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Mesoscale Variability of the Upper Colorado River Snowpack

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In the mountainous regions of the Upper Colorado River Basin, snow course observations give local measurements of snow water equivalent, which can be used to estimate regional averages of snow conditions. We develop a statistical technique to estimate the mesoscale average snow accumulation, using 8 years of snow course observations. For each of three major snow accumulation regions in the Upper Colorado River Basin – the Colorado Rocky Mountains, Colorado, the Uinta Mountains, Utah, and the Wind River Range, Wyoming – the snow course observations yield a correlation length scale of 38 km, 46 km, and 116 km respectively. This is the scale for which the snow course data at different sites are correlated with 70 per cent correlation. This correlation of snow accumulation over large distances allows for the estimation of the snow water equivalent on a mesoscale basis. With the snow course data binned into 1/4° latitude by 1/4° longitude pixels, an error analysis shows the following: for no snow course data in a given pixel, the uncertainty in the water equivalent estimate reaches 50 cm; that is, the climatological variability. However, as the number of snow courses in a pixel increases the uncertainty decreases, and approaches 5-10 cm when there are five snow courses in a pixel.

Introduction

The Colorado River receives 70 per cent of its total annual flow from spring snowmelt (Chang *et al.* 1987), and this water is of major economic importance to the southwestern United States. Managing this vital resource requires synoptic, basin-wide observations of the snowpack extent and equivalent water content. Presently,

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snowpack water equivalent estimates are made from a relatively small number of measurements, considering the size of the Upper Colorado River Basin, that utilize a variety of techniques. The most numerous, and with the longest historical record, are the snow course observations made by the Natural Resources Conservation Service, (formerly the Soil Conservation Service) of the US Department of Agriculture. A snow course measurement consists of an average of approximately 10 point measurements of snow depth and water equivalent along a course that is approximately 100 m long. Monthly observations are made during the snow season. In the Upper Colorado River basin there are over 400 snow courses, but only 137 had records suitable for this study. The others were eliminated because of missing data. Despite this apparently large number of measurements, there are large regions where no observations are available and extrapolation of the distant measurements provide the only means of estimating the snowpack properties.

Satellite-borne passive microwave sensors, with their all-weather capability, are particularly suited to provide the necessary synoptic snowpack observations from remote areas. Passive microwave observations provide information on both the snowpack extent (Josberger 1989) and the internal snowpack properties, including the water equivalent and the grain size (Chang *et al.* 1987). The large footprint of passive microwave sensors, approximately 30 km, gives spatially averaged information over mesoscale-length scales and hence such sensors are ideal for determining large-scale snowpack properties. However, the mesoscale footprint makes interpretation of the observations difficult, especially in the Upper Colorado River Basin with its rugged and variable terrain which results in a snowpack with a strong spatial variability.

This study was motivated by an ongoing research effort to determine snowpack properties for the Upper Colorado River Basin from satellite passive microwave observations made by the Scanning Multichannel Microwave Radiometer (SMMR) on the Nimbus-7 spacecraft from 1979 to 1987 and the Special Sensor Microwave Imager (SSMI) flown by the Defense Meteorological Satellite Program (DMSP). The development and validation of algorithms to determine snowpack properties from passive microwave observations requires the extension of a few point measurements made on the ground to a mesoscale region corresponding to the size of a pixel. Chang *et al.* (1987) have shown that there is a strong correlation of *in situ* point snow water equivalent measurements with changes in brightness temperature as measured by SMMR. Foster *et al.* (1984) give an overview of passive microwave snow research which also has had success in determining snowpack properties for regions which are relatively homogeneous on mesoscale-length scales, particularly the Canadian Plains. In this study we suggest how appropriate mesoscale averages of the *in situ* measurements can be constructed for comparison with satellite data, and determine the uncertainty in these averages. These techniques are also applicable to a variety of hydrologic problems where point measurements must be extrapolated to give larger-scale averages.

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The issue addressed is a sampling question: given a few point measurements of the snow water equivalent, how well can one estimate the average snowpack for a pixel? The answer depends on how variable the snowpack is, and on how representative the point measurement is when compared with other measurements in the pixel. We quantify these properties in terms of the variance of the snow measurement at given locations and the correlation between measurements at different locations. To estimate these quantities, we have used observations from 137 snow course sites in the Upper Colorado Basin for the period fall 1979 through winter 1986. This period corresponds to the passive microwave observations made by SMMR instrument.

