Pulsational $M_V$ versus [Fe/H] relation(s) for globular cluster RR Lyrae variables

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A B S T R A C T

We use the results from recent computations of updated non-linear convective pulsating models to constrain the distance modulus of Galactic globular clusters through the observed periods of first-overtone (RR$_{c}$) pulsators. The resulting relation between the mean absolute magnitude of RR Lyrae stars $\langle M_V(RR) \rangle$ and the heavy element content [Fe/H] appears well in the range of several previous empirical calibrations, but with a non-linear dependence on [Fe/H] so that the slope of the relation increases when moving towards larger metallicities. On this ground, our results suggest that metal-poor ([Fe/H] < −1.5) and metal-rich ([Fe/H] > −1.5) variables follow two different linear $\langle M_V(RR) \rangle - [Fe/H]$ relations. Application to RR Lyrae stars in the metal-poor globular clusters of the Large Magellanic Cloud (LMC) provides an LMC distance modulus of the order of 18.6 mag, thus supporting the ‘long’ distance scale. The comparison with recent predictions based on updated stellar evolution theory is briefly presented and discussed.

Key words: stars: horizontal branch – stars: variables: other – globular clusters: general – distance scale.

1 INTRODUCTION

The intrinsic luminosity of RR Lyrae variables has been for a long time a very popular way to give reasonable estimates of the distance to globular clusters (GCs), both in the Milky Way and in Local Group galaxies (Magellanic Clouds, M31) and, in turn, to constrain the age of these very old stellar systems. However, notwithstanding the large body of work, a general consensus on a precise evaluation of such a luminosity has been not yet achieved. One may notice that a firm knowledge of RR Lyrae luminosities would be of paramount relevance, because it would provide an independent test of the Cepheid distance scale as well as a reliable calibration of several secondary distance indicators (such as e.g. the GC luminosity function or the red giant tip) for external galaxies, thus providing important clues about the value of the Hubble constant $H_0$. On these grounds, RR Lyrae variables could represent relevant milestones on the path to set both a lower and an upper limit to the age of the Universe, playing a fundamental role in several astrophysical problems ranging from stellar evolution to cosmological models.

From the observational side, studies dealing with the absolute magnitude $M_V(RR)$ of RR Lyrae and with the dependence of these magnitudes on the heavy element content [Fe/H] have yielded the well-known debate between the so-called ‘short’ and ‘long’ distance scales. As recently reviewed by Cacciari (1999), empirical estimates of $M_V(RR)$ for RR Lyrae stars at [Fe/H] = −1.6 actually range from about 0.4 to 0.7 mag, thus leaving an uncertainty of $\sim$±0.2 mag on the derived distance moduli (see also Popowski & Gould 1999).

Different estimates have also been given for the dependence of these magnitudes on the star metallicity. As a matter of fact, for the often-assumed linear relation

$$\langle M_V(RR) \rangle = a + b[Fe/H]$$

one finds in the literature evaluations of the coefficient ‘$b$’ mainly in the range $b \sim 0.18 \pm 0.03$ to $\sim$0.30, where the former value is based on the Baade–Wesselink method (see e.g. Fernley et al. 1998b, hereafter Fn98b) and the latter value was suggested earlier by Sandage (1993, hereafter Sa93) when discussing the period–metallicity relation for field and GC RR Lyrae pulsators. However, an even milder slope has been suggested by Fusi Pecci et al. (1996), who investigated eight globular clusters in M31 to derive, over the range $−1.8 < [Fe/H] < −0.4$,

$$\langle M_V(HB) \rangle = (0.13 \pm 0.07)[Fe/H] + (0.95 \pm 0.09).$$

The recent release of Hipparcos statistical and trigonometric parallaxes for halo RR Lyraes ([Fe/H] $\leq −1.30$) has not clarified the issue: one may indeed recall that Fernley et al. 1998a (hereafter Fn98a) and Groenewegen & Salaris (1999, hereafter...
GS99, both assuming the same slope $b = 0.18$, give a zero-point of $1.05 \pm 0.15$ mag and $0.77 \pm 0.26$ mag, respectively, as derived from an identical sample of variables but using different approaches (statistical parallaxes or reduced parallaxes, respectively). In the meantime, McNamara 1999 (hereafter MN99) claims that Baade–Wesselink results for variables with $[\text{Fe}/\text{H}] > -1.5$ yield a quite different relation, as given by 

