Plastic Yielding of a Hard Asperity Sliding on a Soft Flat Surface," *Wear*, Vol. 87, 1983, pp. 151–161.
28 Kayaba, T., Hokkirigawa, K., and Kato, K., "Experimental Analysis of the Yield Criterion for a Hard Asperity Sliding on a Soft Flat Surface," *Wear*, Vol. 96, 1984, pp. 255–265.

APPENDIX

Geometrical Construction of the Domain KBJ

The construction of the field *KBJ* is shown in Fig. 12. From the geometry shown in this figure, it is found that $I\hat{J}B = (\pi/2) - \eta_2 - (\theta/2)$ and, hence, the circle (*Q*) which passes from *I* and *B* is geometrically determined. Thus, the intersection between the circle (*Q*) and the line which passes from I and makes an angle θ with *IB* defines point *J*. Consequently, the bisector of BJ intersects circle (*Q*) at O. The boundary *BJ* of the slip-line field can be obtained by drawing a circle (*S*) with center at O and radius OB.

Two characteristic slip-lines in field KBJ are shown in Fig. 12. Line TP is a β-line and the curve *PR*, which is an arc of a circle centered at *I* with radius *IP*, is an α-line. The angle, ω, that the α-lines meet the boundary *BJ* is not constant. It assumes a value equal to η_2 at *B* and *J* and a slightly higher value between. This implies that the interfacial shear stress, *s*, which is related to the interfacial angle, ω, through the following relation

$$s = k\cos(2\omega)$$

is not constant along the boundary *BJ*. In order to simplify the calculations it is reasonable to assume that $\omega \approx \eta_2$.

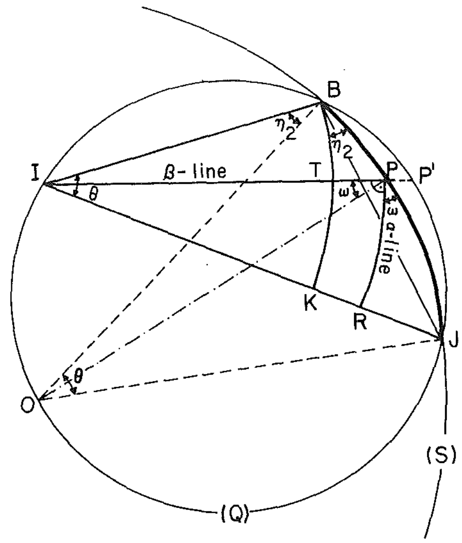

Fig. 12 Geometrical construction of field KBJ

This assumption is fairly correct because the arc *BJ* of circle (*S*) is very close to that of circle (*Q*), i.e., the distance *PP'* is very small so that the approximation $I\hat{P}O \simeq I\hat{B}O$ or $\omega \simeq \eta_2$ is valid. Thus, equation (1) can be used for calculating the shear stress on the boundary *BJ*.

DISCUSSION

F. E. Kennedy, Jr.[1]

The authors should be congratulated for producing a quite complete plasticity analysis of plowing phenomena in sliding contacts. Their model agrees very well with experimental friction coefficients obtained in a variety of tests with conical sliders under both lubricated and unlubricated conditions. Those results point out quite clearly the importance of plastic deformation in plowing and the major contribution of plowing to friction in many sliding contacts involving hard asperities. The application of the model to the authors' dry sliding experiments involving like materials prompts several questions, however. In particular, I hope that the authors might clarify the following points:

1. How were the semi-asperity angles used in Table 3 determined for the cases of like materials in dry sliding? Were the across-the-groove surface profiles of the disk specimens (Figure 1) used, and if so, would those across-the-groove angles be indicative of the angles of plowing asperities on the pin specimens measured in the sliding direction? Was there any evidence from the microscopic observation of the pin surfaces (eg. Figures 2–6) that asperities with α as small as 60° were actually present on the pins?
2. Could third-body particles (wear debris) be responsible for much of the plowing? Either loose debris or debris accumulation on the leading edge of the pins (as a prow) could have attack angles as large as those used in calculations for Table 3. Such debris could easily be work hardened to a hardness greater than that of the material being plowed.
3. It appears that the authors assumed that all plowing asperities, including both the rigid asperity of Figure 7 and the deforming hard asperity of Figure 11, were conical in shape. Would the resulting plastic flow around the cone really be approximated by a plane strain condition?

[1]Thayer School of Engineering, Dartmouth College, Hanover, N.H. 03755.

K. Kato[2]

Several analytical results to estimate the friction force and wear type with slip-line field theory were proposed in the past, some of which are listed by the authors.

The authors should be congratulated in trying to give a more precise analysis for the plowing process. The agreement between the calculated and experimental values of the coefficient of friction seems very good. But the discusser would like to suggest some points to be checked for better understanding:

(1) The question of the wear type: Although the authors' analysis is for the plowing process which should not generate wear debris, Figs. 7 and 8 imply the formation of microchips. Were any microchips of wear debris observed in the sliding tests and, if so, how large were they on the average?

(2) Observations by other researchers: What would be the experimental evidence for the existence of dead zone in their studies?

(3) The question of the quantitative differences between authors' and other researchers' calculated values. Every theoretical model has its own assumptions. In order to know its reliability and usefulness, quantitative comparisons between theoretical values by different methods are necessary. How large would the quantitative differences be between calculated values of authors and Challen et al. [A1] for example?

The discusser obtained good agreement between experimental values and theoretical values calculated with the theory of Challen et al. [A2].

