Acknowledgment

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Reference


APPENDIX

Equations on Internal Stations

The governing equations on internal computing stations have been derived in [1]. Here they are stated for completeness and in order that the terms in the radial equilibrium equation may be identified.

Radial Equilibrium Equation

\[ \frac{1}{\rho} \frac{\partial \rho}{\partial R} = -\tilde{W}_r^2 \cos^2 \lambda \frac{\partial \tan \lambda}{\partial x} \]

Radial pressure gradient

\[ \tilde{W}_r \tan \beta + \Omega R \]

Centrifugal effect

\[ \frac{\sin \lambda \cos \lambda \frac{\partial \rho}{\partial x}}{\rho} + \frac{F_{\theta \theta} (\tan \lambda \tan \delta \cos^2 \lambda)}{R} \]

Axial pressure gradient

\[ + \frac{P \cos^2 \lambda}{R} \]

Blade force effect

\[ \frac{\tilde{W}_r^2}{R} \frac{\partial \tan \lambda}{\partial x} + \frac{\tilde{W}_r^2}{R} \]

Perturbation terms

in which

\[ P = -\tilde{W}_r^2 \frac{\partial \tan \lambda}{\partial x} + \frac{\tilde{W}_r^2}{R} \]

Continuity equation

\[ \dot{m} = 2\pi R \tilde{W}_r \tilde{W}_r \theta \]

Energy equation

\[ \tilde{I} = C_\rho \tilde{I} + \tilde{W}_r^2 (1 + \tan^2 \lambda + \tan^2 \beta) - \frac{\Omega^2 R^2}{2} \]

\[ + \frac{1}{2} \tilde{W}_r^2 (1 + \tan^2 \lambda) + \frac{1}{2} \tilde{W}_r^2 \]

Equation of state

\[ \tilde{p} = \tilde{p} / \tilde{I} \]

DISCUSSION

C. Hirsch

The passage-averaged representation of the turbomachinery flow, as developed in [6, 14] of the paper and applied by the authors, is essentially based on the exact form of the averaged continuity and momentum conservation laws. This leads to the introduction of the tangential blockage factor B in the continuity equation and to the "perturbation terms" in the momentum equation. These terms appear as gradients of a stress tensor, the "secondary stress"

\[ \tilde{T} = \rho \tilde{w}^\omega \times \tilde{w}^\omega \]

and originate from the momentum transport due to the non-axisymmetric components of the flowfield \( \tilde{w}^\omega \) in the same way the Reynolds stresses appear in turbulence.

However, in the present paper, as well as in [6, 14], only approximate forms of the energy and entropy equations are used in the through-flow component of the Quasi-3D system. Since the authors do present a most interesting discussion of the order of magnitude of the different contributions to the radial pressure gradient, including the influence of the secondary stresses, it would be of even great interest to collect similar information for the energy and entropy equations. Applying the passage-averaging procedure as described in the Appendix, the averaged energy equation can be written, for a steady relative flow, as follows:

\[ \nabla \tilde{p} \tilde{w}^\omega IB = 0 \]

where the gradient operator acts on the two-dimensional space \((r, z)\), with the metric coefficients of the three-dimensional cylindrical coordinate system.

Explicitly, one has

\[ \partial_r \tilde{p} \tilde{w}^\omega - \partial_z \tilde{p} \tilde{w}^\omega = 0 \]

Introducing the density weighted, average rothalpy \( \tilde{I} \), the
energy equation becomes, taking into account the continuity equation,
\[ \dot{\rho} \tilde{w}_m \partial_m \tilde{l} = - \frac{1}{B} \nabla (\rho \tilde{w}^* \tilde{T} \tilde{B}) \]  
(D3)

The averaged rothalpy \( \tilde{l} \) is defined by
\[ \tilde{l} = \tilde{h} + \frac{\tilde{u}^2}{2} - \tilde{h} + \frac{\tilde{w}^2}{2} = \tilde{h} + \frac{\tilde{w}^2}{2} - \frac{\tilde{u}^2}{2} + \frac{\rho \tilde{w}^* + \tilde{w}^*}{2} \]
(D4)

The sum of the first three terms of the right-hand side are the total energy of the averaged flow \( \tilde{l} \), while the last term represents the averaged kinetic energy of the large scale nonaxisymmetric fluctuations \( \tilde{w}^* \), \( \tilde{\kappa} \)
\[ \tilde{\kappa} = \frac{\rho \tilde{w}^* + \tilde{w}^*}{2} \]
(D5)

Hence, one can write
\[ \tilde{l} = \tilde{l} + \tilde{\kappa} \]
(D6)

An alternative form of the energy conservation equation for the average flow is obtained by working out the rothalpy fluctuation term \( \tilde{\kappa} \).