$$
M_v(\text{RR}) = 1.06 + 0.32[\text{Fe}/\text{H}].
$$

On the theoretical side, the literature already contains several sets of horizontal branch (HB) evolutionary models computed for wide ranges of the overall metallicity ($Z$ in the range $0.001 - 0.02$) which provide the ‘theoretical route’ to the calibration of the $M_v(\text{RR})$ versus $[\text{Fe}/\text{H}]$ relation. One finds that almost all the recent theoretical predictions concerning the absolute magnitude $M_v(\text{ZAHB})$ of the zero-age horizontal branch (ZAHB) sequence at the RR Lyrae instability strip confirm the non-linear dependence of $M_v(\text{ZAHB})$ on log $Z$ formerly suggested by Castellani, Chieffi & Pulone (1991, hereafter CCP). However, the scaling of the overall metallicity $Z$ to the measured $[\text{Fe}/\text{H}]$ values could be a tricky matter, because the classical assumption of solar-scaled chemical mixtures is likely inappropriate to GC stars. There is indeed a growing observational evidence for a significant enhancement of elements with respect to iron ($\alpha/\text{Fe} \sim 0.3$) in GC and field metal-poor stars (see Carney et al. 1997; Gratton et al. 1997). Moreover, one has to bear in mind that observed RR Lyrae samples do contain stars evolved off their original ZAHB position. Thus, realistic predictions on the average magnitude ($M_v(\text{RR})$) require the evaluation of the evolutionary effects, possibly through synthetic HB simulations (SHB).

The wide grids of SHBs so far published (e.g. Lee, Demarque & Zinn 1990; Lee 1991; Bencivenni et al. 1991; Caputo et al. 1993) have already shown that the predicted mean magnitude of RR Lyrae stars ($M_v(\text{RR})$) significantly depends, with everything else being constant, on the HB morphology. Simulations based on slightly modified CCP models caused Caputo (1997) to suggest

$$
M_v(\text{RR}) = 1.19 + 0.19 \log Z
$$

for RR Lyrae-rich metal-poor GCs with $\log Z \approx -3.0$, whereas for larger metallicities the theory gives

$$
M_v(\text{RR}) = 1.57 + 0.32 \log Z.
$$

Similar results have been found more recently by Demarque et al. (2000), who definitively reject the existence of a unique linear relation covering the metallicities spanned by GCs, confirming that the slope of the predicted $M_v(\text{RR})$–$\log Z$ relation depends on the metallicity range and that the HB morphology of each cluster must be taken into account when using RR Lyrae stars as distance indicators.

On these grounds, one is tempted to conclude that theoretical and observational investigations do show a sort of consistency: the former give warnings against a ‘universal’ linear $M_v(\text{RR})$–$[\text{Fe}/\text{H}]$ relation, the latter fail to reach an agreement on either its slope or its zero-point!

Within such a confusing scenario, one has to mark the seminal attempts made by Sandage (Sa93 and references therein) to use RR Lyrae periods to constrain the luminosity of these stars. This appears a quite relevant approach, because periods are firm and safe observational parameters, independent of distance and reddening. To discuss Sandage’s philosophy, one has to recall that since the pioneering work by Christy (1966) and Stellingwerf (1975, 1984), pulsating models have suggested the existence within the instability strip of a region where both fundamental (RR$_{ab}$) and first overtone (RR$_{c}$) modes are stable (see Bono & Stellingwerf 1994; Bono et al. 1997a; Bono et al. 1997c). The boundaries of this ‘either-or’ region, namely the fundamental blue edge (FBE) at the higher temperature side and the first overtone red edge (FORE) at the lower temperature side, encompass for each given luminosity the range of temperatures (or colours) where the mode shift (i.e. the transition from RR$_{ab}$ to RR$_{c}$) may occur. Assuming that for both Oosterhoff type I (OoI) and Oosterhoff type II (OoII) globular clusters this transition occurs at the blue edge for fundamental pulsation, and using periods and $B - V$ colours of the shortest period RR$_{ab}$ in clusters and in the field, Sa93 derives the star luminosity from the well-established period–mass–luminosity–temperature relation. In this way he obtains the relation

