Acceleration time-scale in an ultrarelativistic shock

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ABSTRACT

The acceleration mechanism in ultrarelativistic shocks is investigated using Monte Carlo simulations. We apply a method of discrete small-amplitude particle momentum scattering to reproduce highly anisotropic conditions at the shock and describe the acceleration mechanism carefully. The obtained acceleration time equals $1.0r_g/c$ if the spectral index reaches the value of 2.2, independent of physical conditions in the shock. Some other parameters of the acceleration process are also provided.

Key words: acceleration of particles – shock waves – cosmic rays – gamma-rays: bursts.

1 INTRODUCTION

Observations carried out by the Burst and Transient Source Experiment show that gamma-ray bursts (GRBs) originate from cosmological sources (Dermer 1992; Meegan et al. 1992). Identification of the host galaxy for the GRB 971214 (Kulkarni et al. 1998) and several other bursts mean that there is little doubt now that some, and most likely all, GRBs are cosmological. These phenomena are surely related to ultrarelativistic shocks with Lorentz factors $\gamma > 10^2$.

Several papers suggested that ultrarelativistic shocks in GRBs could be sources of high-energy cosmic rays (cf. Vietri 1995; Waxman 1995), and simulations performed by Bednarz & Ostrowski (1998) showed that such shocks are able to accelerate charged particles, and values of their energy spectral indices converge to $\sigma = 2.2$ when $\gamma \rightarrow \infty$ and/or magnetic turbulence amplitudes grow. Because the acceleration mechanism is quite different from that in the non-relativistic and mildly relativistic regime, we distinguish a class of ultrarelativistic shocks with Lorentz factors $\gamma \gg 1$.

Observations seem to confirm this mechanism. Waxman (1997) used a fireball model of GRBs and showed from the functional dependence of the flux on time and frequency that $\sigma = 2.3 \pm 0.1$ in the afterglow of GRB 970228. Galama et al. (1998) made two independent measurements of the electron spectrum index in the afterglow of GRB 970508, obtaining a value very close to 2.2.

2 ACCELERATION MECHANISM

A particle crossing the shock to an upstream medium has a momentum vector nearly parallel to the shock normal. Then the particle momentum changes its inclination in two ways: by (1) scattering in an inhomogeneous magnetic field and (2) smooth variation in a homogeneous field component. Hereafter, the mean deflection angles in these two cases will be denoted by $\Delta \Omega_S$ and $\Delta \Omega_H$, respectively. The first process is a diffusive one and the second depends on time linearly. That means that with increasing shock velocity, keeping other parameters constant, $\Delta \Omega_S$ decreases more slowly, as a square root of time, than $\Delta \Omega_H$. The Lorentz transformation shows that with $\gamma \gg 1$ even a tiny angular deviation in the upstream plasma rest frame can lead to a large angular deviation in the downstream plasma rest frame. Let us denote a particle phase by $\phi$ and the angle between momentum and a magnetic field vector by $\theta$, both measured in the downstream plasma rest frame. Values of these parameters at the moment when a particle crosses the shock downstream determine if it is able to reach the shock again in the case of neglected magnetic field fluctuations downstream of the shock. In fact the magnetic field fluctuations upstream of the shock perturbing the momentum direction lead to broadening the $(\phi, \theta)$ range, which allows particles to reach the shock again. Thus, as we show below for efficient scattering, when $\Delta \Omega_H$ becomes unimportant in comparison with $\Delta \Omega_S$, the spectral index and the acceleration time reach their asymptotic values.

The discussed relation between $\Delta \Omega_H$ and $\Delta \Omega_S$ is reproduced in our simulations and presented in Fig. 1. 11 points are shown from $\gamma = 100$ to 320, and three additional for $\gamma = 640$, 1280 and 2560. The expected linear dependence of these quantities can be noticed.

3 NUMERICAL SIMULATIONS

In simulations we combine the procedure used by Bednarz & Ostrowski (1996) with a hybrid approach used in Bednarz & Ostrowski (1998). Monoenergetic seed particles are injected at the shock and then their trajectories are derived in the perturbed magnetic field. The inhomogeneities are simulated by small-amplitude particle momentum scattering within a cone with angular opening $\Delta \theta$ less than the particle anisotropy $\sim 1/\gamma$ (cf. Ostrowski 1991).

A particle is excluded from simulations if it escapes through the free-escape boundary placed far off the shock, or reaches an energy larger than the assumed upper limit. These particles are
considerably for a given t will use downstream in the upstream or downstream plasma rest frame respectively. We Hereafter, subscripts ‘U’ or ‘D’ mean that a parameter is measured in the shock normal rest frame. For the considered case the time and momentum bin depending on particle parameters, as their parameters are transformed to the current plasma rest frame (lower index), and ‘D’ for the formed spectrum shifts toward higher energies with time. The magnetic field inclination to the shock normal upstream of the shock, represented by randomly oriented magnetic cells with field amplitude $B$ below 1, are more adequate for real situations. Additionally, the above authors applied an extremely irregular fluctuations upstream of the shock grow. For $\hat{\sigma}_D \leq 0.1$ the asymptotic value of the acceleration time is close to $r_g/c$. It is found that $r_g/c$ is a good unit provided that the homogeneous magnetic field dominates the random component. Unfortunately, when this condition fails, the meaning of $t_{acc}$ becomes unclear in the simulations. For this reason we will not discuss further the case of $\hat{\sigma}_D = 0.69$.

