An accretion disc model with a magnetic wind and turbulent viscosity

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ABSTRACT
A model is presented for an accretion disc with turbulent viscosity and a magnetically influenced wind. The magnetic field is generated by a dynamo in the disc, involving the turbulence and radial shear. Disc-wind solutions are found for which the wind mass flux is sufficient to play a major part in driving an imposed steady inflow, but small enough for most material to be accreted on to the central object. Constraints arise for the magnetic Reynolds and Prandtl numbers in terms of the turbulent Mach number and vertical length-scale of the disc’s horizontal magnetic field. It is shown that the imposition of a stellar boundary condition enhances the wind mass flux in the very inner region of the disc and may result in jet formation.

Key words: accretion, accretion discs — magnetic fields — MHD — turbulence — ISM: jets and outflows.

1 INTRODUCTION
Accretion discs are of fundamental importance in stellar astrophysics. They frequently occur around compact objects in binary stars, around young stars in T Tauri systems, and are believed to be the main power source in active galactic nuclei. Such discs enable matter of high angular momentum to be accreted on to the central object. The inflow of material is facilitated by the continuous outward transport of angular momentum due to some coupling mechanism, usually of viscous or magnetic origin. One such mechanism, considered by Blandford & Payne (1982), is the effect of a magnetically channelled wind emanating from the disc surfaces. This requires the presence of a suitable magnetic field, and their model had this advected through the disc from an external source. They considered the structure of a wind from a Keplerian disc, treating the disc as an infinitely thin boundary.

There have been many studies of magnetically influenced winds from accretion discs since the paper of Blandford & Payne (1982). Like their paper, most of these studies have concentrated on the detailed wind structure, with the disc treated as a boundary (e.g. Uchida & Shibata 1985; Camenzind 1987; Lovelace, Berk & Contopoulos 1991; Pelletier & Pudritz 1992; Ferreira 1997; Ouyed & Pudritz 1997). Other papers considered the problem of wind launching and local disc structure, but from a diffusionless disc with isorotation holding (e.g. Ogilvie 1997; Ogilvie & Livio 1998). Ogilvie (1997) also considered the principles for a diffusive disc, but with the magnetic and turbulent diffusivities significantly lower than in standard discs, so isorotation still holds to lowest order in the local aspect ratio $h/\sigma$. Lubow, Papaloizou & Pringle (1994) investigated the stability of a simple local wind-driven model. Wardle & Koenigl (1993) considered disc and wind structure from weakly ionized protostellar discs, where ambipolar diffusion is appropriate.

The present paper considers a diffusive disc in which the magnetic field is generated and maintained by a dynamo mechanism. The magnetic field is related to a wind flow and the resulting radial structure of the disc is calculated, also allowing for turbulent viscosity. The dipole-symmetry magnetic field solution of Campbell (1999) is used, with the dynamical and thermal problems determining the radial dependence of the field. The surface wind launching angle is related to the sonic point position, which in turn affects the wind mass flux. The density of material passing through the sonic point is larger the nearer this point is to the disc surface, being controlled by the change in the gravitational–centrifugal potential from this surface. An increased density at the sonic point corresponds to an increased mass flux, while a normal accretion disc model requires the rate of mass loss from the disc surfaces to be small relative to the accretion rate. An increase in poloidal field bending lowers the height of the sonic point, and hence there is a limit on the field inclination to ensure a suitably small wind mass flux. The amount of field bending is affected by the inflow speed through the disc, and this is controlled by the rate of angular momentum extraction. Turbulent viscosity is included in the model in order to investigate its importance relative to angular momentum removal by the magnetic wind.

In Section 2 the magnetohydrodynamic (MHD) disc equations are formulated, describing the dynamical, magnetic and thermal problems. The thin nature of the disc allows some simplification of the problem. Section 3 considers a solution of the induction equation

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satisfying force-free boundary conditions, and uses this to relate the poloidal field bending to the inflow. It is shown from the vertical equilibrium that the resulting magnetic field compresses the disc, and that a lower limit arises for the turbulent magnetic Reynolds number.

In Section 4, angular momentum transport in the wind and disc are related. The sonic point position is found and used to show how the wind mass flux depends on the surface poloidal field inclination and the dynamo properties. A necessary condition is derived for a small wind mass flux, to ensure that most mass is accreted on to the central object. Section 5 considers magnetic and viscous dissipation in the disc, together with radiative transfer.

In Section 6 the radial structure of the disc is found, together with the related properties of the wind. Section 7 derives expressions to test the self-consistency of the solutions. Limits arise for the magnetic Reynolds and Prandtl numbers. Detailed solutions are presented and discussed for a range of turbulent Mach numbers, and the effect of an inner stellar boundary condition is investigated. Section 8 summarizes the conclusions and discusses future work in this area.

2 THE DISC EQUATIONS

A thin, axisymmetric steady disc is considered surrounding a star of mass $M$ and radius $R$. Cylindrical coordinates $(\sigma, \phi, z)$ are used, centred on the star with the disc symmetry plane at $z = 0$.

The $\sigma$-, $\phi$- and $z$-components of the momentum equation are

\begin{equation}
\frac{v_\sigma^2}{\sigma} = \frac{v_\sigma^2}{\sigma} + \frac{1}{\rho} \frac{\partial P}{\partial \sigma} + \frac{1}{\rho \sigma^2 \rho \partial \sigma} \left( \frac{\sigma^2 B_\phi^2}{2 \mu_0} \right) - \frac{1}{\rho} B_\phi J_\phi, \tag{1}
\end{equation}

\begin{equation}
\frac{v_\phi}{\sigma} \frac{\partial}{\partial \sigma} \left( \sigma^2 \Omega \right) + \frac{v_z}{\partial z} \left( \sigma^2 \Omega \right) = \frac{1}{\sigma \rho \partial \sigma} \left( \rho \nu \sigma \frac{\partial \Omega}{\partial \sigma} \right) + \frac{1}{\mu_0 \rho \sigma \partial \sigma} \left( \sigma^2 B_\phi B_\phi \right) + \frac{\sigma}{\mu_0 \rho \partial z} \left( B_\phi B_z \right), \tag{2}
\end{equation}

\begin{equation}
\Omega_k^2 \frac{1}{\partial z} + \frac{1}{\rho} \frac{\partial}{\partial z} \left( P + \frac{B_\phi^2}{2 \mu_0} \right) + \frac{1}{\rho} B_\phi J_\phi = 0, \tag{3}
\end{equation}

where the toroidal current density

\begin{equation}
J_\phi = \frac{1}{\mu_0} \left( \frac{\partial B_\sigma}{\partial z} - \frac{\partial B_z}{\partial \sigma} \right). \tag{4}
\end{equation}

\( \nu \) is the turbulent viscosity and $v_k = \sigma \Omega_k$ with

\begin{equation}
\Omega_k = \left( \frac{GM}{\sigma^3} \right)^{\frac{1}{2}}. \tag{5}
\end{equation}

The poloidal field in the disc is subsonic, and the associated small inertial terms have been dropped in (1)–(3).

For normal disc temperatures the radial gradient of the pressure in (1) is small and, for the dipole symmetry magnetic fields considered here, $B_\phi^2/2 \mu_0 \ll P$ so the first magnetic term is also small. The magnetic field required for wind driving of the inflow has horizontal components of a vertical length-scale of $-h$, the semi-thickness of the disc. It therefore follows that the vertical derivative term dominates in the toroidal current density and hence (4) becomes

\begin{equation}
J_\phi = \frac{1}{\mu_0} \frac{\partial B_\sigma}{\partial z}. \tag{6}
\end{equation}

Since $\partial B_\sigma/\partial z \sim B_\phi/h$, the ratio of the last magnetic term in (1) to the gravitational term can be estimated as

\begin{equation}
\frac{\sigma B_\phi J_\phi}{\rho v_k^2} \sim \frac{B_\phi^2}{\mu_0 \rho_c P_c} \left( \frac{B_\phi}{B_c} \right) \left( \frac{c_s}{\Omega_k h} \right)^2 \frac{h}{\sigma}, \tag{7}
\end{equation}

using the isothermal sound speed $c_s = \sqrt{P_c/\rho_c}$ with the subscript $c$ denoting mid-plane values. This ratio is $\sim (h/\sigma)^2$, as will be confirmed in Section 7. It follows that the azimuthal velocity in the bulk of the disc is close to Keplerian values, so

\begin{equation}
v_\phi = v_k = \left( \frac{GM}{\sigma} \right)^{\frac{1}{2}}. \tag{8}
\end{equation}

Equation (2) relates the radial advection of angular momentum to the viscous and magnetic torques. The poloidal velocity components are also related via the mass conservation equation

\begin{equation}
\frac{\partial}{\partial \sigma} \left( \sigma \rho v_\sigma \right) + \frac{\partial}{\partial z} \left( \sigma \rho v_z \right) = 0. \tag{9}
\end{equation}
Using (6) for \( J_\phi \), the vertical equilibrium (3) becomes

\[
\Omega_k^2 + \frac{1}{\rho} \frac{\partial}{\partial z} \left( P + \frac{B_\sigma^2 + B_\phi^2}{2\mu_0} \right) = 0.
\]

(10)

Since the radial length-scale of \( B \) is \( \sim \sigma \) and the disc inflow is subsonic, the radial diffusion terms and most of the advection terms are small in the mean-field induction equation. The poloidal and toroidal components of this equation then become

\[
v_a B_z + \frac{\partial B_\sigma}{\partial z} - \alpha B_\sigma = 0,
\]

(11)

\[
\eta \frac{\partial^2 B_\phi}{\partial z^2} = -\pi \Omega_k' B_\sigma,
\]

(12)

where \( \eta \) is the magnetic diffusivity, \( \alpha(\sigma, z) \) is the poloidal magnetic field creation function, and the prime in (12) denotes differentiation. As explained in Campbell (1999), these equations give reasonable solutions for \( B \) which contain essential features in agreement with fields generated in discs by magnetic shear instabilities (also see Brandenburg & Campbell 1997). The heat equation relating the divergence of the radiative flux to the viscous and magnetic dissipations in the disc is

\[
\frac{\partial F_R}{\partial z} = \rho \mu (\pi \Omega_k')^2 + \mu_0 \eta J^2.
\]

(13)

For an optically thick disc the flux is given by the radiative diffusion equation

\[
F_R = -\frac{4\sigma_B}{3\kappa \rho} \frac{\partial}{\partial z} (T^4),
\]

(14)

where \( \sigma_B \) is the Stefan–Boltzmann constant, and \( \kappa \) the Rosseland mean opacity. For negligible radiation pressure, the equation of state is

\[
P = \frac{\mathcal{R}}{\bar{\mu}} \rho T,
\]

(15)

where \( \mathcal{R} \) is the gas constant and \( \bar{\mu} \) the mean molecular weight.

The problem is to solve the disc equations and connect their solution to that of a wind flow which extracts angular momentum at a rate sufficient to play a part in driving the radial velocity in the disc.

### 3 Magnetoic Field Generation and the Vertical Equilibrium

#### 3.1 Solution of the Induction Equation

The dynamo equations were solved with force-free boundary conditions in Campbell (1999) to obtain the vertical dependence of the magnetic field. This enables the poloidal field wind launching angle at the disc surface to be related to the inflow and the wind mass flux. The main results will be gathered here for connection with the dynamical and thermal problems, the solution of which determines the radial dependence of \( B \).