$$
M_v(\text{RR}) = 0.94 + 0.30[\text{Fe}/\text{H}],
$$

which accounts for the Oosterhoff dichotomy in Galactic globular clusters as mainly resulting from a luminosity effect. However, a re-analysis by Fernley (1993, hereafter Fr93), using $V - K$ colours and a limited sample of clusters with low and well-known reddening, yields

$$
M_v(\text{RR}) = 0.84 + 0.19[\text{Fe}/\text{H}].
$$

More recently, the assumption of a unique RR$_{ab}$/RR$_{c}$ transition line has been questioned by Bono, Caputo & Marconi (1995), who concluded that the Oosterhoff dichotomy is largely the result of different transition lines in OoI (near FBE) and OoII (near FORE) clusters, as suggested earlier by van Albada & Baker (1973). However, the pulsation theory predicts the limits of the whole instability strip, as given by the first overtone blue edge (FOBE) and the fundamental red edge (FRE), without any ambiguity about the actual pulsation mode. On this basis, Caputo (1997) used a preliminary set of pulsating models to show that theoretical predictions on the pulsator distribution in the period–absolute magnitude $M_v$–$\log P$ plane can constrain the distance to RR Lyrae rich globular clusters.

In recent times, the RR Lyrae pulsating models have been updated and extended to wide ranges of mass and chemical composition, shedding light on the dependence of the instability strip on the metal content. In this paper we will take advantage of these improvements to reconsider the Caputo (1997) analysis. The updated $M_v$–$\log P$ relations at FOBE and FRE are discussed in Section 2, while Section 3 presents the comparison with observation and the derived ‘pulsational’ distance moduli for a selected sample of well-studied GCs. The resulting dependence of our $(M_v(\text{RR}))$ values on the cluster metallicity is discussed in Section 4 in comparison with both empirical relations and recent theoretical HB models. Some concluding remarks will close the paper.

## 2 THE PREDICTED $M_v$–$\log P$ DIAGRAM

The non-linear convective hydrodynamical code used for pulsating models has been already presented in a series of papers (Bono & Stellingwerf 1994; Bono, Caputo & Marconi 1995; Bono et al. 1997c) and it will be not discussed further. With respect to previous computations, the new models differ in the adopted opacity tables, using the most updated compilations by Iglesias & Rogers (1996) and extending in such a way the preliminary results presented in Caputo et al. (1999) for $Z = 0.001$ and $M = 0.65\,M_\odot$.  

Table 1 presents temperatures, absolute magnitudes and periods of stars at the FOBE or FRE, for selected choices of $Z$ and suitable values of the star mass ($M$), luminosity ($L$) and helium abundance ($Y$). Absolute magnitudes are derived using the bolometric corrections provided by Castelli, Gratton & Kurucz (1997a,b). The adopted $Y$ values account reasonably for the extra helium brought to the stellar surface by the first dredge-up, as well as galactic enrichment as given by $\Delta Y/\Delta Z \sim 2.5$. As discussed in Caputo, Marconi & Santolamazza (1998), mild variations of $Y$, also caused by uncertainties in the efficiency of element sedimentation (see Cassisi et al. 1998, 1999, hereafter Cs99), have negligible effects on the temperature of the instability edges and, in turn, on the related pulsational periods. Stellar masses and luminosities have been chosen in such a way as to encompass reasonably the available expectations about these evolutionary parameters for GC RR Lyrae pulsators.