Additional References

A1 Challen, J. M., and Oxley, "An Explanation of the Different Regimes of Friction and Wear Using Asperity Deformation Models," *Wear*, Vol. 53, 1979, pp. 229–243.
A2 Kato, K., and Hokkirigawa, K., "Abrasive Wear Diagram," *Proceedings of Eurotrib 85 Congres International de Tribologie*, Lyon, France, Sept. 9–12, 1985.

[2]Tohoku University, Sendai, Japan; Currently at NASA Lewis Research Center, Cleveland, Ohio.

Authors' Closure

We wish to express our appreciation to Professors Kennedy and Kato for their generous comments on the paper and for the critical questions. We hope that our response will help clarify the points raised by the discussers.

Reply to Professor F. E. Kennedy

The angles in Table 3 were determined from the surface profiles obtained perpendicular to the sliding direction. We have assumed that groove formation resulted from the microcutting action of the entrapped wear particles and/or hard asperities, and that the angle α is the same in directions perpendicular and parallel to the sliding direction. It is possible that some of the wear debris adhered initially to the surfaces and agglomerated to form wedges (prows), such as those reported in references [3] through [8], before it became loose eventually. The micrographs of the worn surfaces have indicated, however, that in most cases long plowing grooves formed parallel to the sliding direction. This experimental evidence suggests that plowing takes place primarily due to the wear debris entrapped at the interface. Because microscopic observation of the interface during sliding is not possible, and the orientation of the wear particles trapped at the interface cannot be determined, it was decided to use the values of α from the transverse surface profiles. Different kinds of experiments, such as with wedges of known cutting angles α or with abrasive papers where the slopes of the abrasive grits can be measured, are necessary. Nevertheless, a comparison of the theoretical values of the coefficient of friction with experimental values obtained from cutting experiments with conical tools of known angles α has shown that the aggreement was very good (see Fig. 10 and Table 2).

With respect to the question about the plane strain assumption for the plastic flow, plane strain conditions can be assumed to prevail when the width of cut is much larger than the depth of cut. The plane strain condition is an appropriate assumption when the rigid asperity (or wear particle) has the shape of a wedge. The flow is then confined to planes normal to the edge of the wedge and the problem can be analyzed fairly accurately with two-dimensional slip-line fields such as those shown in Figs. 7 and 11. Moreover, the plane strain condition is a reasonable approximation even when the wear debris and the asperities are idealized with spheres or cones, for example, provided that the depth of penetration is significantly less than the width of the formed groove. This is typically the case in metal sliding where the penetration depth to groove width ratio is much less than one. In the present study the depth-to-width ratio assumed values less than 0.3 (for lubricated sliding it was below 0.2) and thus the plane strain assumption for the plastic flow seems reasonable. Because the theoretical friction coefficients obtained from the slip-line analysis were in good agreement with the experimental friction results the plane-strain assumption, we think, is justified.

Reply to Professor K. Kato

The term plowing has been reserved for the plastic flow of a soft surface when a loaded rigid asperity slides over it. From the wear point of view, the plastically deforming material flows upwards and sideways of the microcutting edge resulting in the formation of wear debris (microchips) and ridges. Under certain conditions, e.g., for very soft metals such as indium and lead, the amount of wear debris produced is small and the deformed material is displaced along the sides of the groove. This type of wear, where very small amount of material is removed, has been referred to in the past as plowing. Alternatively, when the amount of wear debris is significant the material is removed as in metal grinding, i.e., in discontinuous microchips. Under these conditions the type of wear is microcutting.

From the friction point of view, however, plowing is referred to as the friction mechanism responsible for the formation of grooves due to the entrapped wear particles or hard asperities. Consequently, the plowing friction mechanism should not be confused with the particular type of wear. Numerous studies in the past have shown that wear debris and microchips are generated when plowing friction conditions prevail at the interface. This was also found to be the case in the present study. Wear debris formation was observed in all experiments. Because the focus of this study was on friction, systematic characterization of the debris was not attempted. However, wear particles in the range 1-10 μm were observed and the calculated wear coefficients were in the range 10^{-4} to 10^{-2}.

Furthermore, the analysis based on the slip-line model of Figs. 7 and 8 does not depend on the formation of wear debris and microchips. The friction coefficient is determined only from the deformation field defined by the boundaries HI, IA, ABJC, and HGFEDC, and thus it is appropriate for any plowing conditions, independent of the kind of wear.

It is difficult to identify the existence of a dead zone during sliding because observation at the interface when plowing occurs is not possible. The existence of a dead zone, however, has been observed in numerous plowing experiments in the past with cutting tools and in metal grinding. Because plowing in metal sliding is essentially similar to metal grinding [18], but on a smaller scale, it is expected that a dead zone will also form in front of the plowing wear particles and asperities during sliding.

As regards to the quantitative agreement between the theoretical friction values, our approach to the problem was to consider primarily all the qualitative aspects in plowing friction. The proposed models are in agreement with the experimental evidence obtained for plowing conditions. In addition, the remarkably good agreement between theoretical and experimental friction values of our work and other studies, indicates that the theoretical models are also quantitatively correct. The qualitative differences between our model and those most commonly used in the past are apparent from the details given in the paper. Because most of these models are in poor qualitative agreement with the experimental evidence, any correlation between friction values calculated from those models and experimental results must be fortuitous.

In particular, the qualitative limitations of the models proposed by Challen and Oxley [17] have been addressed in the introduction. Hence, a quantitative comparison between our model and those of reference [17] is unrealistic. In reference [A2], however, a comparison between experimental values and theoretical values obtained from [17] was attempted. Although an artificial correction factor was introduced to bridge the gap between the theoretical and experimental friction values of that study, the agreement is poor. In fact, it was found that the experimental friction coefficient values were higher than the theoretical friction coefficients by 30 to 70 percent.