Since detailed expressions are not presently available for \( \alpha \) and \( \eta \) generated by magnetic shear instabilities, it is reasonable to represent them by simple parametrized forms. Taking \( \eta \) as independent of \( z \) in the disc enables an analytic solution of the induction equation to be found. Eliminating \( B_\sigma \) between the induction equation components (11) and (12) gives

\[
\frac{\partial^3 B_\phi}{\partial z^3} + \frac{\pi \Omega_k' \alpha}{\eta^2} B_\phi = \frac{\pi \Omega_k'}{\eta} v_a B_z.
\]

(16)

For an effective wind, the Alfvén surface lies well beyond the disc surface, and consequently the wind magnetic field will be nearly force-free close to \( z = \pm h \). Such a field obeys the equation

\[
B_\phi \cdot \nabla (\pi B_\phi) = 0.
\]

(17)

A dipole symmetry field has horizontal components which are antisymmetric about the disc mid-plane, and hence

\[
B_\sigma(\sigma, 0) = B_\phi(\sigma, 0) = 0.
\]

(18)

The slow variation of the aspect ratio \( h/\sigma \) and the form used below for \( \alpha \) enable the toroidal field in and near the disc to be expressed in the separable form

\[
B_\phi(\sigma, z) = B_\phi(\sigma) f_\phi(\zeta),
\]

(19)

where $B_{\phi}(\sigma) = B_{\phi}(\sigma, h)$ and $\zeta = z/h$. A simple form is used for $\alpha$, given by
\[ \alpha(\sigma, z) = \begin{cases} -\hat{\alpha}(\sigma), & 0 < z < h, \\ 0, & z = 0, \\ \hat{\alpha}(\sigma), & -h < z < 0, \end{cases} \]
where $\hat{\alpha} > 0$. This incorporates the necessary antisymmetry, and the use of negative $\alpha$ above the mid-plane is consistent with simulation results for turbulence generated by magnetic shear instabilities (Brandenburg & Donner 1997).

Substitution of (19) for $B_{\phi}$ in (16) leads to the equation
\[ f'''_{\phi} + Df_{\phi} = -DF, \]
where the dynamo number is
\[ D = \frac{3\Omega_k \hat{\alpha} h^3}{2\eta^2} \]
and
\[ F = \frac{\psi_{\sigma} B_z}{\hat{\alpha} B_{\phi}}. \]

For a dipole-symmetry field with a vertical length-scale $\geq h$, it follows that $B_{\phi} = B_{\phi}(\sigma, h) = B_{\phi}(\sigma)$ to lowest order in $h/\sigma$. It will be shown that steady mass inflow solutions are consistent with $D$ and $F$ being constant. The general solution of (21) is then
\[ f_{\phi} = \gamma - F, \]
where $\gamma(\xi)$ satisfies the homogeneous equation
\[ \gamma''' + Dy = 0. \]

The force-free condition (17) at the disc surface and the antisymmetry conditions (18) lead to the three boundary conditions
\[ y'(1) = 0, \quad y(0) = F, \quad y''(0) = 0. \]

The solution of (25) satisfying these conditions is
\[ y = \frac{2}{\sqrt{3} Q} \left\{ e^{-K(\xi - 1)} \cos \left( \frac{\sqrt{3}}{2} K + \frac{\pi}{6} \right) + e^{K(\xi - 1)/2} \left[ \cos \left( \frac{\sqrt{3}}{2} K \xi - \frac{\pi}{6} \right) + e^{3K/2} \cos \left( \frac{\sqrt{3}}{2} K(\xi - 1) + \frac{\pi}{6} \right) \right] \right\}, \]
where
\[ Q = 1 + 2e^{3K/2} \cos \left( \frac{\sqrt{3}}{2} K \right) \]
and
\[ K = D^{1/3} = \frac{h}{\ell_{z}} \]
with $\ell_z$ the vertical length-scale of $B_{\phi}$ in the disc. Equation (19) requires $f_{\phi}(1) = 1$, then (24) and (27) give
\[ F(K) = \frac{1 + 2e^{3K/2} \cos(\sqrt{3}K/2)}{2e^K \sinh K - 2 \sinh(K/2) \cos(\sqrt{3}K/2)}. \]

This function vanishes at critical $K$-values given by
\[ K_c = (2m - 1) \frac{\pi}{\sqrt{3}}, \quad m = 1, 2, 3, \ldots \]

However, (23) shows that $F$ must be finite for a non-vanishing inflow and magnetic field so the critical values of $K$ cannot occur, but $K$ could be close to $K_c$. The simplest vertical variation of $B_z$, with $B_{\phi}$ and $B_{\phi}$ monotonically increasing in magnitude from the central plane to the disc surface, occurs for $K < \pi/\sqrt{3}$ with $\pi/\sqrt{3}$ being the first critical eigenvalue in (31). Fig. 1 shows the variation of $F(K)$ for a range of relevant $K$-values. It is noted from (29) that very small values of $K$ cannot be considered, since $\ell_z \ll \sigma$ is a necessary condition for ignoring the radial derivatives of field components in the induction equation. The solution $y(\xi)$, given by (27), is a good approximation for $K > 0.3$. 

3.2 Field ratios at the disc surface

The inclination of the poloidal field at the disc surface is an important quantity, being related to the position of the sonic point and the associated wind mass flux. Consideration of the poloidal induction equation (11) in Campbell (1999) led to the surface field ratio

\[ \frac{B_{\infty}}{B_z} = \frac{[\nu_{\infty}h]}{\eta} I_1(K), \]

where

\[ I_1(K) = \frac{e^{K/2}[e^{3K/2} - 2 \cos(\sqrt{3}K/2 + \pi/3)]}{K[1 + 2e^{3K/2} \cos(\sqrt{3}K/2)]}. \]

This function of \( K \) is related to the vertical variation of \( B_{\infty} \) by

\[ I_1(K) = \langle \frac{\partial B_{\infty}}{\partial z} \rangle \bigg|_{z=0}, \]

where the angle brackets denote the vertical mean. The functions \( I_1(K) \) and \( F(K) \) are related to \( y''(1) \) by

\[ I_1(K) = -\frac{y''(1)}{K^2 F}. \]

Fig. 2 shows the variation of \( I_1(K) \).

Simple parametrized forms are taken for \( \alpha \) and \( \eta \) as

\[ \alpha = \epsilon c_s, \]

and

\[ \eta = \epsilon_m c_s h, \]

where the dimensionless quantities \( \epsilon \) and \( \epsilon_m \) are < 1 for subsonic turbulence, and

\[ c_s = \left( \frac{P_c}{\rho_c} \right)^{1/2} = \left( \frac{\mathcal{R}}{\mu T_c} \right)^{1/2} \]

is the isothermal sound speed in the mid-plane. It will be shown that self-consistent disc-wind solutions always have \( B_{\infty}^2/2\mu_0 \) values significantly below \( P_c \), and hence quenching modifications are not used here.

Taking \( \alpha \) as the rms turbulent speed \( v_T \), (36) gives the turbulent Mach number

\[ \epsilon = \frac{v_T}{c_s}. \]
A turbulent magnetic Reynolds number can be defined as

\[ N_A = \frac{\tilde{\alpha} h}{\eta} = \frac{\tau_D}{\tau_T} \]  

(40)

This gives a characteristic value for the ratio of the vertical diffusion time \( \tau_D = h^2/\eta \) to the turbulent time-scale \( \tau_T = h/\tilde{\alpha} \). Using (36) and (37) for \( \alpha \) and \( \eta \) in (40) yields

\[ N_A = \frac{\epsilon}{\epsilon_m}. \]  

(41)

The turbulent diffusivity can then be expressed as

\[ \eta = \frac{\epsilon}{N_A c_s h}. \]  

(42)

Use of (36) and (42) in (22) and (29) gives the dynamo number

\[ K^3 = \frac{3}{2} N_A^2 \Omega \kappa h. \]  

(43)

Equations (12), (19), (24), (42) and (43) yield the surface ratio of the horizontal field components as

\[ \frac{B_{r s}}{B_{\theta s}} = \frac{f(K)}{N_A}. \]  

(44)

where

\[ f(K) = \frac{K^3}{\gamma^a(1)} = -2Ke^{K/2} \frac{[\sinh K - 2\sinh(K/2)\cos(\sqrt{3}K/2)]}{e^{3K/2} - 2\cos(\sqrt{3}K/2 + \pi/3)}. \]  

(45)

Fig. 3 shows the variation of this function.

A surface poloidal field angle can be defined by

\[ \tan i_s = \frac{B_{r s}}{B_{\theta s}} = \frac{1}{I_1[\nu]|h|}. \]  

(46)

where the last equation follows from (32). The amount of field bending is determined by a balance between distortion arising from the inflow and field slippage due to diffusion.

### 3.3 The vertical equilibrium

Vertical integration of (10), employing \( P_s \ll P_c \) and the field conditions (18), gives the aspect ratio

\[ \frac{h}{\sigma} = (2\pi)^{1/2} \frac{c_s}{\nu_k} \left( 1 - \frac{B_{\theta s}^2 + B_{r s}^2}{2\mu_0 P_c} \right)^{1/2}. \]  

(47)
where \( P_c = P(\sigma), 0 \). The vertical mean \( \langle z \rho \rangle = \rho_c h / 2n \) is used, where \( n \) is related to the vertical dependence of the density (see Section 6.4). It is seen from (47) that the horizontal magnetic field compresses the disc. A finite-thickness disc requires

\[
\frac{B_{\text{ms}}^2 + B_{\text{os}}^2}{2 \mu_0 P_c} < 1. \tag{48}
\]

Eliminating \( \Omega_K h / c_s \) between (43) and (47) leads to

\[
\frac{B_{\text{ms}}^2 + B_{\text{os}}^2}{2 \mu_0 P_c} = 1 - \frac{2}{9n} \frac{e^2}{N_a^2} K^6, \tag{49}
\]

ensuring that (48) is satisfied.

Using (44) to eliminate \( B_{\text{ms}}^2 \) in (49) gives the relation

\[
\frac{B_{\text{os}}^2}{\mu_0} = A P_c, \tag{50}
\]

where

\[
A(K, N_a, \epsilon) = 2 \left( 1 - \frac{2}{9n} \frac{e^2}{N_a^2} K^6 \right) \left( 1 + N_a^2 \right)^{-1}, \tag{51}
\]

with \( f(K) \) given by (45). A condition for finite \(|B_{\text{os}}|\) values then follows as \( A > 0 \), and hence

\[
K < \left( \frac{9n}{2} \right)^{\frac{1}{2}} \left[ \frac{2}{\epsilon} \right]^{\frac{1}{2}} \left[ \frac{N_a^2}{\epsilon} \right]^{\frac{1}{2}}, \tag{52}
\]

is required. Use of (29) for \( K \) then yields

\[
\epsilon_c > \left( \frac{2}{9n} \right)^{\frac{1}{2}} \frac{1}{\epsilon} \frac{1}{\epsilon} h. \tag{53}
\]

It will be shown later how the vertical equilibrium affects the wind mass flux and hence the angular momentum transport through the disc.

### 4 Angular Momentum Transport

#### 4.1 Wind angular momentum and mass flux

Beyond the disc in the wind region the diffusivity \( \eta \) becomes small, and highly conducting material flows along the magnetic field. The poloidal velocity and magnetic field are therefore related by

\[
\mathbf{v}_p = s(r) \mathbf{B}_p. \tag{54}
\]
Conservation of mass and poloidal flux then gives

\[ \mathbf{B}_p \cdot \nabla \left( \frac{\rho v_z}{B_z} \right) = \mathbf{B}_p \cdot \nabla \left( \frac{\rho v_z}{B_z} \right) = 0, \]  

(55)

so \( \rho v_z/B_z \) is conserved along the field-streamlines. Angular momentum conservation for the wind flow can be written

\[ \frac{\rho v_z}{B_z} \sigma^2 \Omega - \frac{1}{\mu_0} \sigma v_B = \frac{\rho v_z}{B_z} \sigma^2 \Omega_{\kappa}(\sigma) \]  

(56)

(e.g. Campbell 1997), where \( \sigma_{\kappa} \) denotes the Alfvén point and \( \sigma \), the surface coordinate where a field line cuts the disc at \( z = h \). Well inside the Alfvén surface magnetic stress dominates material stress and (56) becomes

\[ \frac{1}{\mu_0} \sigma v_B = - \frac{\rho v_z}{B_z} \sigma^2 \Omega_{\kappa}(\sigma) \]  

(57)

to a good approximation. This is consistent with the force-free condition (17).