It should be noted that, for each given set of entry parameters, the computations have been performed using steps of 100 K and that we adopt as limits of the instability region the average effective temperature between the last pulsating model and the first non-pulsating one. It follows that the intrinsic uncertainty of the FOBE and FRE temperatures in Table 1 is $\pm 50$ K, which in terms of period means $\delta \log P \sim \pm 0.01$ (see Caputo et al. 1998). From the data in Table 1 one derives analytical expressions connecting the absolute $V$ magnitude at the instability edges with the pulsator period, mass and metallicity, as given by

\[
M_V(\text{FOBE}) = -0.685 - 2.255 \log P(\text{FOBE}) - 1.259 \log M/M_\odot + 0.058 \log Z, \tag{1}
\]

\[
M_V(\text{FRE}) = +0.552 - 2.018 \log P(\text{FRE}) - 1.348 \log M/M_\odot + 0.108 \log Z, \tag{2}
\]

with an rms scatter $\sigma_V = 0.027$ mag.

However, a further source of uncertainty in the predicted pulsation edges results from the efficiency of convection in the external layers. The lack of a rigorous treatment of superadiabatic convection is indeed a well-known fault in the whole stellar evolution theory, and almost all the evolutionary sequences are calculated within the so-called ‘mixing-length’ scenario, which involves an adjustable parameter $\alpha = 1.5$, in reasonable agreement with the values generally used for evolutionary computations. As the effect of convection is to quench pulsation, variations of $\alpha$ lead to variations in the effective temperature of the boundaries for instability, with the amount of this effect decreasing from the red to the blue edge.

To shed light on such an uncertainty, we performed suitable numerical experiments, finding that, decreasing $\alpha$ down to 1.1 (i.e. decreasing the efficiency of convection and thus increasing the local temperature gradients), the FOBE periods decrease by $\delta \log P \sim 0.029$, while with $\alpha = 2.0$ these periods increase by $\delta \log P \sim 0.017$. On this ground one can estimate that mixing-length values in the range $1/\alpha = 1.3 \sim 1.8$, as widely adopted in the relevant literature, yield an additional uncertainty on FOBE periods by $\delta \log P \sim \pm 0.01$. At the red side of the instability strip the mixing length affects the predicted periods much more significantly, and therefore in the following we will rely on theoretical predictions concerning FOBE only, tentatively placing the red edge of the pulsation region at $\Delta \log P = 0.45$ with respect to FOBE.

As a relevant point, when constraining the luminosities of ZAHB pulsators in the range covered by current evolutionary predictions for various metallicities and for ages in the range from 8–18 Gyr, one finds that the data in Table 1 predict FOBE effective temperatures very close to $log T_\text{eff} = 3.85$, without

significant variation with the metal content. This constant value results from the balancing effects of metallicity on the FOBE temperature (which decreases if only increasing the luminosity) and ZAHB luminosity: when decreasing Z only, the FOBE will become hotter, but the contemporary increase of the ZAHB luminosity eventually leaves the effective temperature unchanged. More generally, we can use such evidence to take safely from evolutionary theories the predicted masses for HB pulsators at the blue side of the instability strip. Luckily enough, at variance with the luminosities, the evolutionary masses have remained substantially unchanged despite the many improvements affecting HB models in the last years, thus representing a rather firm and trustworthy prediction. As a relevant point, one finds that such an evolutionary prediction appears in close agreement with independent mass estimates from the period ratios of double-mode RR Lyrae pulsators (see Cox 1991; Bono et al. 1996).

Thus, by inserting into equation (1) the predicted mass of the ZAHB model at log$T_e=3.85$ as presented in Table 2 (from Bono et al. 1997b), one finally obtains the period–luminosity–metallicity relation for evolutionary FOBE pulsators, as given by

$$M_V(\text{FOBE}) = -0.178 - 2.255 \log P(\text{FOBE}) + 0.151 \log Z, \quad (3)$$

with a total intrinsic dispersion (including the above uncertainty of $\pm 50$ K, the mixing-length effects and mass variations by 5 per cent with respect to the values in Table 2) of $\sigma_P = 0.065$ mag.