In the basin there are also daily water equivalent measurements obtained from the SNOTEL network which telemeters the data from remote mountain sites. However, we used only the snow course data rather than the SNOTEL data because even though there are now about 100 SNOTEL sites, there were far fewer in the early part of the study period. Also, at most of the SNOTEL sites, there is a snow course. Therefore the snow course data set provides the greatest spatial and temporal coverage of snow conditions in the Upper Colorado River Basin.

The study region is bounded by 36° N to 44° N and 105° W to 113° W. It is divided into 1/4° latitude by 1/4° longitude areas which correspond to the picture elements, pixels, of the gridded satellite brightness temperature maps produced by NASA. Within a pixel, the number of snow courses ranges from zero to nine. For this study, we divided the basin into its three major mountainous regions (each containing many pixels), which accumulate most of the snow: the Colorado Rocky Mountains, Colorado; the Uinta Mountains, Utah; and the Wind River Mountains, Wyoming.

Analysis

To analyze the snowpack variability, we use the method of kriging as described by Cressie (1991). Let z_{xpq} represent the amount of snow on the ground at location x for month p of year q, expressed in centimeters of water. The overbar, $\bar{z}_{xp}$, denotes the climatological average over many years for particular month and location. Let the departures from the climatological average be denoted by

$$w_{xpq} = z_{xpq} - \bar{z}_{xp} \tag{1}$$

The upper case Z_{pq} indicates an area average for month p; of year q over a rectangle with area A

$$Z_{pq} = A^{-1}\iint z_{xpq} d^2x \tag{2}$$

Since they remain fixed in what follows, the subscripts p and q are suppressed. We

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use the notation d^2x to indicate an infinitesimal area $dxdy$, or dA, in the rectangle A.

We seek an estimate of Z in terms of z_i observations of the snow accumulation at measurement sites $x = x_i$. Take the linear estimator

$$\hat{Z} = \tilde{Z} + \sum_{i=1}^{i=N} a_i w_i = \tilde{Z} + \sum_i a_i w_i \tag{3}$$

which estimates the average snowpack for an area to be equal to the climatological areal average $\bar{Z}(= A^{-1} \iint \bar{z}_{xp} d^2x)$ plus a weighted sum of the departures from the climatological average at N point measurement sites. Note that the symbol $\sum_i$ has been used to represent summation over i from 1 to N, for brevity. The weights, a_i, are chosen to minimize the variance of the estimation error

$$\Psi = E(\hat{Z}-Z)^2 \tag{4}$$

Here $E(\,)$ is the expectation operator. The minimization proceeds as follows (Eq. (5) through Eq. (8))

$$\frac{\partial \Psi}{\partial a_j} \equiv 0 \; ; \qquad j = 1, \ldots, N \tag{5}$$

or

$$E\{2(\hat{Z} - Z)(\frac{\partial \hat{Z}}{\partial a_j} - \frac{\partial Z}{\partial a_j})\} = 0 \tag{6}$$

Since $\partial Z/\partial a_j = 0$, $\partial \bar{Z}/\partial a_j = 0$, and $\partial(\sum_i a_i w_i)/\partial a_j = w_j$, we have

$$E(\frac{\hat{Z}\partial \hat{Z}}{\partial a_j} - \frac{Z\partial \hat{Z}}{\partial a_j}) = E\{(\hat{Z} + \sum_i a_i w_i) w_j - Z w_j\} = 0$$

or

$$E(\bar{Z} w_j) + \sum_i a_i E(w_i w_j) = E(Z w_j) \; ; \qquad j = 1, \ldots, N \tag{7}$$

but

$$E(\bar{Z} w_j) = \bar{Z}\, E(w_j) = 0$$

and

$$E(Z w_j) = E(A^{-1} \iint z_x w_j d^2x) = A^{-1} \iint E(z_x w_j d^2x)$$

$$= A^{-1} \iint E(w_x w_j d^2x)$$

therefore

$$\sum_i a_i E(w_i w_j) = A^{-1} \iint E(w_x w_j \; d^2x) \; ; \qquad j = 1, \ldots, N \tag{8}$$

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The above equation is the condition for minimizing the estimation error, which we express more compactly as

$$\sum_i a_i R_{ij} = S_j \tag{9}$$

where

$$R_{ij} = E(w_i w_j) \quad \text{and} \quad S_j = A^{-1} \iint E(w_x w_j)\, d^2x \;; \quad j = 1, \ldots, N \tag{10}$$