The vertical dependence of this is contained in \( \sigma_{\kappa} \) and, since \( \sigma_{\kappa} \) is nearly constant through the disc and hence \( \sigma v_B/\sigma_z \) does not change sign, this implies that

\[ B_{\text{mr}}(\sigma, z) = B_{\text{mr}}(\sigma_0) \frac{z}{h}, \]  

(58)

where \( \sigma_0 \) denotes points in the mid-plane. The use of (58) in \( \nabla \cdot \mathbf{B}_p = 0 \) then shows that to leading order

\[ B_z(\sigma, z) = B_z(\sigma_0, 0) = B_{\text{mr}}(\sigma_0). \]  

(59)

Another way of demonstrating the nearly linear variation of \( B_{\text{mr}} \) with \( z \) in the disc is to note that the toroidal component of the induction equation, given by (12), yields

\[ \frac{\partial B_{\text{mr}}}{\partial z} = \frac{2}{3} \frac{\eta}{\Omega_{\kappa}} \frac{\partial^3 v_\phi}{\partial z^3}. \]  

(60)

The use of (19), (24) and (25) for \( B_\phi \) then gives

\[ \frac{\partial B_{\text{mr}}}{\partial z} = \frac{2}{3} K_3 \frac{\eta B_{\text{mr}}}{\Omega_{\kappa} h^3 y(\zeta)}. \]  

(61)

The vertical dependence of this is contained in \( y(\zeta) \) and through the disc (i.e. \( 0 \leq \zeta \leq 1 \)) it follows from \( y(0) = F(K) \), together with \( f_\phi = y - F \) and \( f_\phi(1) = 1 \), that

\[ F \leq y(\zeta) \leq F + 1. \]  

(62)

Equation (30) gives \( F(K) \approx 10 \) for \( K \approx 0.7 \), and hence the variation of \( y(\zeta) \) through the disc is small (<10 per cent) for such values of \( K \). It follows from (61) that \( \partial B_{\text{mr}}/\partial z \) is nearly constant through the disc and hence \( B_{\text{mr}} \) is close to a linear function of \( z \).

Provided that the sonic point vertical coordinate satisfies \( z_{\text{sn}}/\sigma_0 \ll 1 \), the poloidal field components (58) and (59) can be used up to the sonic surface. These components give the local field-line equation

\[ z^2 = 2h \tan i_s(\sigma - \sigma_0), \]  

(63)

where \( \tan i_s \) is the surface field ratio given by (46).

It is noted that Ogilvie (1997) derived a locally straight field-line geometry for the wind region close to the disc. However, this was a consequence of taking zero magnetic diffusivity and exact isorotation everywhere (i.e. \( \mathbf{B}_p \cdot \nabla \Omega = 0 \)). Then \( B_\phi = 0 \) and hence in the nearly force-free region, where \( \mathbf{J} \) is essentially parallel to \( \mathbf{B} \), it follows that \( \mu_0 J_\phi = \partial B_{\text{mr}}/\partial z = 0 \) and the poloidal field is locally straight. However, there is then no magnetic wind torque on the disc, since \( B_{\text{mr}} B_z = 0 \), and hence no inflow driving. Ogilvie (1997) also considered the case of a low-diffusivity disc, with \( \eta \) a factor of \( h/\sigma_0 \) smaller than in the standard case used here. This leads to \( B_{\text{dr}}/B_z \sim h/\sigma_0 \) and a locally straight poloidal field to lowest order, while the standard diffusive disc considered here gives \( B_{\text{dr}}/B_z \sim 1 \) and a locally curved poloidal field.

The mass flux through the sonic point is

\[ \dot{m} = (\rho v_z)_{\text{sn}}. \]  

(64)
and it was shown in Campbell (1999) that the coordinates of this point are
\[
\alpha_{sn} = \alpha_0 + \left( \frac{1}{3} \tan \iota_s + \frac{1}{2} \tan \iota_s \right) h, \tag{65}
\]
\[
z_{sn} = \left( 1 + \frac{2}{3} \tan^2 \iota_s \right) \frac{1}{2} h. \tag{66}
\]

The inclination of the poloidal field to the horizontal at the sonic point was found to be
\[
i_s = \sin^{-1} \left( \frac{\tan \iota_s}{\left( 1 + \frac{2}{3} \tan^2 \iota_s \right)^{\frac{1}{2}}} \right). \tag{67}
\]

The mass flux given by (64) can then be expressed as
\[
m = (\rho v_z)_s \frac{\sin i_s}{\sin i_{sn}} = \left( 1 + \frac{2}{3} \tan^2 \iota_s \right)^{\frac{1}{2}} (\rho v_z)_s, \tag{68}
\]

using (67) for \( \sin i_s \).

Evaluating (57) at the sonic point, and eliminating \( (\rho v_z)_s \) between this and (68), relates the magnetic field stress \( B_o B_z \mu_0 \) to the mass flux through the sonic point by
\[
\frac{1}{\mu_0} (\alpha B_o B_z)_s = -\frac{\tan \iota_s}{\left( 1 + \frac{2}{3} \tan^2 \iota_s \right)^{\frac{1}{2}}} m \sigma^3 \Omega_k (\alpha_s). \tag{69}
\]

The force-free field condition (17) shows that \( \alpha B_o \) is constant along \( B_p \), while the leading-order approximation (59) gives \( B_z \) constant. Hence, to this approximation, \( \alpha B_o B_z \) is conserved along \( B_p \) and so (69) gives
\[
\frac{1}{\mu_0} (\alpha B_o B_z)_s = -\frac{\tan \iota_s}{\left( 1 + \frac{2}{3} \tan^2 \iota_s \right)^{\frac{1}{2}}} m \sigma^3 \Omega_k (\alpha_s). \tag{70}
\]

The conservation of \( \alpha B_o B_z \) along \( B_p \) close to the disc surface applied to (57) shows that \( \rho v_z \) is also conserved along the poloidal field in this region, and hence an alternative to (68) is
\[
m = \left( 1 + \frac{2}{3} \tan^2 \iota_s \right)^{\frac{1}{2}} (\rho v_z)_s. \tag{71}
\]

It was shown in Campbell (1999) that the mass flux \( m \) is related to the surface poloidal field inclination by
\[
m = a \rho_s \exp \left( -\left[ \Omega_k^2 (\alpha_0) h^2 / 6 \sigma^2 \right] \tan^2 \iota_s \right), \tag{72}
\]

where \( a \) is the isothermal sound speed in the gas between the disc surface and the sonic point, and \( \rho_s \) is the density at the disc surface. It follows from (43) that
\[
\frac{\Omega_k^2 h^2}{a^2} = \frac{4}{9} \frac{e^2}{N_o K^s} \frac{\left( \alpha_0 \right)^2}{\alpha_s}. \tag{73}
\]

It is noted that (63) relates the \( \alpha \)-coordinate where a poloidal field line cuts the disc surface to that at its footpoint in the mid-plane by
\[
\alpha_s = \left( 1 + \frac{1}{2} \tan \iota_s \frac{h}{\sigma_0} \right) \alpha_0. \tag{74}
\]

It is essential to distinguish between \( \Omega_k (\alpha_s) \) and \( \Omega_k (\alpha_0) \) in deriving the sonic point coordinates (65) and (66), and the wind mass flux (72) (see Campbell 1999). Equations (5) and (74) relate the angular velocities by
\[
\Omega_k (\alpha_s) = \Omega_k (\alpha_0) \left( 1 + \frac{1}{2} \tan \iota_s \frac{h}{\sigma_0} \right)^{-\frac{1}{2}}. \tag{75}
\]

However, it is sufficient in (70) to use the lowest order expansions in \( h/\sigma_0 \) of (74) and (75). To this order \( \sigma_s = \sigma_0 \) and \( \Omega_k (\alpha_s) = \Omega_k (\alpha_0) \), and (70) becomes
\[
\frac{1}{\mu_0} \sigma_0 B_o B_z \sigma = -\frac{\tan \iota_s}{\left( 1 + \frac{2}{3} \tan^2 \iota_s \right)^{\frac{1}{2}}} m \sigma^3 \Omega_k (\alpha_s). \tag{76}
\]
4.2 Angular momentum advection in the disc

If most mass is accreted through the disc rather than lost in the wind, a restriction relating the wind mass flux \( \dot{m} \) to the accretion rate \( \dot{M} \) is necessary. For clarity, the cylindrical coordinate \( \sigma_0 \) will now be used for points in the disc. Vertical integration through the disc of the continuity equation (9), noting the antisymmetry of \( v_z \), gives

\[
\frac{d}{d\sigma_0} \left( \int_{-h}^{h} \sigma_0 \rho v_z \, dz \right) = -2 \sigma_0 (\rho v_z)_h. \tag{77}
\]

The mass inflow rate through the disc is

\[
\dot{M} = -2\pi \int_{-h}^{h} \sigma_0 \rho v_z \, dz,
\]

and hence (77) becomes

\[
\frac{d\dot{M}}{d\sigma_0} = 4\pi \sigma_0 (\rho v_z)_h. \tag{78}
\]

Use of (71) to eliminate \( (\rho v_z)_h \) then gives

\[
\frac{d\dot{M}}{d\sigma_0} = \frac{4\pi \tan i_s}{\left(1 + \frac{1}{2} \tan^2 i_s\right)^{\frac{3}{2}}} \sigma_0 \dot{\mu}_i. \tag{79}
\]

Small mass loss via the wind is consistent with \( \dot{M} \) varying on a radial length-scale \( \gg \sigma_0 \), and hence

\[
\frac{\sigma_0}{M} \frac{d\dot{M}}{d\sigma_0} = \frac{4\pi \sigma_0^2 \dot{\mu}_i}{\left(1 + \frac{1}{2} \tan^2 i_s\right)^{\frac{3}{2}}} M < 1
\]

is required. This is satisfied by

\[
\frac{4\pi \sigma_0^2 \dot{\mu}_i}{M} \approx 1, \tag{80}
\]

and then \( \dot{M} \) can be taken to be essentially constant through the disc.