Here one should note that the mass of HB models does depend on the abundance of a elements. However, one can benefit from the principle of correspondence, for which the evolutionary behaviour of HB stars (and the predicted pulsator masses) depends on the overall metallicity Z, independently of the internal ratio between $\alpha$ and heavy elements (see e.g. Bencivendi et al. 1991; Salaris, Chieffi & Straniero 1993). In other words, the comparison between the predicted period–luminosity–metallicity relation given in equation (3) and the GC RR Lyrae distribution in the observed $V$–log$P$ plane only requires that the scaling between the measured [Fe/H] and the overall metallicity $Z$ is properly evaluated as

$$\log Z = [\text{Fe/H}] - 1.70 + \log(0.638f + 0.362), \quad (4)$$

where $f$ is the $\alpha$-enhancement factor with respect to iron (Salaris et al. 1993).

### 3 THE OBSERVED V–LOG P DIAGRAM

As our analysis will be focused on the predicted FOBE, we selected well-studied clusters with statistically significant numbers of RR$_c$ stars. The sample of the GCs used is presented in Table 3, which gives for each cluster the adopted iron content [Fe/H] (from Carretta & Gratton (1997) or from Zinn & West (1984) and Rutledge, Hesser & Stetson (1997) values transformed into the Carretta & Gratton metallicity scale), HB type (Harris 1996) and mean visual magnitude ($V_{\text{M}}$) of RR Lyrae stars. For each assumption about the globular cluster distance modulus, one obtains the distribution of the cluster RR Lyraes in the $M_V$–log$P$ plane. We derive a ‘pulsational’ evaluation of the cluster distance modulus by constraining the observed RR$_c$ distribution to match the predicted blue limit of the pulsation region in order to have no variables in the hot stable region. Fig. 1 shows the result of such a procedure, by assuming for the cluster a solar-scaled chemical composition, i.e. $f=1$ in equation (4). As already stated, our analysis is focused on RR$_c$ stars and the right edge of the instability strip has been simply placed at $\Delta \log P = 0.45$ with respect to the left edge. However, the fair agreement found between the predicted FRE and the RR$_{ab}$ distribution also seems worthy of notice.

The derived GC apparent distance moduli $D_M$ are summarized in Table 4, together with the resulting mean absolute magnitude ($M_V(\text{RR})$) of RR Lyrae stars. The total errors in ($M_V(\text{RR})$) listed in Table 4 account for the observed dispersion $\sigma_{(\text{RR})}$ (see Table 3) and the predicted total uncertainty ($\pm 0.07$ mag) of the FOBE period–luminosity–metallicity relation. The last column gives the