From
\[ \tilde{l} = \tilde{l} + \tilde{\kappa} \]
(D7)

one has
\[ \tilde{l} = h + \frac{w^* + \tilde{w}^*}{2} \]
(D8)

This leads to the following form of the conservation equation for the total energy of the averaged flow, \( \tilde{l} \)
\[ B \dot{\rho} \tilde{w}_m \partial_m \tilde{l} = - \nabla (\rho \tilde{w}^* \tilde{h}^*) - \nabla (B \tilde{w} \tilde{w}^*) + \nabla (B \tilde{w} \tilde{\kappa}) \]
(D9)

The first term on the right-hand side represents the contributions from the transport of fluctuating enthalpy by the nonaxisymmetric flow field. This term contributes to energy mixing together with the second term, which describes the transport of the kinetic energy of the nonaxisymmetric flow components
\[ \kappa^* = \frac{w^* + \tilde{w}^*}{2} \]

by the total flow field \( w \). The last term represents the work of the secondary stresses against the average velocity field.

In [6, 14] of the paper, the energy equation (D9) is applied with a vanishing right-hand side, assuming conservation of the total energy of the averaged flow \( \tilde{l} \). The authors, on the other hand, apply equation (D3) with a vanishing right-hand side, implying the conservation of the total energy \( \tilde{l} \).

Since these two quantities differ by the kinetic energy term \( \kappa^* \), according to equation (D6), it would be interesting to estimate the magnitude of this term for the examples treated by the authors. In addition, since the equations (D3) and (D9) show that one cannot define an average flow that satisfies all the conservation equations, a discussion of the approximations involved in using an approximate form of the energy equations would be welcome.

Another approximation applied by the authors, and also by Smith and Hirsch in [6, 14], concerns the entropy equation. Assuming, in accordance with the concept of a distributed friction force \( F_m \), that the entropy field remains axisymmetric, that is
\[ s^* = 0 \text{ and } s = \tilde{s} \]
(D10)

one obtains the following form for the entropy equation
\[ \dot{\rho} \tilde{w}_m \partial_m \tilde{s} = - \rho \tilde{w} \tilde{F}_m - (\rho \tilde{w}^* + \tilde{T}^*) \nabla \tilde{s} \]
(D11)

The last term on the right-hand side contributes to the redistribution of losses by the nonaxisymmetric flow components, and is generally neglected. Although this term is probably small for low-speed flows, this might not be the case anymore at higher speeds and higher blade loadings. Here also, the authors might be able to derive from their calculations some estimations of the approximations involved in neglecting this term.

An interesting aspect of the authors' work concerns the transfer of data from the blade-to-blade surfaces to the meridional surface. Two elements are essential in this transfer and are related to the deviation angles and to the losses. The authors determine the outlet angles in the blade-to-blade calculations by applying a Kutta condition at the trailing edge. This replaces the introduction of correlations. However, it is known that this leads to large uncertainties and variations in the outlet angle, especially for rounded trailing edges, depending on the way this Kutta condition is applied. A discussion of the accuracy of this procedure would be interesting, particularly with regard to the three-dimensional effects on the outlet flow angles arising from the overall three dimensionality of the flow and from secondary flow effects.

The second aspect concerns the compatibility of the axial variation of stagnation pressure inside the blade rows, as applied in the through-flow calculation and as obtained, or imposed, in the blade-to-blade calculations. Since the inviscid blade-to-blade codes assume isentropic flow conditions for continuous flows, the stagnation pressure at any chordwise position is equal to the inlet relative stagnation pressure (in absence of shock waves). This is however not the case in the corresponding position in the meridional plane. A clarification of the method used by the authors for the estimation of losses and of the way the compatibility of the total pressure variations are ensured would be most interesting.

Authors' Closure

For the vane presented in Paper II the maximum value of \( \tilde{\kappa} \) was approximately 20 percent of the dynamic energy (\( w^*/2 \)) of the flow. We have seen values as high as 40 percent on some rotors. The effect of this term on the temperature of the flow is, however, small, the static enthalpy being around 98 percent of the rothalpy for this rig case.