Multiplying the angular momentum equation (2) by \( \sigma_0 \rho \) and combining it with the continuity equation (9) yields

\[
\frac{\partial}{\partial \sigma_0} (\sigma_0 \rho \sigma_0^2 \Omega_K) + \frac{\partial}{\partial z} (\sigma_0 \rho \sigma_0^2 \Omega_K) = \frac{\partial}{\partial \sigma_0} (\rho \nu \sigma_0^2 \Omega_K') + \frac{1}{\mu_0} \frac{\partial}{\partial \sigma_0} (\sigma_0^2 B_\sigma B_z) + \frac{1}{\mu_0} \frac{\partial}{\partial z} (\sigma_0^2 B_\sigma B_z). \tag{81}
\]

This relates the poloidal advection of angular momentum through the disc to the viscous and magnetic torques. Vertical integration of (81) through the disc, using (78) for \( \dot{M} \), gives

\[
-\frac{M}{2\pi} \frac{d}{d\sigma_0} (\sigma_0^2 \Omega_K) + 2 \sigma_0^2 \Omega_K' (\rho v_z)_h = \frac{d}{d\sigma_0} (\sigma_0^2 \Omega_K' \rho \Sigma) + \frac{d}{d\sigma_0} \left[ \frac{\sigma_0^2}{\mu_0} \int_{-h}^{h} B_\sigma B_d \, dz \right] + \frac{2}{\mu_0} \sigma_0^2 (B_\sigma B_d)_h, \tag{82}
\]

where

\[
\Sigma = \int_{-h}^{h} \rho \, dz. \tag{83}
\]

The ratio of the magnitude of the second to the first term in (82) is

\[
\frac{4\pi \sigma_0 \sigma_0^2 \Omega_K' (\rho v_z)_h}{M d(\sigma_0^2 \Omega_K)/d\sigma_0} = \frac{2 \tan i_s}{\left(1 + \frac{1}{2} \tan^2 i_s\right)^{\frac{3}{2}}} \frac{4\pi \sigma_0^2 \dot{\mu}_i}{M} \ll 1,
\]

using (5) and (71) for \( \Omega_K \) and \( \dot{m} \) and then condition (80). Hence the second term in (82) is negligible. Using the mean-value definition in the magnetic integral, the ratio of the magnetic torque terms is

\[
\frac{d(\sigma_0^2 (B_\sigma B_d)_h)/d\sigma_0}{\sigma_0^2 (B_\sigma B_d)_h} = \frac{B_\sigma h}{B_d B_z} \approx \frac{B_\sigma h}{B_d \sigma_0} \ll 1,
\]

since \( B_\sigma h / B_d \approx 1 \) for wind launching. Equation (82) therefore becomes

\[
-\frac{M}{2\pi} \frac{d}{d\sigma_0} (\sigma_0^2 \Omega_K) = \frac{d}{d\sigma_0} (\sigma_0^2 \Omega_K' \rho \Sigma) + \frac{2}{\mu_0} \sigma_0^2 (B_\sigma B_d)_h. \tag{84}
\]

The use of (76) in (84) to express the magnetic torque in terms of the wind mass flux then yields the disc angular momentum...
equation as

\[ \frac{M}{2\pi} \frac{d}{d\varpi_0} (\sigma^2_0 \Omega_K^2) = \frac{3}{2} \frac{d}{d\varpi_0} (\sigma^2_0 \Omega_K \nu) + 2 \tan \hat{i}_a \mu_0 \sigma_0 \Omega_K (\varpi_0). \]  

(85)

This gives the radial advection of angular momentum through the disc due to the turbulent viscous torque and the magnetic wind torque.

## 5 DISSIPATION AND ENERGY TRANSPORT

### 5.1 Viscous and magnetic dissipation

The thermal problem involves calculating the viscous and magnetic dissipations in the disc and relating these to the radiative flux for optically thick gas. The use of an opacity and an equation of state then enables \nu to be expressed in terms of the central temperature and disc height.

Vertical integration of the heat equation (13) over the range 0 < z < h, noting that \nu (\varpi_0, 0) = 0, gives the disc radiative surface flux as

\[ F_{\nu} = \frac{1}{2} (\sigma_0 \Omega_K^2)^2 \nu \Sigma + \mu_0 \eta \int_0^h J^2 \, dz. \]  

(86)

It can be shown that the \( J^2 \) term makes the major contribution to the magnetic dissipation integral. To prove this, note that (19) and (24) for \( B_\phi \) give the poloidal components of the current density in the disc as

\[ J_x = -\frac{1}{\mu_0} \frac{1}{\varpi_0} \frac{d}{d\varpi_0} (\sigma_0 B_\phi) = \frac{1}{\mu_0} \frac{d}{d\varpi_0} (\sigma_0 B_\phi) f_x. \]  

(87)

\[ J_y = \frac{1}{\mu_0} \frac{1}{\varpi_0} \frac{d}{d\varpi_0} (\sigma_0 B_\phi) = \frac{1}{\mu_0} \frac{d}{d\varpi_0} (\sigma_0 B_\phi) f_y. \]  

(88)

The toroidal current density follows from (6) and (61) as

\[ J_\phi = \frac{1}{\mu_0} \frac{1}{\varpi_0} \frac{d}{d\varpi_0} (\sigma_0 B_\phi) = -\frac{2}{3} \frac{K^3}{\mu_0} \frac{\eta B_{\phi0}}{h^2} y. \]  

(89)

Equations (37) and (43) for \( \eta \) and \( K \) yield

\[ \frac{\eta K^3}{\Omega_k h^2} = \frac{3}{2} N_a, \]

and hence (89) becomes

\[ J_\phi = -\frac{N_a B_{\phi0}}{\mu_0} \frac{h}{h} y. \]  

(90)

Equations (24) and (27) for \( y(z) \) give \( f_y \sim K f_\phi \), except close to the disc surface where \( f_y \sim h/\varpi_0 \) due to the force-free boundary condition (see Campbell 1999). It follows from (87) and (88) that

\[ \int_0^h J_x^2 \, dz \sim \frac{1}{K^2} \left( \frac{h}{\varpi_0} \right)^2 \int_0^h J_y^2 \, dz \]  

(91)

in the magnetic dissipation integral. Hence, since \( K \ll 1 \) for a monotonic vertical variation of the components of \( B \) in the disc, the \( J^2 \) contribution to the dissipation is ignorable. Noting that \( f_\phi = y / Ky \), (87) and (90) yield

\[ \int_0^h J_\phi^2 \, dz \sim \frac{N_{\phi0}^2}{K^2} \int_0^h J_\phi^2 \, dz. \]  

(92)

A shear-based magnetic Reynolds number can be defined as

\[ N_{\phi0} = \frac{\sigma_0 \Omega_k h^2}{\eta} = -\frac{3 v_k h^2}{2 \varpi_0 \eta}. \]  

(93)

Using this, together with (40) for \( N_\alpha \) and (36) for \( \alpha \), gives

\[ \frac{|N_\omega|}{N_\alpha} = \frac{K^3}{N_{\phi0}}. \]  

(94)

The condition for the validity of an \( \alpha \omega \)-dynamo is \( |N_\omega|/N_\alpha \gg 1 \), and hence

\[ N_{\phi0}^2 \ll K^3 \]  

(95)

is required. It then follows that \( N_{\phi0}^2 / K^3 \ll K \), and since \( K \ll 1 \) this gives \( N_{\phi0}^2 / K^2 \ll 1 \), therefore (92) shows that the contribution of \( J_\phi^2 \) is ignorable in the magnetic dissipation integral.

Substituting for $J_4$ in (86) from (87), and using (5) for $\Omega_K$, gives the surface radiative flux as

$$F_{Rs} = \frac{9}{8} \Omega_K^2 v_\Sigma + \frac{\eta B_\phi^2}{\mu_0 h} \int_0^1 y'' d\zeta. \quad (96)$$

Defining the integral

$$I_2(K) = \int_0^1 y'' d\zeta, \quad (97)$$

this can be expressed in terms of the derivatives of $y(\zeta)$ at $\zeta = 0$ and 1. Equations (25) and (29) give

$$y''' + K^3 y = 0. \quad (98)$$

Multiplying by $y''$, this can be written as

$$\frac{1}{2} \frac{d}{d\zeta} (y'')^2 = -K^3 y'' y'. \quad (99)$$

Integrating over $0 < \zeta < 1$, and using (26) for the boundary conditions on $y$, yields

$$\int_0^1 y'' d\zeta = \frac{y''(1)}{2K^3} - F(K) y'(0), \quad (100)$$

where $F(K)$ is given by (30). Using (99) in (97) gives the surface flux (96) as

$$F_{Rs} = \frac{9}{8} \Omega_K^2 v_\Sigma + \frac{\eta B_\phi^2}{\mu_0 h} I_2(K). \quad (101)$$

Differentiation of (27) yields

$$y'(0) = \frac{KF}{Q} \left[ 1 - 2e^{3K/2} \cos \left( \frac{\sqrt{3}}{2} K + \frac{\pi}{3} \right) \right], \quad (102)$$

where $Q$ is given by (28), while (35) gives

$$y''(1) = -K^3 F(K) I_1(K). \quad (103)$$

Equations (97), (99), (101) and (102) then yield

$$I_2(K) = K F^2 \left\{ \frac{1}{2} K^2 I_1 - \frac{1}{Q} \left[ 1 - 2e^{3K/2} \cos \left( \frac{\sqrt{3}}{2} K + \frac{\pi}{3} \right) \right] \right\}. \quad (104)$$

Fig. 4 shows the variation of $I_2(K)$.

The magnetic dissipation term in (100) can be expressed as a multiple of the viscous dissipation, by using the vertical equilibrium. It follows from (42) and (50) for $\eta$ and $B_\phi$, that

$$\frac{\eta B_\phi^2}{\mu_0 h} = \epsilon \frac{v}{N_a} A c_P c. \quad (105)$$

A standard parametrized turbulent viscosity can be defined as

$$\nu = \epsilon \sigma c_s h, \quad (106)$$

where $\epsilon < 1$ for subsonic turbulence and $c_s$ is given by (38). The magnetic Prandtl number is then

$$N_P = \frac{\nu}{\eta} = \frac{\epsilon \sigma}{\epsilon} N_a, \quad (107)$$

where the last expression follows from (42) and (105). Using this to eliminate $\epsilon \sigma$ in (106) gives

$$\nu = \epsilon N_P c_s h. \quad (108)$$

Taking the vertical average $\langle \rho \rangle = \rho_c / \sqrt{n}$ (see Section 6.4) with $n > 1$, (83) gives

$$\Sigma = \frac{2}{\sqrt{n}} \rho_c h. \quad (109)$$

It follows from (107) and (108) that

$$\Omega_K^2 v_\Sigma = \frac{2}{\sqrt{n}} \epsilon N_P c_s \rho_c \Omega_K^2 h^2 = \frac{8}{9 \sqrt{n}} \epsilon N_P K^6 c_s P_c. \quad (110)$$
using (43) to eliminate $\Omega_k h$, and $P_c = \rho_c c_s^2$. Substituting for $c_s P_c$ in (104) from (109) gives

$$\frac{\eta B_{\text{rs}}}{\mu_0 h} = \frac{9}{8} \sqrt{\frac{\pi}{e}} \frac{N_a}{N_p} \frac{A \Omega_k^2}{K^6} \nu \Sigma.$$  \hfill (110)

Equation (100) for the surface radiative flux then becomes

$$F_{\text{rs}} = \frac{9}{8} W \Omega_k^2 \nu \Sigma,$$  \hfill (111)

where

$$W = 1 + \sqrt{\frac{\pi}{e}} \frac{N_a}{N_p} \frac{A I_2}{K^6}.$$  \hfill (112)

The last term in this equation is the ratio of magnetic to viscous dissipation in the disc.