<table>
<thead>
<tr>
<th>Name</th>
<th>[Fe/H]</th>
<th>HB</th>
<th>$\langle V_{\text{Rg}} \rangle$</th>
<th>$\sigma_{(V_{\text{Rg}})}$</th>
<th>Ref. (RR Lyrae)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 1851</td>
<td>-1.05</td>
<td>-0.36</td>
<td>16.03</td>
<td>0.09</td>
<td>Walker (1998)</td>
</tr>
<tr>
<td>NGC 4147</td>
<td>-1.54</td>
<td>+0.55</td>
<td>16.98</td>
<td>0.08</td>
<td>Newburn (1957)</td>
</tr>
<tr>
<td>IC449</td>
<td>-1.26</td>
<td>+0.11</td>
<td>17.64</td>
<td>0.08</td>
<td>Walker &amp; Nemec (1996)</td>
</tr>
<tr>
<td>NGC 4590 M68</td>
<td>-1.99</td>
<td>+0.17</td>
<td>15.64</td>
<td>0.04</td>
<td>Walker (1994)</td>
</tr>
<tr>
<td>NGC 4833</td>
<td>-1.58</td>
<td>+0.93</td>
<td>15.33</td>
<td>0.15</td>
<td>Demers &amp; Wehlau (1977)</td>
</tr>
<tr>
<td>NGC 5053</td>
<td>-1.98</td>
<td>+0.52</td>
<td>16.64</td>
<td>0.11</td>
<td>Nemec, Mateo &amp; Schombert (1995)</td>
</tr>
<tr>
<td>NGC 5272 M3</td>
<td>-1.34</td>
<td>+0.08</td>
<td>15.63</td>
<td>0.10</td>
<td>Carretta et al. (1998)</td>
</tr>
<tr>
<td>NGC 5466</td>
<td>-2.14</td>
<td>+0.58</td>
<td>16.46</td>
<td>0.07</td>
<td>Corwin, Carney &amp; Nifong (1999)</td>
</tr>
<tr>
<td>NGC 5904 M5</td>
<td>-1.11</td>
<td>+0.31</td>
<td>15.06</td>
<td>0.08</td>
<td>Caputo et al. (1999)</td>
</tr>
<tr>
<td>NGC 6171 M107</td>
<td>-0.91</td>
<td>-0.73</td>
<td>15.60</td>
<td>0.10</td>
<td>Dickens (1970)</td>
</tr>
<tr>
<td>NGC 6333 M9</td>
<td>-1.56</td>
<td>+0.87</td>
<td>16.26</td>
<td>0.07</td>
<td>Clement &amp; Shelton (1999)</td>
</tr>
<tr>
<td>NGC 6362</td>
<td>-0.96</td>
<td>-0.58</td>
<td>15.25</td>
<td>0.07</td>
<td>Walker (in preparation)</td>
</tr>
<tr>
<td>NGC 6809 M55</td>
<td>-1.61</td>
<td>0.87</td>
<td>14.37</td>
<td>0.06</td>
<td>Olech et al. (1999)</td>
</tr>
<tr>
<td>NGC 7006</td>
<td>-1.35</td>
<td>-0.28</td>
<td>18.79</td>
<td>0.14</td>
<td>Wehlau, Slawson &amp; Nemec (1999)</td>
</tr>
<tr>
<td>NGC 7078 M15</td>
<td>-2.12</td>
<td>+0.67</td>
<td>15.83</td>
<td>0.06</td>
<td>Bingham et al. (1984)</td>
</tr>
<tr>
<td>NGC 7089 M2</td>
<td>-1.31</td>
<td>+0.96</td>
<td>15.97</td>
<td>0.09</td>
<td>Lee &amp; Carney (1999)</td>
</tr>
</tbody>
</table>
weight $W$ of our $\langle M_V(\text{RR}) \rangle$ estimates, as simply derived from the number of RR$_c$ stars matching the predicted FOBE.

Note that the results in Table 2 refer to solar-scaled chemical compositions. If the chemical mixtures are $\alpha$-enhanced, then for each cluster the nominal metallicity $Z$ increases (see equation 4) and the derived distance modulus decreases (see equation 3). As a matter of example, with $f = 3$ all the distance moduli in Table 4 have to be decreased by 0.05 mag, with a consequent increase of $\langle M_V(\text{RR}) \rangle$.

4 THE $\langle M_V(\text{RR}) \rangle$ VERSUS METALLICITY RELATION(S)

The final correlation between our $\langle M_V(\text{RR}) \rangle$ and the cluster metallicity $[\text{Fe/H}]$ is presented in Fig. 2 for the two cases $f = 1$ and $f = 3$, assuming for each $[\text{Fe/H}]$ an error of $\pm 0.15$ dex. The same figure shows the already quoted observational calibrations based on RR Lyrae periods (Sa93, Fn93), Hipparcos data for field RR Lyraes (Fn98a) and the Baade–Wesselink method (Fn98b).

Figure 1. Globular cluster RR Lyraes in the $M_V$–log $P$ plane. Each group of figures represents a ‘class’ of metallicity. In each panel, filled and open circles are RR$_c$ and RR$_ab$ pulsators, respectively, while the solid lines are the theoretical boundaries of the instability strip (see text for details). The labelled apparent distance moduli are obtained by constraining the observed RR$_c$ distribution to match the predicted blue limit of the pulsation region, under the assumption of solar-scaled chemical composition.
with respect to our results. However, one derives the result that between pulsational and other empirical calibrations, except the 0.03 mag fainter than Cr99, is not presented, for the sake of clarity.

sequence fitting procedure, while the GS99 relation, which is only Hipparcos on We add the result by Carretta et al. [1999 (Cr99), 2000], as based on stellar evolution theoretical predictions. We show in Table 5 that a bare linear best fit to the data in Table 4, starting from the four metal-poorest clusters with [Fe/H] ~ −2.0 (NGC 4590, 5053, 5466, 7078) and regularly increasing the metallicity range, yields a $M_V(\text{RR})$ ~ [Fe/H] relation that becomes steeper and steeper when moving towards metal-rich clusters, suggesting a change in slope at [Fe/H] ~ −1.5. It seems worthy of notice that this result agrees with the recent analysis of RR Lyrae variables in the field (MN99) and the globular cluster ω Centauri (Rey et al. 2000).

