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discussion

A New View of the Modal Control Technique[1]

B. PORTER.[2] It has already been admitted by Guerin and Rabins [1][3] that the "new view of the modal control technique" presented by Ju, Rabins, and Han [2] "does not result in a true modal control." However, the conditions under which the technique of Ju, Rabins, and Han [2] *does* result in a true modal control have still not been established by Guerin and Rabins [1] even though such conditions were essentially given by Porter and Crossley [3] in 1970.

The computation of the $p \times m$ proportional feedback controller matrix K_{fb} is reduced by Ju, Rabins, and Han [2] to the solution of the equation

$$TK_{fb}{}^{*}T^{-1} = BK_{fb}D \qquad (1)$$

where T is an $n \times n$ nonsingular eigenvector matrix, $K_{fb}{}^{*}$ is a prescribed $n \times n$ diagonal matrix, B is an $n \times p$ matrix of rank $p(p \leq n)$, and D is an $m \times n$ matrix of rank $m(m \leq n)$. The solution of equation (1) proceeds [2] by pre- and post-multiplying equation (1) by B' and D', respectively, so that

$$B'TK_{fb}{}^{*}T^{-1}D' = B'BK_{fb}DD'. \qquad (2)$$

Since $B'B$ and DD' are nonsingular $p \times p$ and $m \times m$ matrices, respectively, the solution continues [2] by pre- and post-multiplying equation (2) by $(B'B)^{-1}$ and $(DD')^{-1}$, respectively, so that

$$K_{fb} = (B'B)^{-1}B'TK_{fb}{}^{*}T^{-1}D'(DD')^{-1} \qquad (3)$$

which is the result given by Ju, Rabins, and Han [2].

However, it is obvious by substituting from equation (3) into equation (1) that the value for K_{fb} given in equation (3) will in fact only be a solution of equation (1) in cases where the matrices B, D, T, and $K_{fb}{}^{*}$ satisfy the equation

$$TK_{fb}{}^{*}T^{-1} = B(B'B)^{-1}B'TK_{fb}{}^{*}T^{-1}D'(DD')^{-1}D. \qquad (4)$$

It is clear that, except in the trivial case $m = n = p$, equation (4) will be satisfied only for a very restricted class of matrices $\{K_{fb}{}^{*}\}$ which will contain diagonal members only in very special circumstances. In particular, in the two cases

$$K_{fb}{}^{*} = \text{diag}\ (0.43, 1.35, 0.79, 1.39, 1.12, 2.0) \qquad (5a)$$

and

$$K_{fb}{}^{*} = \text{diag}\ (20, 20, 20, 20, 20, 20) \qquad (5b)$$

considered in detail by Ju, Rabins, and Han [2], it is a matter of simple computation to demonstrate that equation (4) is falsified in both cases.

The "new view of the modal control technique" [1, 2] is accordingly defective in the following two fundamental respects:

(i) in any design the set $\{K_{fb}{}^{*}\}$ of admissible matrices must be determined by nontrivial computations from equation (4);

(ii) there is absolutely no guarantee that any member of the resulting set $\{K_{fb}{}^{*}\}$ corresponds to satisfactory closed-loop time-domain behavior.

[1]By H. Ju, M. J. Rabins, and C. D. Han, published in the September, 1975, issue of the Journal of Dynamic Systems, Measurement, and Control, Trans. ASME, Series G, Vol. 97, pp. 300–308.

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[3]Numbers in brackets designate Additional References at end of discussion.

Additional References

1 Guerin, J. P., and Rabins, M. J., "A New View of the Modal Control Technique," (Letter to the Editor), Journal of Dynamic Systems, Measurement, and Control, Trans. ASME, Series G, Vol. 97, No. 4, Dec. 1975, pp. 456–457.

2 Ju, H., Rabins, M. J., and Han, C. D., "A New View of the Modal Control Technique," Journal of Dynamic Systems, Measurement, and Control, Trans. ASME, Series G, Vol. 97, No. 3, Sept. 1975, pp. 300–308.

3 Porter, B., and Crossley, T. R., "Simple Method of Closed-Loop System Matrix Assignment," *Electronics Letters*, Vol. 6, No. 3, Feb. 1970, pp. 79–80.

Authors' Closure

We thank Professor Porter for bringing to our attention his 1970 article [3]. The conditions he notes in his discussion are precisely those that Guerin and Rabins had in mind in their 1975 letter to the editor [1] when they wrote that the approach of Ju, Han, and Rabins [2] "does not result in a true modal control." The goal of that latter study was an approximate approach based upon the pseudo-inverse technique. There probably could have been a better title for the paper [2] to avoid any misunderstanding on theoretical rigor. Since control engineers are dealing with real systems for which rigorous linear equations never precisely apply, they must always check the performance of their design by a computer simulation that includes the various nonlinearities—especially end effects. This is especially true when they *approximate* a true system by a lumped-parameter linear model and use rigorous modal-control theory. Since the pseudo-inverse approach itself is an approximate technique, it is even more important that a check by computer simulation is absolutely necessary. A satisfactory closed-loop time domain behavior, in a practical sense, can be clamined at a later stage where various "nonideal" conditions are taken into account in a more detailed simulation, or even better, in a test of the real plant.

Unlike classical analog control, there are no hardware constraints on control algorithms when a mini-computer is used so that in many cases control engineers may retain flexibility to search for a satisfactory closed-loop response by testing a control algorithm on a real plant, by trimming control gains or even by changing control algorithms.