### 5.2 Thermal energy transport

As will be confirmed, the disc gas is optically thick so radiative diffusion holds. The vertical integration of (14) gives the central temperature from

$$\frac{4}{3} \sigma_b T_c^4 = \int_0^h \kappa \rho F_R \, dz.$$  \hfill (113)

A Kramers opacity is used, so

$$\kappa = K \rho T^{-\frac{1}{2}},$$  \hfill (114)

where $K$ is a constant. Taking the vertical mean $\langle \kappa \rho F_R \rangle = \kappa_c \rho_c F_{\text{rs}}$ in (113), and using (114), relates the central temperature and density by

$$T_c^{15} = \frac{3K}{4\sigma_b} \rho_c^2 h F_{\text{rs}}.$$  \hfill (115)

Using (38) for the isothermal sound speed, it follows from (115) that

$$c_s^{15} = \frac{3K}{4\sigma_b} \left( \frac{N_a}{N_p} \right)^{\frac{15}{2}} \rho_c^2 h F_{\text{rs}}.$$  \hfill (116)

Equations (107) and (108) for $\nu$ and $\Sigma$ give

$$(\nu \Sigma)^2 = \frac{4}{n} \frac{e^2 N_p^2}{N_a} c_s^2 h^3 \rho_c^2 h.$$
and eliminating $p_c h$ between this and (116) leads to

$$c_s^2 = \frac{3n}{16} \frac{\hat{K}}{\sigma_B} \left( \frac{\epsilon}{\mu} \right)^{\frac{1}{2}} \frac{N_a^2}{e^2 N_p^2} \frac{(\Sigma \sigma_s)^2}{h^3} F_{K4}. \quad (117)$$

Substituting for $F_{K4}$ in (117) from (111) then gives

$$\nu \Sigma = \frac{4}{3} \frac{2 \sigma_B}{n K} \left( \frac{\mu}{\Sigma \sigma_s} \right)^{\frac{1}{2}} \frac{e^2 N_p^2}{\nu s} \frac{c_s^2 h}{s} \frac{2}{N_p W \Omega K} \frac{2}{\Omega K}. \quad (118)$$

6 DISC STRUCTURE AND WIND PROPERTIES

6.1 The disc solution

The inflow speed $v_{in}$ is taken to be weakly dependent on $z$ in the main bulk of the disc, since there is no cause of significant vertical shears in this region. This is supported by numerical calculations of vertical structure in magnetic discs by Shalybkov & Ruediger (2000). Equations (78) and (83) then give the mass accretion rate through the disc as

$$M = 2 \pi \sigma_0 |v_{in}| \Sigma. \quad (119)$$

It is noted that $v_{in}$ will vary rapidly with height through a narrow region in the tenuous surface layers where the wind flow begins, with the inflow reversing sign to become an outflow. Equation (46) relates the surface wind launching angle $i_s$ to the disc inflow speed. Using (106) and (119) to eliminate $\eta$ and $|v_{in}|$ in (46) gives

$$\nu \Sigma = \frac{1}{2 \pi} N_p I_1 M \tan i_s \frac{h}{\sigma_0}. \quad (120)$$

Equating this to (118), and using (38) for $c_s$, gives the central temperature as

$$T_c = \left( \frac{3}{8 \pi} \right)^{\frac{6}{17}} \frac{n \mu G}{2 \sigma_0 R} \left( \frac{\epsilon}{\Sigma \sigma_s} \right)^{\frac{2}{17}} \frac{N_a^2 N_p^2}{e^2} I_1^6 W \Omega K M^{\frac{1}{17}} (\tan i_s)^{\frac{6}{17}}. \quad (121)$$

The dynamo number equation (43), together with (38) for $c_s$, relates $h$ to $T_c$ via

$$h = \frac{2}{3} \left( \frac{\epsilon}{\Sigma \sigma_s} \right)^{\frac{1}{3}} N_a^2 K^\frac{3}{2} \Omega K. \quad (122)$$

Substituting for $T_c$ from (121) then yields

$$h = \frac{2}{3} \left( \frac{3}{8 \pi} \right)^{\frac{1}{17}} \left( \frac{n K}{2 \sigma_0} \right)^{\frac{1}{17}} \left( \frac{n G}{\mu} \right)^{\frac{15}{17}} \frac{N_a^2}{e^2} K^\frac{1}{17} W \frac{1}{\Omega K M^{\frac{1}{17}}} (\tan i_s)^{\frac{3}{17}} \sigma_0^{\frac{20}{17}}. \quad (123)$$

Using this in (120) gives

$$\nu \Sigma = \frac{1}{3 \pi} \left( \frac{3}{8 \pi} \right)^{\frac{1}{17}} \left( \frac{n \hat{K}}{2 \sigma_0} \right)^{\frac{1}{17}} \left( \frac{n G}{\mu} \right)^{\frac{15}{17}} \frac{N_a^2}{e^2} K^\frac{1}{17} W \frac{1}{\Omega K M^{\frac{1}{17}}} (\tan i_s)^{\frac{20}{17}} \sigma_0^{\frac{6}{17}}. \quad (124)$$

The central density can be found by noting that (107) and (108) for $\nu$ and $\Sigma$ give

$$\nu \Sigma = \frac{2}{\sqrt{n}} \frac{N_a}{N_p} c_s \rho_c h^2. \quad (125)$$

Then equating this to (120) gives

$$\rho_c = \frac{\sqrt{n}}{4 \pi} \frac{N_a}{N_p} I_1 M \tan i_s \frac{h}{\sigma_0 c_s h}. \quad (126)$$

Equation (43) yields

$$c_s h = \frac{3}{2} \frac{N_a^2}{\epsilon} \frac{1}{K^3} \Omega K h^2. \quad (127)$$
and hence (124) becomes

$$\rho_c = \frac{\sqrt{n}}{6\pi N_a} K^3 I_1 \tan i \frac{M}{v_k h^2}.$$  

(125)

Use of (122) for $h$ then gives the central density as

$$\rho_c = \left( \frac{3}{8\pi} \right)^{11} (nG)^{13} \left( \frac{2\sigma_b}{K} \right)^{17} \left( \frac{K}{\mu} \right)^{15} \frac{N^2_0}{e^{17} N_P^2 K^6} M^{13} I^{11} \frac{(\tan i_c)^{11}}{\eta_0^{17}}.$$

(126)

Equation (110) relating $B_{\phi_0}$ to $v \Sigma$ was derived from the dynamo equations and vertical equilibrium. Using (42) for $\eta$ in (110) gives

$$B_{\phi_0} = \frac{9\sqrt{n}}{8} \mu_0 N_a^3 A \Omega_k h / c_s.$$  

Substituting for $\Omega_k h / c_s$ from (43) and for $v \Sigma / h$ from (120) then leads to

$$B_{\phi_0} = -\left( \frac{3\mu_0}{8\pi} \right)^{17} (nG)^{13} \left( \frac{\sigma_b}{K} \right)^{17} \left( \frac{1}{K} \right)^{16} \frac{A_i I_1}{M^2 M^2} \frac{1}{\tan i_c^{12} / \eta_0^{17}}.$$

(127)

Equations (44) and (46) give

$$B_{c_s} = \frac{N_a}{\sqrt{n}} \frac{\tan i_c}{f(K)} B_{\phi_0}.\tag{128}$$

The function $f(K)$ given by (45) is negative for the relevant $K$-values and, since $\tan i_c > 0$ for wind launching, it follows from (128) that $B_{\phi_0}$ and $B_{c_s}$ have opposite signs. This is consistent with $B_{\phi_0} B_{c_s} < 0$, corresponding to the wind extraction of disc angular momentum in (84). Eliminating $B_{\phi_0}$ between (127) and (128) yields

$$B_{c_s} = \left( \frac{3\mu_0}{8\pi} \right)^{17} (nG)^{13} \left( \frac{\sigma_b}{K} \right)^{17} \left( \frac{1}{K} \right)^{16} \frac{A_i I_1}{M^2 M^2} \frac{1}{\tan i_c^{12} / \eta_0^{17}}.$$

(129)

The remaining surface field component is then found from $B_{\infty} = B_{c_s} / \tan i_c$.

The bulk inflow speed through the disc can be found from (38), (42) and (46) as

$$v_{\infty} = \frac{\varepsilon}{N_a} \left( \frac{\eta_1}{\mu} \right)^{17} \frac{\sigma_b}{I_1 \tan i_c}.$$  

Use of (121) for $T_c$ then gives

$$v_{\infty} = -\left( \frac{3}{8\pi} \right)^{17} (nG)^{13} \left( \frac{\sigma_b}{K} \right)^{17} \left( \frac{1}{K} \right)^{16} \frac{A_i I_1}{M^2 M^2} \frac{1}{\tan i_c^{12} / \eta_0^{17}}.$$

(130)

### 6.2 Normalized disc solutions

The foregoing disc solutions can be expressed in convenient normalized forms using parameters typical for discs around white dwarfs. This gives

$$h = 3.7 \times 10^3 n^{0.15} \frac{K^{0.17} I^{0.13} (\tan i)^{0.17} \sigma}{M^{0.31}} \text{ m},$$

(131)

$$T_c = 3.0 \times 10^5 n^{0.15} N^{0.17} \frac{N_a^{0.25} I^{0.17} M^{0.10} (\tan i)^{0.17}}{e^{17} N_P^{0.25} K^{0.51}} \text{ K},$$

(132)

$$\rho_c = 2.1 \times 10^{-6} n^{0.15} \frac{N_a^{0.25} I^{0.17} M^{0.10} (\tan i)^{0.17}}{e^{17} N_P^{0.25} K^{0.51} \sigma} \text{ kg m}^{-3},$$

(133)

$$v_{\infty} = -6.5 \times 10^{0.15} n^{0.15} \frac{K^{0.17} I^{0.17} M^{0.10} (\tan i)^{0.17}}{N_a^{0.15} I^{0.17} \sigma} \text{ m s}^{-1},$$

(134)

\[
B_{\phi s} = -10^{-2} n^2 \frac{5}{\epsilon} N_0 \left( \frac{A_1}{K^3} \right)^{\frac{1}{2}} M_0^{\frac{1}{2}} \frac{1}{\sigma_A^2} \left( \frac{\tan i_s}{\tan i_s} \right)^{\frac{3}{2}} \text{ tesla},
\]

\[
B_{zs} = 10^{-2} n^2 \frac{5}{\epsilon} N_0 \left( \frac{A_1}{K^3} \right)^{\frac{1}{2}} M_0^{\frac{1}{2}} \frac{1}{\sigma_A^2} \left( \frac{\tan i_s}{\tan i_s} \right)^{\frac{3}{2}} \text{ tesla},
\]

where \( M_1 = M / M_{\odot} \), \( M_{10} = M / 10^{-10} M_{\odot} \text{ yr}^{-1} \) and \( \sigma_A = \sigma_0 / 10^8 \text{ m} \). The weak explicit dependence on the vertical structure measure \( n \) is noted. It will be shown how \( \tan i_s \) and \( n \) can be calculated, and also what effect an inner boundary condition has on the \( \sigma_0 \) variation of \( \tan i_s \) and hence on the disc structure.