On this ground, the least-squares solutions performed through our weighted $\langle M_V(\text{RR}) \rangle$ values with [Fe/H] < −1.5 and [Fe/H] > −1.5 yield

\[
\langle M_V(\text{RR}) \rangle = 0.71(\pm 0.10) + (0.17 \pm 0.04)[\text{Fe/H}] + 0.03f
\]

and

\[
\langle M_V(\text{RR}) \rangle = 0.92(\pm 0.12) + (0.27 \pm 0.06)[\text{Fe/H}] + 0.03f
\]

respectively, in agreement with the empirical calibrations by Fn93, GS99 and Cr99 (metal-poor clusters) and MN99 (metal-rich clusters).

As shown in Fig. 3, where OoI and OoII clusters are depicted with different symbols, around [Fe/H] ~ −1.5 the Oosterhoff dichotomy shows off, leading us to guess that the two above $\langle M_V(\text{RR}) \rangle$ ~ [Fe/H] relations hold for the two Oosterhoff groups. More interestingly, the same figure suggests that OoII clusters have brighter RR Lyraes than OoI clusters with similar metal content, evidence that, coupled with their blue HB morphology (see HB types in Table 4), confirms that the RR Lyrae evolutionary stage is more important than metallicity in triggering the Oosterhoff dichotomy, as suggested earlier by Lee, Demarque & Zinn (1990) and recently supported by independent investigations (Lee & Carney 1999; Clement & Shelton 1999).

Fig. 4 finally presents the pulsational $\langle M_V(\text{RR}) \rangle$ results as a function of log Z, as derived through equation (4), together with selected theoretical predictions based on stellar evolution theory. The lines drawn in the figure refer to recent $M_V(\text{ZAHB})$ ~ log Z calibrations as given by Cassisi & Salaris (1997, hereafter CS); Cs99; Caloi, D’Antona & Mazzitelli (1997, hereafter CDM) and Ferraro et al. (1999, hereafter Ff99), with the predicted ZAHB magnitude decreased by 0.1 mag to account for the luminosity excess of actual RR Lyrae stars over the ZAHB level.

We add the result by Carretta et al. [1999 (Cr99), 2000], as based on Hipparcos parallaxes for field subdwarfs and the main-sequence fitting procedure, while the GS99 relation, which is only 0.03 mag fainter than Cr99, is not presented, for the sake of clarity.

Inspection of Fig. 2 reveals that there is a general agreement between pulsational and other empirical calibrations, except the Fn98a relation, which definitely suggests magnitudes that are too faint. If $f = 1$, then the Fn98b relation also appears fainter with respect to our results. However, one derives the result that none of the empirical linear calibrations is able to match our pulsational results fully over the whole range of metal content.

The lack of a full agreement is largely caused by the fact that our data foreseeable a non-linear relation between $(M_V(\text{RR}))$ and metallicity, in agreement with stellar evolution theoretical predictions. We show in Table 5 that a bare linear best fit to the data in Table 4, starting from the four metal-poorest clusters with [Fe/H] ~ −2.0 (NGC 4590, 5053, 5466, 7078) and regularly increasing the metallicity range, yields a $M_V(\text{RR})$ ~ [Fe/H] relation that becomes steeper and steeper when moving towards metal-rich clusters, suggesting a change in slope at [Fe/H] ~ −1.5. It seems worthy of notice that this result agrees with the recent analysis of RR Lyrae variables in the field (MN99) and the globular cluster ω Centauri (Rey et al. 2000).
similar behaviour seems present among RR Lyrae stars in towards the metal-richer variables. On observational grounds, that suggests that the slope of the relation increases when moving empirical linear calibrations, but with evidence for non-linearity...‰

range of metal-poor stars Centauri (Rey et al. 2000). Moreover, we notice that over the predictions fully agrees with our pulsational results with \( f = 1 \), as the luminosities provided by Cs99 and CDM are systematically too bright, while those by CS and Fr99 match our results only with \( \log Z < -3.0 \). If \( f = 3 \), then the CS and Fr99 calibrations appear the most consistent with our data, but with a tendency to overestimate the luminosity of the most metal-rich variables. It seems worth noticing that the Cs99 relation reproduces all our data with \( f = 3 \) well, but with an overluminosity of about 0.08 mag.