Using Eqs. (1), (2), and (3), the minimum variance of the estimation error is

$$\Psi_{\min} = E(\hat{Z} - Z)^2 = E(\bar{Z} + \sum_i a_i w_i - \bar{Z} - A^{-1} \iint w_x d^2x)^2$$

$$= E(\sum_i a_i w_i - A^{-1} \iint w_x d^2x)^2$$

$$= E(\sum_i a_i w_i)^2 - E\{2(\sum_i a_i w_i) A^{-1} \iint w_x d^2x\} + E\{A^{-2}(\iint w_x d^2x)^2\} \tag{11}$$

but

$$E(\sum_i a_i w_i)^2 = E(\sum_{ij} a_i a_j w_j w_j) = \sum_{ij} a_i a_j E(w_i w_j) = \sum_j a_j S_j$$

$$E\{2(\sum_i a_i w_i)\; A^{-1} \iint w_x d^2x\} = 2 \sum_i a_i A^{-1} \iint E(w_x w_i) d^2x$$

$$= 2 \sum_i a_i S_i \equiv 2 \sum_j a_j S_j$$

and

$$E\{A^{-2}(\iint w_x d^2x)^2\} \equiv E\{A^{-2} \iiiint w_x w_{x'} d^2x d^2x'\} \equiv A^{-2} \iiiint E(w_x w_{x'}) d^2x d^2x'$$

$$= A^{-2} \iiiint R_{xx'} d^2x d^2x'$$

Therefore

$$\Psi_{\min} \equiv A^{-2} \iiiint R_{xx'} d^2x\, d^2x' - \sum_i a_i S_i \tag{12}$$

where the variables of integration x and x' both range over the entire rectangle A.

To carry out this procedure, we have estimated the climatological averages and the covariances R and S from the original observations z_{ipq} where $i \equiv 1$ to N measurement sites, $p \equiv 1$ to 4, for the months January through April, and $q = 1$ to 8 years. Thus we have taken

$$\bar{z}_{ip} = \frac{1}{8} \sum_{q=1}^{8} z_{ipq} \tag{13}$$

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$$\bar{Z}_p = \frac{1}{N}\frac{1}{8}\sum_{i=1}^{N}\sum_{q=1}^{8} z_{ipq} \quad (14)$$

To define the covariances, we calculated the sample covariance for each pair of observation stations

$$r_{nm} \equiv \frac{1}{4}\frac{1}{8}\sum_{p=1}^{p=4}\sum_{q=1}^{q=8} (z_{npq} - \bar{z}_{np})(z_{mpq} - \bar{z}_{mp}) \quad (15)$$

The correlations were computed by dividing by the sample variances $\sigma^2 = r_{nn}$ *which is assumed to be the same for each station of the same area*, and we also assumed that the covariance (& variance) does not change with month although the mean does change with month. Furthermore, the eight year sample means are assumed to be the same as the climatological average. Fig. 1a,b,c shows the results for the three areas. Despite the scatter in these sample correlations, they show the following general behavior. The correlations are quite high for very small separation distances and slowly decrease with increasing distance. As the distance approaches zero the correlations should approach one. To imitate this behavior, we approximate the correlation dependence on distance with the following expression

$$R_{ij} = \sigma^2\{\exp -0.357(\frac{d_{ij}}{L})^n\} \quad (16)$$

where d_{ij} is the distance between x_i and x_j. This equation contains two parameters n and L where n is a shape parameter for the curve and L is the characteristic length at which the average correlation becomes 70%. In other words, when $d_{ij} = L$, $R_{ij}/\sigma^2 = 0.7$. Table 1 gives the standard deviations, σ, and the values of n and L obtained by curve fitting through the data for the three regions. As shown in the table, the corresponding characteristic lengths are 46 km, 38 km, and 116 km respectively for the Uinta Mountains, Colorado, and Wyoming. For separation distances less than the characteristic length, the correlation would be higher than 70 per cent.

Table 1 – Values of the standard deviation σ, and parameters n and L.