### 6.3 Wind properties

The wind properties consistent with the foregoing disc solution are now derived. Equation (55) expresses the conservation of mass and poloidal magnetic flux in the wind flow. Applying this to the disc surface and Alfvén point gives

\[
(p v). v(h) = (p v). v(\Lambda). \frac{B_{zs}}{B_{z\Lambda}}.
\]

The wind mass flux given by (71) can then be expressed as

\[
\dot{m} = \frac{1}{\tan i_s} \frac{1}{\tan i_s} \frac{B_{zs}}{B_{z\Lambda}} \rho_A \nu_A.
\]

By definition

\[
u_{\Lambda}^2 = \frac{B_{z\Lambda}^2}{\mu_0 \rho_A},
\]

while fast rotator wind theory gives

\[
u_{\Lambda} = 0.5 \sigma_A \Omega_K \sigma_0
\]

(e.g. Campbell 1997, Mestel 1999), using \( \sigma_s = \sigma_0 \) to a good approximation. Equations (139) and (140) yield

\[
\rho_A \nu_A = \frac{2 B_{z\Lambda}^2}{\mu_0 \sigma_A \Omega_K},
\]

and use of this in (138) leads to

\[
\dot{m} = 2 \frac{1}{\tan i_s} \frac{1}{\tan i_s} \frac{B_{zs}}{B_{z\Lambda}} \frac{\sigma_0}{\sigma_A} \frac{\sigma_0}{\sigma_0} \hat{f},
\]

where

\[
\hat{f} = \frac{B_{z\Lambda}}{B_{zs}}.
\]

Another expression for \( \dot{m} \) follows from the wind angular momentum equation (76) as

\[
\dot{m} = - \left( \frac{1}{\tan i_s} \frac{1}{\tan i_s} \frac{B_{\phi s}}{B_{zs}} \frac{\sigma_0}{\sigma_A} \right)^2.
\]

Equating this to (141) gives

\[
\frac{B_{\phi s}}{B_{zs}} = -2 \frac{\sigma_A}{\sigma_0} \hat{f}.
\]

This quantity can be related to \( v \Sigma \) by using the disc angular momentum equation (85), which can be written as

\[
\frac{M}{2\pi} \frac{d}{d \sigma_0} \left[ \sigma_0^2 \Omega_K \left( 1 - 3 \pi v \Sigma \frac{M}{M} \right) \right] = \frac{2 \tan i_s}{\left( 1 + \frac{5}{2} \tan^2 i_s \right) \pi} \frac{\nu_A}{\sigma_0} \sigma_A \sigma_0 \Omega_K.
\]
Noting that (123) gives
\[
\frac{d}{d\sigma_0} (v\Sigma) = \frac{5}{34} \frac{v \Sigma}{\sigma_0},
\]
while \(d(\sigma_0 \Omega_k)/d\sigma_0 = \sigma_0 \Omega_k/2\), use of (141) for \(m\) in (145) leads to
\[
\frac{M}{4\pi} \left( 1 - \frac{66\pi \Sigma}{17 M} \right) = 4 \sigma_0^2 \frac{B_{02}^2}{\mu_0 v_k} \frac{\sigma_A}{\sigma_0} \tilde{f}.
\]
Substituting for \(B_{02}\) from (129) then yields
\[
\tilde{f} = \frac{1}{6 \sqrt{n} N_a^3 A_i \tan^2 i_s} \left( 1 - \frac{66\pi \Sigma}{17 M} \right) \frac{\sigma_0}{\sigma_A}.
\]
Eliminating \(B_{02}/B_3\) between (128) and (144) gives
\[
\frac{1}{N_a \tan i_s} = \frac{2 \sigma_A}{\sigma_0} \tilde{f},
\]
and then using (148) to eliminate \(\tilde{f}\) gives
\[
\tan^2 i_s = \frac{1}{3 \sqrt{n} N_a^4 A_i} \left( 1 - \frac{66\pi \Sigma}{17 M} \right) \frac{\sigma_0}{\sigma_A}.
\]
An equation determining \(\tan i_s\) then results by substituting \(\Sigma\) from (123) into (150), giving
\[
\tan^2 i_s + C(\tan i_s)^{20} - \frac{1}{3} \frac{\epsilon^2 K_i^3 |f|}{\sqrt{n} N_a^4 A_i} = 0,
\]
where
\[
C = \frac{2.4 \times 10^{-3}}{n^{1/2} T_{10}^{3/2}} \frac{30}{N_a^{1/2} K_i} \frac{1/3}{1} \frac{W}{1/2} \frac{M_i}{1} \frac{1}{1/2} \frac{\sigma_i^0}{\sigma_i^A},
\]
employing the same normalization as used in (131)–(136), and taking \(\bar{\mu} = 0.6\) and the opacity constant \(K = 10^{19} \text{ m}^2 \text{ kg}^{-2} \text{ K}^{-2/3}\). It is found that \(\tan i_s\) is very weakly dependent on \(\sigma_0\), as a result of the weak spatial dependence of \(C\). Noting that \(\tan i_s \sim 1\) for suitable wind launching angles, then the use of \((\tan i_s)^2 = 1\) is a good approximation in (151), which consequently becomes a quadratic equation with solution
\[
\tan i_s = \frac{1}{2} \left[ \left( C^2 + \frac{4}{3} \frac{\epsilon^2 K_i^3 |f|}{\sqrt{n} N_a^4 A_i} \right)^{1/2} - C \right].
\]

The Alfvén coordinate \(\sigma_A\) can be found by considering the wind mass flux \(m\). The isothermal sound speed in the gas between the disc surface and the sonic point and the density at the disc surface are taken to be
\[
\alpha = \xi_c c_s,
\]
\[
\rho_s = \xi_d \rho_c,
\]
where the constants \(\xi_c\) and \(\xi_d\) are both < 1. The justification for these forms is given below. Equation (72) then becomes
\[
\dot{m} = \xi_d \xi_c \rho_c \exp[-(\Omega_k^2 h^2/6c^2) \tan^2 i_s].
\]
Another expression for \(m\) can be derived by writing (143) as
\[
\dot{m} = -\left( 1 + \frac{2}{3} \tan^2 i_s \right)^{1/2} \frac{B_{02}^2 B_{2s}}{\mu_0 v_k} \frac{\sigma_0}{\sigma_A} \left( \frac{1}{|f|} \right)^{1/2} \frac{A P_i c_s}{\sigma_i^0} \frac{\sigma_0}{\sigma_A}
\]
using (50) for \(B_{02}^2\) and (128) for \(B_{2s}\). Equating this to (155), noting that \(P_i = \rho_c c_s^2\), gives
\[
\left( \frac{\sigma_A}{\sigma_0} \right) = \left( \frac{\sigma_0}{\xi_c \xi_d} \right)^{1/2} \left( \frac{1}{|f|} \right)^{1/2} \left( \frac{1}{2} \right) \left( \frac{\alpha}{c_s} \right) \exp[(\Omega_k^2 h^2/6c^2) \tan^2 i_s].
\]
Substituting for \(h\) in (120) from (43) yields
\[
\frac{c_s}{v_k} = \frac{3\pi N_a^2}{e N_p K_i^3 A_i \tan i_s M}.
\]

Equation (150) gives
\[
\frac{\nu_S}{M} = \frac{17}{66\pi} \left( 1 - 3\sqrt{n} \frac{N_a^2}{e^2} \frac{A_1}{K^3|p|} \tan^2 i_c \right). \tag{158}
\]

Using (157) and (158) to eliminate \(c_s/v_K\) in (156) gives the wind Alfvén \(\sigma\)-coordinate as
\[
\sigma_{\lambda} = \frac{0.9}{(\xi_s \xi_d)^2} \frac{N_a^3}{e^2 N_f^2} \left( \frac{A}{K^2 T_c|p|} \right)^{\frac{1}{2}} \left( 1 + \frac{e^2 \tan^2 i_c}{\tan^2 i_c} \right)^{\frac{1}{2}} \left( 1 - 3\sqrt{n} \frac{N_a^2}{e^2} \frac{A_1}{K^3|p|} \tan^2 i_c \right)^{\frac{1}{2}} \exp\left( [\Omega_{\lambda} h^2 / 12a^2] \tan^2 i_c \right) \xi_0. \tag{159}
\]

It follows from (73) and (154a) that
\[
\frac{\Omega_{\lambda} h^2}{a^2} = \frac{4 e^2 K^6}{9 e^2 N_a^3}. \tag{160}
\]

### 6.4 Evaluation of disc surface quantities

The dimensionless quantities \(\xi_s\) and \(\xi_d\), defined in (154), will be determined by the detailed vertical structure of the disc. The present model does not calculate the detailed vertical dependence of \(\rho\) but the quantity \(n_s\), introduced in the vertical equilibrium integral (47), is used to gauge the effect of the vertical density profile on the solutions. A reasonable estimate for \(n\) can be made by employing the density function corresponding to an isothermal vertical structure in the absence of a magnetic field, giving
\[
\rho(\xi_0, z) = \rho_c(\xi_0) e^{-z^2/\lambda^2}, \tag{161}
\]

where \(\lambda = h(\xi_0) = \sqrt{n_\lambda}\),

\[
\rho(\xi_0, z) = \rho_c(\xi_0) e^{-z^2/\lambda^2}. \tag{162}
\]

relating the disc surface height to the width of the Gaussian profile. It will be shown that \((B_{\sigma_0}^2 + B_{\xi_0}^2)/2\mu_0 P_c\) is significantly less than unity for a self-consistent wind-influenced disc, while the surface temperature is typically \(T_s = 0.5 T_c\). Hence the ansatz (161) should give reasonable results. Its vertical average leads to
\[
\left( \frac{\rho}{\rho_c} \right) = \frac{1}{\sqrt{n}}. \tag{163}
\]

showing how \(n_s\) is related to the vertical central condensation in the disc. The form (161) also gives the result \(\langle zho \rangle = \rho_c h/2n\) used in the derivation of the aspect ratio (47). The density ratio \(\xi_d\) can now be related to \(n_s\), since (161) and (162) yield
\[
\xi_d = \frac{\rho_c}{\rho_s} = e^{-n_s}. \tag{164a, b}
\]

It follows from (154a) that
\[
\xi_s = \left( \frac{T_s}{T_c} \right)^{\frac{1}{2}}, \tag{165}
\]

where \(T_s\) is the disc surface temperature. Taking this to be the effective temperature gives
\[
F_{Rs} = \sigma_B T_s^4, \tag{166}
\]

where \(\sigma_B\) is the Stefan–Boltzmann constant and \(F_{Rs}\) the radiative surface flux. Equations (114) and (115) yield
\[
F_{Rs} = \frac{4 \sigma_B T_s^4}{3 \kappa_c \rho_c h}. \tag{167}
\]

Equating (166) and (167) leads to
\[
\left( \frac{T_s}{T_c} \right)^{\frac{1}{2}} = \left( \frac{4}{3 \kappa_c \rho_c h} \right)^{\frac{1}{2}}. \tag{168}
\]

The optical depth through the disc is given by
\[
\tau_D = \int_0^h \kappa \rho dz = K \int_0^h \rho^2 T^{-3/2} dz. \tag{169}
\]
where the last expression follows from the Kramers opacity (114). Taking $\rho T^{-7/2}$ as approximately independent of $z$ yields

$$\tau_0 = K \rho_c T_c^{-7/2} (\rho) h,$$

and hence the use of (163) for $\langle \rho \rangle$ leads to

$$\tau_0 = \frac{K}{\sqrt{n}} \rho_c T_c^{-7/2} h.$$ (170)

This can also be expressed as

$$\tau_0 = \frac{1}{\sqrt{n}} \kappa \rho_c h.$$ (171)

Equations (165), (168) and (171) then give

$$\xi = \frac{1}{n^{1/6}} \left( \frac{4}{3 n_0^2} \right)^{1/8}.$$ (172)

Using (38) and (108) to eliminate $T_c$ and $\rho_c$ in (170), together with the form of $\nu$ from (107), enables the disc optical depth to be expressed as

$$\tau_0 = \sqrt{n} \left( \frac{\mu}{\bar{\mu}} \right)^{1/2} \frac{2 K N_a^2 (\nu \Sigma)^2}{e^2 N_p^2 h^2 e_c^2}.$$ (173)

Substituting for $c_s$ and $\nu \Sigma$ from (43) and (118) leads to

$$\tau_0 = 64 \frac{\alpha^2}{n^4} \left( \frac{\mu}{\bar{\mu}} \right)^{1/2} \frac{2 K N_a^2}{e^2 N_p^2 h^2 e_c^2}.$$ (174)

Use of (131) for $h$ then gives

$$\tau_0 = \frac{11}{n^{3/2}} \frac{N_a^{1/2}}{\epsilon_{17}^{3/2} N_p^{11/2}} \frac{4}{I_{17}^{1/2} (\tan i_{17})^{1/2}} \frac{4}{M_{10}^{1/2}} \frac{\Omega h^{1/2}}{M_{10}^{1/2}} \frac{1}{\sigma_8},$$ (175)

where the normalizations of Section 6.2 are employed. The very weak dependence on $\sigma_8$ shows that the optical depth is essentially constant through the disc.