5 CONCLUDING REMARKS

In this paper we have used results from the most recent and updated computations of non-linear convective pulsating models to constrain the distance modulus of Galactic globular clusters through the observed periods of RR\(_p\) pulsators. The resulting \( \langle M_V(\text{RR}) \rangle - [\text{Fe/H}] \) relation appears in the range of several empirical linear calibrations, but with evidence for non-linearity that suggests that the slope of the relation increases when moving towards the metal-richer variables. On observational grounds, similar behaviour seems present among RR Lyrae stars in \( \omega \) Centauri (Rey et al. 2000). Moreover, we notice that over the range of metal-poor stars ([Fe/H] < -1.5) our pulsational calibration is in good agreement with the relations given by Fernley (1993), Groenewegen & Salaris (1999) and Carretta et al. (1999, 2000), while with [Fe/H] > -1.5 it agrees with McNamara’s (1999) results.

Application of our results to RR Lyrae stars of metal-poor globular clusters in the Large Magellanic Cloud (see data in GS) would give a distance modulus of 18.61 ± 0.12 mag \( (f = 1) \) and 18.56 ± 0.12 mag \( (f = 3) \), thus supporting the ‘long’ distance scale (see also Romaniello et al. 2000).

By relying on the present pulsational RR Lyrae absolute magnitudes, one derives that the non-linearity of our \( \langle M_V(\text{RR}) \rangle - [\text{Fe/H}] \) relation is well reproduced by current predictions based on stellar evolution theory. However, in the case of solar-scaled chemical compositions, none of the evolutionary predictions published in the recent literature appears in satisfactory agreement, supporting observational evidence for \( \alpha \)-enhanced chemical mixtures in metal-poor stars. With the \( \alpha \) elements enhanced by a factor of 3 with respect to iron, the predictions by CS and Fr99 agree with our pulsational magnitudes even though they have a tendency to overestimate the luminosity of metal-rich pulsators. Interestingly enough, one finds that the Cs99 relation reproduces the general dependence of \( M_V(\text{RR}) \) on \( \log Z \) well, but with an overluminosity of about 0.08 mag. On the basis of Cs99 results, beautiful agreement with our data would be achieved by systematically increasing the cluster metallicity by \( \sim 0.2 \) dex, an occurrence that is unlikely to be acceptable.

To discuss this point further, one has to remember that differences in stellar models are mainly, if not only, the result of differences in the adopted input physics. Discussing RR Lyrae stars in the globular cluster M5 (Caputo et al. 1999), we have already reported pulsational evidence suggesting that models with the ‘most updated’ input physics (as in Cs99) give HB stars that are too luminous. Such evidence has been supported further by independent estimates based on Hipparcos parallaxes for clumping field He-burning stars (Castellani et al. 2000). Data seen in Fig. 4 reinforce such evidence, suggesting that the ‘most updated physics’ is probably far from being the most adequate. In summary, we have the tantalizing evidence that Cs99 models give the right metal dependence but not the right luminosities, whereas those of Fr99 and CS give much better luminosities but a slightly worse slope.
The role played by the various physical ingredients in determining the predicted luminosity of HB structures has been recently discussed in several papers (see Cassisi et al. 1999; Castellani & Degl’Innocenti 1999; Castellani 1999) and cannot be repeated here. However, the reader may notice that the most recent theoretical predictions displayed in Fig. 4 all agree within a range of luminosity of about $\pm 0.05$ mag. This, in our view, should be regarded as evidence of the high standard reached by evolutionary theories, as well as a warning that better precision should require a corresponding level of accuracy in the input physics, not yet reached by currently available evaluations.

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