	σ	n	L
Uinta Mountains	36 cm	0.43	46 km
Colorado	41	0.26	38
Wyoming	52	0.20	116

Results and Discussion

First, the departures from the 8-year climatological means are highly correlated across each of the three mountainous regions, as shown in Fig. 1a,b,c. Thus, typic-

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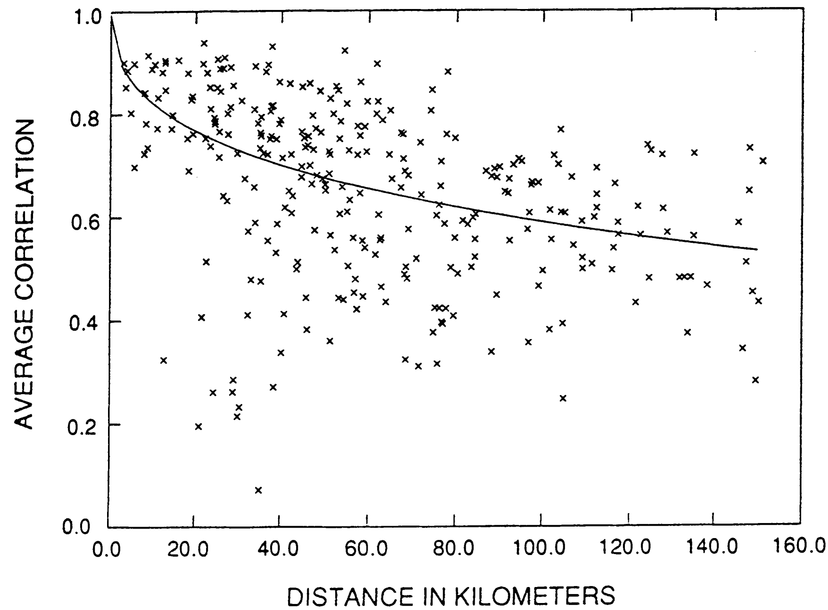

Fig. 1a. Average correlation as a function of separation distance for the Uinta Mountains, Utah. The solid line approximates the spatial variation of the correlation, Eq. (16).

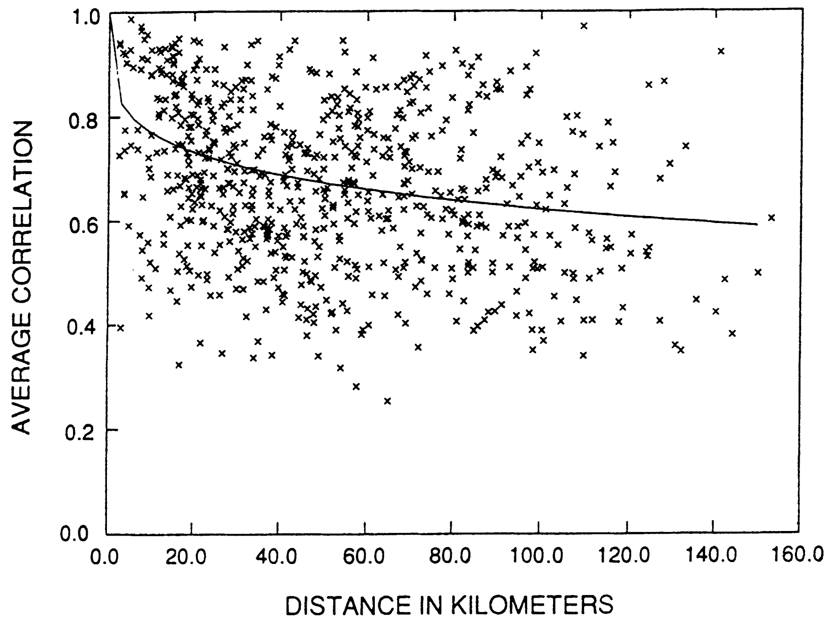

Fig. 1b. Average correlation as a function of separation distance for the Colorado Mountains. The solid line approximates the spatial variation of the correlation, Eq. (16).