The ratio of sound speeds, $\xi_s$, follows from (172) and (174) as

$$\xi_s = a_s = 0.77 \frac{31}{n^{1/6}} \frac{N_a^{5/2}}{\epsilon_{17}^{5/2} N_p^{5/2}} \frac{K^{3/2} \Omega^{3/2}}{I_{17}^{1/2} (\tan i_{17})^{1/2}} \frac{M_{10}^{3/2}}{M_{10}^{3/2}},$$ (176)

where the dependence on $\sigma_8$ is put equal to unity owing to its very small fractional power dependence.

The density ratio $\xi_d$ can be related to $\xi_s$ by taking the disc surface as the photospheric base and using the standard result for the optical depth of a photosphere of

$$\tau_p = \kappa \rho_c h = \frac{2}{3}$$ (177)

(e.g. Cox & Giuli 1968). This, together with $\tau_0$ from (171) and the Kramers opacity (114), gives

$$\frac{\tau_p}{\tau_0} = \sqrt{n} \left( \frac{\rho_c}{\bar{\mu}} \right)^{1/2} \frac{T_s}{T_c}.$$ (178)

Use of the definitions of $\xi_d$ and $\xi_s$ from (164a) and (165) then leads to

$$\xi_d = \frac{1}{n_{1/3}} \left( \frac{\tau_0}{\tau_0} \right)^{1/2} \xi_s^7.$$ (179)

Then using $\tau_p = 2/3$, and (172) to eliminate $\tau_{Dp}$ gives

$$\xi_d = \frac{1}{\sqrt{2} \xi_s^7}.$$ (180)
Taking the surface $z = h$ to be the photospheric base is consistent with the use of the radiative diffusion equation in the optically thick disc, and an isothermal structure in the optically thin wind. The surface temperature $T_s$ is sufficient to lead to values of the wind mass flux $\dot{m}$ high enough to play a major part in driving the inflow.

An equation determining $n$ can be found by noting that (164b), (172) and (177) yield

$$e^n = \sqrt{2} \left( \frac{1}{4} \right) \left( \frac{15}{16} \right)^{1/2} \left( \frac{n}{\tau_0} \right)^{15/16},$$

which can also be written as

$$n = 0.08 + \frac{15}{16} \ln \left( \frac{1}{\tau_0} \right).$$

Equation (174) shows that $\tau_0$ has an $n$-dependence explicitly and via $W$ and $\tan i_s$, given by (112) and (153) respectively. Equation (179) can be solved numerically to give $n$ for specified values of $\epsilon, N_a$ and $N_P$.

An upper limit arises for the magnetic Prandtl number, corresponding to $B_{fs}^2 = 0$. Eliminating $\tau_0$ between (174) and (178), using (112) for $W$, yields

$$N_P = \frac{66N_a^2 M_{10}^2 \frac{1}{2} \exp \left( -\frac{136}{75} \nu \right)}{e^{10} K^{10} M_{10}^{4/3}},$$

where $A$ is given by (51). It follows that $N_P$ increases as $n$ decreases and reaches a maximum when $A$ vanishes, with (51) giving the corresponding minimum $n$-value of

$$n_{\text{min}} = \frac{2}{9} \frac{e^{2} K^{6}}{N_a^{2/3}}.$$

Substituting this in (180) gives the maximum magnetic Prandtl number as

$$\left( N_P \right)_{\text{max}} = \frac{23N_a^2 M_{10}^2 \frac{1}{2} \exp \left( -\frac{2}{5} \frac{e^{2} K^{6}}{N_a^{2/3}} \right)}{e^{10} K^{10} M_{10}^{4/3}}.$$

$N_P$ must be beneath this limit to ensure finite $B_{fs}$.

7 THE NATURE OF THE SOLUTIONS

7.1 Self-consistency conditions

Condition (80) must be satisfied to ensure a small mass-loss rate in the wind, so most material is accreted on to the star. The disc angular momentum equation (145) gives

$$\frac{4 \pi \sigma_d^2 \dot{m}}{M} = \left( 1 + \frac{5}{3} \tan^2 i_s \right) \left\{ \frac{d}{d \sigma_0} \left[ \sigma_0 \Omega_k \left( 1 - 3 \frac{\nu \Sigma}{M} \right) \right] \right\} \left( \frac{\sigma_0}{\sigma_k} \right)^2.$$

Use of (5) for $\Omega_k$ and (146) for the derivative of $\nu \Sigma$ then leads to

$$\frac{4 \pi \sigma_d^2 \dot{m}}{M} = \left( 1 + \frac{5}{3} \tan^2 i_s \right) \left( \frac{1}{2} \tan i_s \right) \left( 1 - \frac{66 \pi \nu \Sigma}{17} \right) \left( \frac{\sigma_0}{\sigma_k} \right)^2. \tag{183}$$

Equation (84) relates the radial advection of angular momentum to the viscous and magnetic torques per unit length. The ratio of these torques is

$$\frac{T_v}{T_m} = \frac{2 \sigma_d^2 B_{fs} B_{zs}}{d \left( \sigma_0 \Omega_k \nu \Sigma \right)/d \sigma_0}. \tag{184}$$

Equations (5), (84) and (146) yield

$$\frac{2}{\mu_0} \frac{\sigma_d^2 B_{fs} B_{zs}}{M} = \frac{4 \pi}{M} \left( 1 - \frac{66 \pi \nu \Sigma}{17} \right) \nu_k. \tag{185}$$
and
\[
\frac{d}{d\sigma_0} (\sigma_0^3 \Omega_K^4 \nu \Sigma) = - \frac{33}{34} v_k \nu \Sigma.
\] (186)

It follows that (184) becomes
\[
\frac{T_m}{T_v} = \frac{17}{66\pi} \frac{M}{\nu \Sigma} - 1.
\] (187)

Using this to eliminate \( \nu \Sigma/M \) in (183) leads to
\[
\frac{4\pi \sigma_0^3 \nu \Sigma}{M} = \left( 1 + \frac{1}{2} \tan^2 i_c \right) \left( \frac{T_m}{T_v} \right) \left( \frac{\sigma_0}{\sigma_A} \right)^2.
\] (188)

The torque ratio term is \( \leq 1 \) and hence most of the mass is accreted provided that \( \sigma_A > \sigma_0 \).

A condition for the validity of using a Keplerian disc can now be derived. First, consider the radial momentum equation (1) in the mid-plane \( z = 0 \). The thermal pressure term is \( \sim c_s^2/\sigma_0 \), and hence is ignorable since \( c_s^2 \ll v_K^2 \). The magnetic term involving \( B^2 \phi \) vanishes because \( B^2 \phi (\sigma_0, 0) = 0 \). Hence the only remaining term that might cause a significant deviation from Keplerian rotation in the mid-plane is the last magnetic term in (1). The relevant ratio of magnetic to gravitational force in the mid-plane is therefore
\[
U_c = \frac{\sigma_0 B_j J_\phi}{\rho_c v_K^2}.
\] (189)

Use of (59) and (90) for \( B_z \) and \( J_\phi \) then gives
\[
U_c = \frac{N_n}{\mu_0} \left( F(K) \frac{\sigma_0 B_\phi B_z}{\rho_c h v_K^2} \right),
\] (190)

noting that \( \gamma(0) = F(K) \). Eliminating \( \Sigma \) between (108) and (119) yields
\[
\rho_c h = \frac{\sqrt{n}}{4\pi \sigma_0 |\nu| v_K} M.
\]

Employing this, and (185) for \( B_\phi B_z \), in (190) gives
\[
U_c = \frac{N_n}{2\sqrt{n}} \left( F(K) \left( 1 - \frac{66\pi \nu \Sigma}{17 M} \frac{|\nu|}{v_K} \right) \right).
\] (191)

Equations (42) and (46) lead to
\[
|\nu| = \frac{\epsilon c_s}{N_n I_1 \tan i_c},
\] (192)

and using this in (191) yields
\[
U_c = \frac{\epsilon}{2\sqrt{n}} \left( F(K) \left( 1 - \frac{66\pi \nu \Sigma}{17 M} \frac{c_s}{v_K} \right) \right).
\]

Finally, substituting for \( c_s/v_K \) from (157), and using (187) to eliminate \( \nu \Sigma/M \) in terms of the torque ratio, gives
\[
U_c = \frac{17}{44\sqrt{n}} \frac{N_n^2}{N_\rho K^2 T_i^2 \tan^2 i_c} \left( \frac{F}{17 M} \right) \frac{T_m}{T_v}.
\] (193)

As will be shown, this always gives \( U_c \leq 10^{-4} \), so justifying the use of a Keplerian angular velocity in the disc mid-plane.

Away from the mid-plane, as \( |z| \) is increased the thermal pressure term decreases since the temperature, and hence the sound speed, decreases. Hence this term remains small in (1) as the disc surface is approached. It follows from (90) for \( J_\phi \) that the ratio of the last magnetic force term in (1) to the gravitational term \( v_K^2/\sigma_0 \) can be expressed at the disc surface as
\[
U_s = \frac{F + 1}{F - U_c},
\] (194)

which yields \( U_s \leq 10^{-3} \). Hence the last magnetic term in (1) remains ignorable as the disc surface is approached.

This leaves the magnetic force term involving \( B^2 \phi \), which becomes finite away from \( z = 0 \). The ratio of the surface value of this term to the gravitational term follows as
\[
\frac{U_s}{F} = \frac{1}{\epsilon_0 \rho_0 v_K^2} \frac{d}{d\sigma_0} \left( \frac{\sigma_0^3 B^2_\phi}{2\mu_0} \right) \sim \frac{1}{\xi_0} \frac{B^2_\phi}{2\mu_0 P_v} \frac{c_s^2}{v_K^2}.
\] (195)
Equations (157) and (158) yield
\[
c_{\nu K} = \frac{17 N_a^2}{22 \varepsilon N_P K^3 L_1 \tan i_s} \left( 1 - 3\sqrt{\frac{N_e^2}{\varepsilon^2 K^3} A_{\nu i} \tan^2 i_s} \right),
\]
and hence (195) becomes
\[
\dot{U}_s \sim \frac{1}{2} \left( \frac{17}{22} \right)^2 \frac{N_a^2}{\varepsilon N_P K^3 L_1 \tan^2 i_s} A \left( 1 - 3\sqrt{\frac{N_e^2}{\varepsilon^2 K^3} A_{\nu i} \tan^2 i_s} \right)^2.
\]

This is found to be \( \leq 10^{-3} \) for all self-consistent disc solutions (see Tables 1–6). Hence the \( B_s^2 \) magnetic force term is also ignorable in (1) and the angular velocity is therefore close to Keplerian values for \( |z| \leq h \).

It is noted that Wardle & Koenigl (1993) and Ogilvie (1997) found deviations from \( \Omega_K \) in the disc of order \( h/\sigma_0 \). This is because these authors considered ambipolar diffusion and vanishing diffusion, respectively. The case considered here with turbulent diffusion and a dynamo-generated magnetic field has deviations from \( \Omega_K \) of order \( (h/\sigma_0)^2 \).