To clarify the position regarding our use of equation (D3) there are two points to be made. The first is that the only physical argument in the literature for neglecting \( \tilde{\kappa} \) is given by Sehra and Kerrebrock [R1]. They intuitively postulate that the energy of fluctuations is unavailable to the mean flow, and define their rothalpy (\( \tilde{l} \)) in accordance with equation (D6). This would be true if the analogy between these apparent stresses and the Reynolds stresses of turbulent flow theory could be shown to be valid. We doubt that this is true, especially considering the size of the term involved and the mechanism causing the nonuniformity. Secondly, we construct our energy equation as equation (D3) to be consistent with our blade-to-blade programs. While Sehra and Kerrebrock obtain their blade-to-blade information from our efforts at producing a fast design system capable of operating in an industrial environment, these considerations dictate the use of an inviscid mainstream and coupled(redacted for space).
boundary layer approach for the blade-to-blade analysis. While it may be possible to construct examples where $s^*$ or $h^*$ have some importance, it is difficult to imagine that these terms could be more important than the already ignored effects of streamsurface distortion and radial mixing. These latter effects dictate the caution needed in using the system. Components such as fans and high-pressure turbine vanes are amenable to the present approach, while multistage compressors would be viewed with much more caution.

Professor Hirsch is correct when he comments about the Kutta condition. Strictly this condition can only be applied at a sharp trailing edge but we apply the condition to blunt trailing edges in the knowledge that the answers we obtain agree with our experimental data. The condition itself, equalization of pressure or velocity, is applied in each blade-to-blade program depending on the numerical scheme employed. As we are not attempting to include secondary flow or three-dimensional wake effects into the scheme at present this treatment is adequate. If the flow being examined was felt to have these effects then a three-dimensional code would be used to examine them.

The whole subject of loss in a Quasi-3D system is deserving of far greater discussion than we have room for here. Inside Rolls-Royce we have numerous ways of including loss in our calculations. The ways depend on the physical processes to be modeled and the accuracy of our calculation/correlation procedures. However most of these ways can be put into one of two categories:

(i) Use the blade-to-blade program to predict the loss from boundary layers, shock waves, wakes, etc., and impose this loss on the through-flow calculation.

(ii) Prescribe the loss by using an outside correlation or test data. This is the procedure adopted by Wang et al. [R2].

In the latter case the blade-to-blade program accepts the prescribed loss from the through-flow by an adjustment of the streamtube contraction through the row. In this way we effectively interchange blockage and loss in the calculations.

References


J. W. Railly

It would be most valuable to know the extent to which the perturbation terms in the through-flow problem solution influence the distribution of pitch-wise-averaged whirl and meridional velocities. To this end, would it be possible to make available a comparison between solutions to the (direct) through-flow problem as given in the paper and the same problem in which the perturbation terms are left out? Such a comparison should permit, in the latter case, the choice of a suitable (single) $S_2$ surface such as the surface (as quoted by the authors) which divides the mass flow (from the $S_1$ solution) into two equal parts. Alternatively, the choice of a surface which, at the trailing edge, blends into the average outlet angle of the $S_1$ solutions would be suitable. Blade thickness effects would be included in the usual way.

Authors' Closure

Professor Railly has asked an interesting question to which we can only supply part of the answer here. If we were to set all the perturbation terms to zero and converge the quasi-3D system then the resulting aerodynamic variables from the BBP and TFP would not agree. This is because the blade-to-blade solution would inherently contain the perturbation terms due to its nonuniform circumferential flow, while the through-flow solution would not. We could alternatively set the perturbation terms to zero for the fully converged solution and rerun just the TFP. This would give an indication as to the value of the perturbation terms due to its nonuniform circumferential flow, while the through-flow solution would not. We could alternatively set the perturbation terms to zero for the fully converged solution and rerun just the TFP. This would give an indication as to the value of the perturbation terms for this one step in the solution scheme. The results of doing this are shown in terms of the meridional velocity in Fig. 1, the whirl velocity showing similar trends as the whirl angle is fixed. The maximum difference between the two solutions is about 3 percent, which is considerably more than our convergence criterion. A paper the authors are at present writing should help answer Professor Railly's question, as it itemizes each perturbation term with respect to its fellows and indicates their relative importance for a vane and a rotor.

We agree that, in principle, a passage-averaged approach could well be used to test the various $S_2$ surfaces chosen by other authors.