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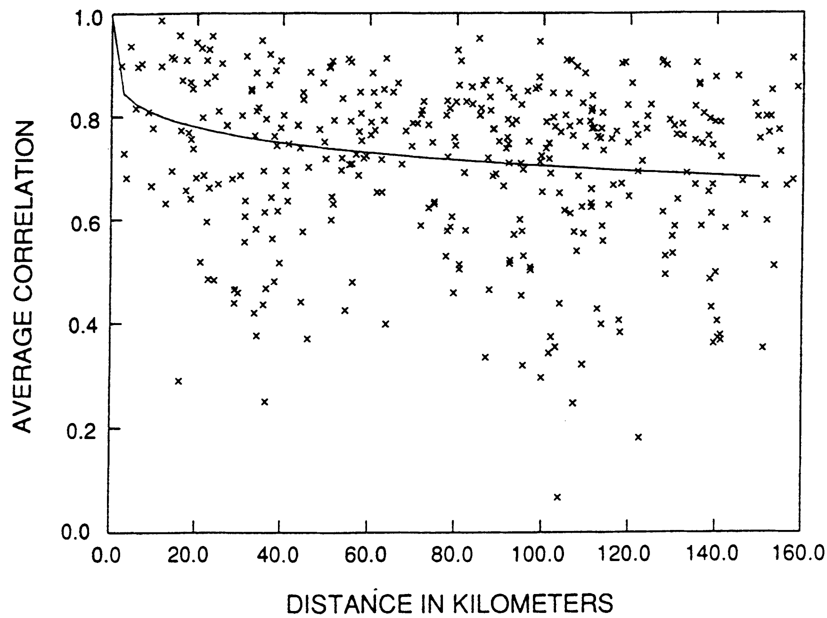

Fig. 1c. Average correlation as a function of separation distance for the Wind River, Salt River and Wyoming Mountains, Wyoming. The solid line approximates the spatial variation of the correlation, Eq. (16).

ally, when one snow course records anomalously high snow accumulation, so do other sites in the same region. This is a result of the scale of the storms that deposit the snow which is comparable to or greater than the extent of each mountain region. Therefore the results from even a single site can be taken as representative of a large region. The Wyoming region generally shows the highest correlations over all separation distances, with a global average correlation of approximately 0.75. The term "average correlation" is used here to represent the "average" correlation of all correlation points for each separation distance, approximated by the exponential curves shown on Fig. 1a,b,c. The "global average correlation" is the average of all correlation points regardless of separation distance (a horizontal line instead of an exponential curve in the figure). The Colorado region, which is larger and more diverse than either of the other two regions, had a global average correlation of approximately 0.7. The Uinta region, which is the only mountain range in this study that trends East-West, showed the strongest correlation dependence on separation distance.

The standard deviations of the departures from the climatological mean, σ given in Table 1, range from 36 to 52 cm of water equivalent. This is indicative of the year-to-year variations of the snowpack for these 8 years. It is the error associated with estimating the water equivalent of a pixel with no surface snow observations in it,

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Table 2 – Average estimation error, in cm of water, for increasing numbers of snow courses in a pixel, for the three mountainous regions

Number of snow courses	0	1	2	3	4	5
Uinta Mountains	36	12	9	7	6	5.5
Colorado	41	16	12	10	9	8
Wyoming	52	20	15	12	10	9

from observations outside of the pixel. Table 2 shows the reduction of the estimation error as the number of snow courses within a pixel increases. With only one measurement, the uncertainly in the estimated water equivalence is reduced from ±40 cm to ±12-20 cm. The error further decreases as the number of snow courses in each pixel increases. When there are four or five snow courses in a pixel, the error reduces to 6-10 cm of water. The reduction in error as the number of observation sites is increased (Table 2) is a consequence of the sharp rise in correlation as the separation decreases, because with more sites, the distance between sites decreases, as modeled by the behavior of Eq. (16) using the parameter estimates in Table 1. As the number of evenly distributed observation sites N approaches infinity, the estimation error approaches zero. This is shown below

$$a_i = \frac{1}{N} \tag{17}$$

$$\sum_i a_i S_i = \sum_i \sum_j a_i a_j R_{ij} = (\frac{1}{N})^2 \sum_i \sum_j R_{ij} \quad ; \quad i,j = 1,\ldots,N \text{ and } N \to \infty$$

$$= (\frac{1}{A})^2 \iiiint R_{xx'} d^2x d^2x' \tag{18}$$

which leads to

$$\Psi_{min} = A^{-2} \iiiint R_{xx'} d^2x d^2x' - \sum_i a_i S_i = 0 \quad ; \quad i = 1,\ldots,N \qquad N \to \infty \tag{19}$$

In conclusion, this statistical technique of kriging provides a viable method for estimating the water equivalent of a mesoscale area from a few point observations. The technique uses only the statistical characteristics of the observations to estimate the uncertainties. It may be possible to improve on this purely statistical approach by incorporating some aspects of the physical processes that control the deposition of snow.

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