is significant. The present writers had published a nonideal gas flow paper (Leung and Epstein, 1988) earlier. This paper treats only the critical flow through convergent nozzles and not the divergent section where supersonic flow prevails. Using also the R-K EOS, generalized solutions in terms of reduced pressure, reduced temperature and the specific heat ratio at zero pressure (ideal gas specific heat ratio k^+ or $\gamma = C_p{}^+/C_v{}^+$) were presented. Moreover, these were algebraic equations from which maximum flow and throat conditions can be easily obtained. Generalized charts, such as Fig. 1 for γ of 1.2, were most useful in displaying the conditions under which large departures from the ideal gas solution can be expected. We should point out that a constant $C_p{}^+$ has been assumed, as is common practice in ideal gas treatment and which allows the simplicity in results. The success of such a generalized treatment, admittedly less precise than the subject paper, was demonstrated by comparison with both experimental data (Hendricks, 1974), and critical flow data (calculated with very complex virial-type EOS) published by Johnson (1964, 1965, 1968, 1970, 1972, 1974) at NASA Lewis Research Center. To provide a further comparison, we take the two cases discussed in the subject paper for methane gas and tabulate the following results based on our generalized solution approach as well as the interpolated results from the Johnson's publication (1972). For the nonideal case (Case I), our solution is in good agreement with the subject paper, and our mass flux is only 4 percent lower than Johnson's data which were calculated using NASA's extensive computer program (Johnson, 1971). As for the near-ideal case (Case II), our generalized chart of Fig. 1 clearly demonstrates that nonideal gas effects are unimportant for the given reservoir conditions.

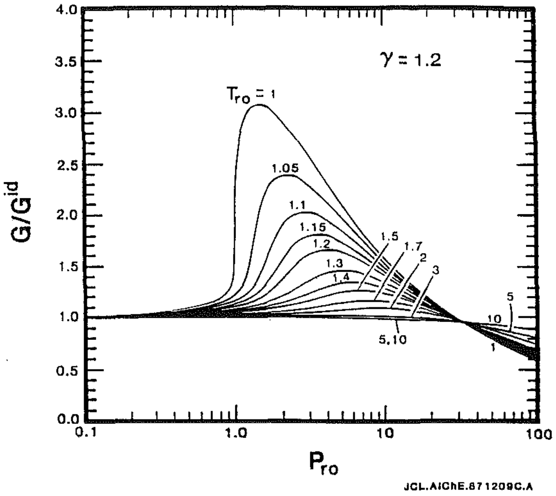

Fig. 1 Generalized chart for $\gamma = 1.2$ (G^{id} denotes ideal gas mass flux based on γ)

Table 1

Critical flow parameters	$\frac{\dot{m}\sqrt{RT_0}}{A\ P_0}$	$\frac{P_t}{P_0}$	$\frac{T_t}{T_0}$
Case I (Nonideal condition) $P_{r0}=5.0 \quad T_{r0}=1.4$ $C_p{}^+=2124$ J/kg-K $\quad k^+=1.32$			
Bober and Chow (1990)	0.93[(a)]	0.406	0.85[(b)]
Leung and Epstein (1988)	0.926	0.418	0.846
Johnson (1972)	0.964	0.416	0.843
Case II (Near ideal condition) $P_{r0}=0.8 \quad T_{r0}=3.0$ C_p+3150 J/kg-K $k^+=1.20$			
Bober and Chow (1990)	0.650[(a)]	0.463[(c)]	0.905[(c)]
Leung and Epstein (1988)	0.6505	0.560	0.907
Johnson (1972)	0.6506	0.560	0.905

(a)From Fig. 14 of Bober and Chow.
(b)From Fig. 1 of Bober and Chow.
(c)From ideal gas evaluation using $k = 1.209$.

Additional References

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[1] By W. Bober and L. W. Chow published in the December, 1990 issue of the JOURNAL OF FLUIDS ENGINEERING, Vol. 112, pp. 455–461.
[2] Fauske & Associates, Inc., Burr Ridge, Ill.