Approximate calculations of Gallant & Achterberg (1999) showed that $t_U^D/t_D^U = 1$, where $t_U^D$ is the particle mean residence time upstream of the shock (upper index) as measured in the upstream plasma rest frame (lower index), and ‘D’ in $t_D^U$ stands for the downstream residence time. However, they were not able to consider the anisotropic particle momentum distribution, and our results in Fig. 4 transformed to the upstream plasma rest frame with $t_U^D/t_D^U$ within the range 0.01–0.1, are more adequate for real situations. Additionally, the above authors applied an extremely irregular magnetic field upstream of the shock, represented by randomly oriented magnetic cells with field amplitude $B$, and they measured time in the upstream unit of $r_g(B)/c$. As a result they obtained the result that $t_U^D/t_D^U$ could be much larger than 1 in this case.
Just before the spectral index reaches its minimal value (cf. Fig. 2), $DV_S$ stabilizes near the limit, after which its value does not depend further on the magnetic field inclination, as is seen in Fig. 5. Momentum vectors of particles crossing downstream of the shock have similar distributions measured in the downstream plasma rest frame if $DV_S$ approaches the maximum value. Then, it follows that parameters we consider below depend only on $t_D$.

For growing $t_D$ ($\gamma = 0, 1.0 \times 10^{-3}, 1.1 \times 10^{-2}, 0.11$), the acceleration time is constant and accompanied by a slow increase of the mean energy gain in one cycle downstream–upstream–downstream ($\Delta n/n = 0.89, 0.94, 1.0, 1.1, 1.1, 1.1$) and a slight decrease of the fraction of particles that reach the shock again after crossing it downstream ($\Delta n/n = 0.51, 0.50, 0.48, 0.44$). Simultaneously the mean time a particle spends downstream of the shock grows as $t_D^\gamma = 0.96, 1.0, 1.2, 1.35$. The time that a particle spends upstream of the shock can be neglected in this rest frame, as it is visible in Fig. 4. It implies, approximately, $t_{acc} = t_D^\gamma / (\Delta n/n)$ if one neglects correlations between these quantities (cf. Bednarz & Ostrowski 1996). Similarly, we can roughly estimate the value of the energy spectral index of accelerated particles as $\sigma = 1 - \ln(\Delta n/n)/\ln(\Delta n/n + 1)$.

The simulated maximum distance, in the downstream medium, that the particle is able to depart from the shock and reach it again is, respectively, $d_M = 0.84, 1.5, 2.5, 4.0$ (the values were derived from $\sim 10^5$ events). We calculated the average values of $\sin^2 \theta$ for returning particles wandering downstream of the shock and found, respectively, $\langle \sin^2 \theta \rangle = 0.676, 0.658, 0.650, 0.651$.

5 DISCUSSION

Because of a newly found acceleration mechanism in ultrarelativistic shock waves, we propose that some part of the GRB radiation could arise as a result of synchrotron radiation of electrons or electron pairs accelerated across the mechanism. We follow the internal shocks model of GRBs (cf. Kobayashi, Piran & Sari 1997 for example). In the model, two different shells have different Lorentz factors. The inner shell overtakes a slower outer shell and forms a shock. The Lorentz factor of the shock as
measured in the frame at rest with respect to the outer shell is
assumed to be $\sim 2$. With only a part of kinetic energy converted
into the internal energy, the particle energy distribution down-
stream of the shock will be non-thermal with, possibly, a
substantial fraction of relativistic particles. As a result an amount
of relativistic particles will be present in the shock. The particles
can be accelerated across the mechanism presented in the paper,
and could be observed in the afterglow (cf. Waxman 1997;
Galama et al. 1998).

We derived some parameters of the process that could be used
in GRB models. The acceleration time $t_{acc} = 1.0r_g/c$, measured
in the downstream plasma rest frame in units of the particle
gyroradius in the homogeneous magnetic field component divided
by the speed of light, is the second important parameter, besides
the spectral index $\sigma = 2.2$. The values of $d_M^D$ and $t_D^D$ define
the dimensions of the shock that allow the process to be effective, and
the values of $\left< \sin^2 \theta \right>$ show how the synchrotron radiation can
influence the process.

In the ejecta of relativistic matter in the GRB model, the outer
shells can be faster than the following ones. In this case, separated
shocks with Lorentz factors reaching $\gamma \sim 10^3$ will be generated.
The leading shock could produce seed protons with energies of
$10^{14}$–$10^{16}$ eV. These protons downstream of the first shock can
interact with the following one, to be reflected with energy gains
$\sim \gamma^2$ (cf. Gallant & Achterberg 1999; Bednarz & Ostrowski 1999).
For a constant reflection probability, the spectrum of these
reflected highest energy particles, above $10^{20}$ eV, will be only the
spectrum of seed particles, shifted in energy, with the universal
spectral index $\sim 2.2$.

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