In deriving the coordinates of the sonic point, the wind material was taken to be corotating with the angular velocity distribution of the disc surface, \( \Omega_K(\sigma_0) \). The accuracy of this can be checked by noting that the induction equation yields
\[
\frac{\Delta \Omega}{\Omega_K(\sigma_0)} = \frac{\Omega_{m} - \Omega_K(\sigma_0)}{\Omega_K(\sigma_0)} = \frac{(v_i)_m}{(v_i)_0} \left( \frac{B_\theta}{B_z} \right)_{sn},
\]
which must be small to justify the approximation \( \Omega = \Omega_K(\sigma_0) \) along a field line between the disc surface and the sonic point. It was shown

<table>
<thead>
<tr>
<th>Table 1. Disc and wind quantities for ( \epsilon = 10^{-3} ) and ( N_P = 0.2 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_s = 7.4 \times 10^{-3} )</td>
</tr>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>3.9</td>
</tr>
<tr>
<td>( Q_m/Q_v )</td>
</tr>
<tr>
<td>0.17</td>
</tr>
</tbody>
</table>

| \( N_s = 7.5 \times 10^{-3} \) |
| \( n \) | \( \xi_s \) | \( \xi_d \) | \( \tau_D \) | \( (N_P)_{\max} \) | \( (N_s)_{\min} \) | \( i_s \) | \( T_m/T_v \) |
| 3.9 | 0.61 | 1.8 \times 10^{-2} | 29.8 | 0.71 | 7.2 \times 10^{-3} | 57.5 | 80.6 |
| \( Q_m/Q_v \) | \( B_{s0}^2 U_c U_s + \dot{U}_s \frac{\sigma_0}{\sigma_i} \) | \( \frac{4\pi \sigma_0^2 \dot{\rho}_m}{M} \) | | | | | |
| 0.25 | 0.16 | 1.5 \times 10^{-5} | 1.4 \times 10^{-3} | 7.9 | 1.1 \times 10^{-2} | 1.9 \times 10^{-2} |

<table>
<thead>
<tr>
<th>Table 2. Disc and wind quantities for ( \epsilon = 10^{-3} ) and ( N_P = 1.0 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_s = 7.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>( n )</td>
</tr>
<tr>
<td>3.1</td>
</tr>
<tr>
<td>( Q_m/Q_v )</td>
</tr>
<tr>
<td>1.6 \times 10^{-2}</td>
</tr>
</tbody>
</table>

| \( N_s = 7.8 \times 10^{-3} \) |
| \( n \) | \( \xi_s \) | \( \xi_d \) | \( \tau_D \) | \( (N_P)_{\max} \) | \( (N_s)_{\min} \) | \( i_s \) | \( T_m/T_v \) |
| 3.1 | 0.68 | 3.8 \times 10^{-2} | 14.7 | 1.8 | 7.6 \times 10^{-3} | 61.1 | 12.5 |
| \( Q_m/Q_v \) | \( B_{s0}^2 U_c U_s + \dot{U}_s \frac{\sigma_0}{\sigma_i} \) | \( \frac{4\pi \sigma_0^2 \dot{\rho}_m}{M} \) | | | | | |
| 3.6 \times 10^{-2} | 0.11 | 1.6 \times 10^{-5} | 6.5 \times 10^{-4} | 4.0 | 4.1 \times 10^{-2} | 2.1 \times 10^{-2} |
Table 3. Disc and wind quantities for $\varepsilon = 10^{-3}$ and $N_p = 2.0$.

<table>
<thead>
<tr>
<th>$N_\nu$</th>
<th>$\xi_s$</th>
<th>$\xi_d$</th>
<th>$\tau_D$</th>
<th>$(N_p)_{\text{max}}$</th>
<th>$(N_\nu)_{\text{min}}$</th>
<th>$i_*$</th>
<th>$T_m$</th>
<th>$T_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7.9 \times 10^{-3}$</td>
<td>2.8</td>
<td>0.70</td>
<td>5.0 $\times$ 10$^{-2}$</td>
<td>10.4</td>
<td>2.4</td>
<td>7.8 $\times$ 10$^{-3}$</td>
<td>4.4</td>
<td>71.1</td>
</tr>
<tr>
<td>$5.5 \times 10^{-3}$</td>
<td>2.8</td>
<td>0.71</td>
<td>5.2 $\times$ 10$^{-2}$</td>
<td>10.6</td>
<td>2.7</td>
<td>7.8 $\times$ 10$^{-3}$</td>
<td>4.4</td>
<td>65.6</td>
</tr>
</tbody>
</table>

Table 4. Disc and wind quantities for $\varepsilon = 10^{-2}$ and $N_p = 0.2$.

<table>
<thead>
<tr>
<th>$N_\nu$</th>
<th>$\xi_s$</th>
<th>$\xi_d$</th>
<th>$\tau_D$</th>
<th>$(N_p)_{\text{max}}$</th>
<th>$(N_\nu)_{\text{min}}$</th>
<th>$i_*$</th>
<th>$T_m$</th>
<th>$T_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.46 \times 10^{-2}$</td>
<td>3.1</td>
<td>0.68</td>
<td>3.8 $\times$ 10$^{-2}$</td>
<td>14.0</td>
<td>0.34</td>
<td>2.4 $\times$ 10$^{-2}$</td>
<td>66.4</td>
<td>62.9</td>
</tr>
<tr>
<td>$0.12$</td>
<td>7.2 $\times$ 10$^{-2}$</td>
<td>1.1 $\times$ 10$^{-4}$</td>
<td>3.2 $\times$ 10$^{-3}$</td>
<td>45.2</td>
<td>3.3 $\times$ 10$^{-4}$</td>
<td>4.5 $\times$ 10$^{-3}$</td>
<td>4.5</td>
<td>3.3</td>
</tr>
</tbody>
</table>

Table 5. Disc and wind quantities for $\varepsilon = 10^{-2}$ and $N_p = 1.0$.

<table>
<thead>
<tr>
<th>$N_\nu$</th>
<th>$\xi_s$</th>
<th>$\xi_d$</th>
<th>$\tau_D$</th>
<th>$(N_p)_{\text{max}}$</th>
<th>$(N_\nu)_{\text{min}}$</th>
<th>$i_*$</th>
<th>$T_m$</th>
<th>$T_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.62 \times 10^{-2}$</td>
<td>2.3</td>
<td>0.75</td>
<td>7.9 $\times$ 10$^{-2}$</td>
<td>7.0</td>
<td>1.2</td>
<td>2.6 $\times$ 10$^{-2}$</td>
<td>70.8</td>
<td>9.2</td>
</tr>
<tr>
<td>$1.4 \times 10^{-2}$</td>
<td>2.3</td>
<td>0.75</td>
<td>8.1 $\times$ 10$^{-2}$</td>
<td>7.1</td>
<td>1.3</td>
<td>2.6 $\times$ 10$^{-2}$</td>
<td>67.5</td>
<td>11.6</td>
</tr>
</tbody>
</table>

in Section 4.1 that $\sigma_B \rho_B$ and $\rho v_c$ are conserved along $B_p$ and $B_c = B_{c_0}$, for $|z| \leq 3h$. It follows that (198) can be written

$$\frac{\Delta \Omega}{\Omega_K(\sigma_c)} = \frac{v_{c_b} B_{c_0}}{v_{c_1} B_{c_0}} \frac{\sigma_c}{\sigma_{c_0}} \frac{\rho_c}{\rho_{c_0}}.$$  (199)
Equations (64) and (71) for m, together with (154a) for \( a = (v_p)_m \), give

\[
v_{\infty} = \frac{\xi_c c_s \tan i}{(1 + \frac{s}{2} \tan^2 i_s)^{\frac{1}{2}}} \frac{p_m}{P_i}.
\]

(200)

Using this, together with (128) for \( B_{d\phi}/B_{c\phi} \), the magnitude of (199) becomes

\[
\frac{|\Delta \Omega|}{\Omega_K} = \frac{\xi_c}{N_s} \frac{|f|}{(1 + \frac{s}{2} \tan^2 i_s)^{\frac{1}{2}}} \frac{c_s}{v_K},
\]

noting that (65) and (74) enable \( \sigma_v/\sigma_m = 1 \) and \( v_K(\sigma_m) = v_K(\sigma_0) \) to be used in (199) to good approximations. The use of (196) for \( c_s/v_K \) then yields

\[
\frac{|\Delta \Omega|}{\Omega_K} = \frac{17 \xi_s N_s}{22 e N_p} \frac{|f|}{K^3 I_1 \tan \left(1 + \frac{s}{2} \tan^2 i_s\right)^{\frac{1}{2}}} \left(1 - 3 \sqrt{n} \frac{1}{e^2} \frac{A_1}{K^4 |f|} \tan^2 i_s\right).
\]

(202)

This gives \( |\Delta \Omega|/\Omega_K \ll 1 \) (see Tables 1–6).

7.2 Disc-wind solutions

Values must be chosen for the quantities \( \epsilon, N_a \) and \( N_K \) representing the turbulent Mach number, the magnetic Reynolds number and the magnetic Prandtl number respectively. A detailed knowledge of the turbulence would connect these quantities. However, although this is not presently available, the disc model constrains the range of values.

Tables 1–6 show the main disc and wind properties for two values of the turbulent Mach number \( \epsilon \) and different magnetic Prandtl numbers. The ratio of the magnetic to viscous torque on disc material is given by (187). Using (120) to eliminate \( M/vS \) in this yields

\[
\frac{T_m}{T_v} = \frac{17}{33} \frac{1}{N_p} \frac{\sigma_0}{\sigma_m} c_i - 1.
\]

(203)

Valid wind solutions have \( \tan i_s \sim 2 \), and hence for \( N_p \leq 1 \) it follows that \( T_m/T_v \geq \sigma_0/4h > 1 \), so the magnetic contribution to driving the inflow is at least comparable to turbulent viscosity for such Prandtl numbers, as Tables 1–6 illustrate. The viscous torque can become comparable to the magnetic torque for \( N_p = 2 \). The values of the magnetic Reynolds number \( N_a \) satisfy (52), while the value of \( K \) ensures that \( B_m \) and \( B_{\phi} \) are monotonically increasing between \( z = 0 \) and \( z = h \), as explained in Section 3.1.

It is noted that the disc is always optically thick, but has values of \( \tau_d \) lower than those of the standard viscous disc without a wind. The main reason for this is that the enhanced inflow speed owing to the magnetic wind torque results in a lower density disc of smaller mass.

In all cases the viscous dissipation dominates the magnetic dissipation in the disc. The value of \( B_{d\phi}^2/2\mu_0 P_c \) is sufficiently small to justify ignoring \( \alpha \)-quenching effects. Disc solutions satisfying \( 4\pi \sigma_0 \rho_m \ll M \) are only possible for \( B_{d\phi}^2/2\mu_0 P_c \approx 0.1 \), corresponding to at most moderate magnetic compression of the disc. Large values of this pressure ratio lower \( h \) and, in accordance with (155), lead to larger values of \( m \). Equation (160) shows that the quantity \( \Omega_K h^3/a^2 \) is sensitive to \( N_a \) and \( K \), so it follows from (155) that \( m \) is very sensitive to these quantities, owing to its exponential dependence. Values of \( N_a \) must not be much above the lower limit given by (52) or wind evaporation of the disc results.

The value of \( n \) decreases with increasing \( N_p \) as expected from (180). Hence lower values of \( N_p \) give lower vertical central condensation in the disc, in accordance with (163).

All solutions presented are self-consistent since \( 4\pi \sigma_0 \rho_m M/c_i + U_c + U_\phi \) and \( |\Delta \Omega|/\Omega_K |\sigma_v| \) are small, so ensuring that most mass is accreted, the disc angular velocity is close to a Keplerian distribution, and corotation of wind material is a good approximation up to the sonic point.

7.3 The effect of an inner boundary condition

A Keplerian angular velocity was shown to be valid for the foregoing disc solution. However, as the stellar surface is approached the
rotational motion of the accretor is liable to affect the angular velocity in the disc. For accretion to occur, the stellar rotation rate must be smaller than the Keplerian angular velocity at the surface $\sigma_0 = R$. In the standard model the disc angular velocity is taken to reach a maximum at the edge of a boundary layer of width $\delta$, and hence the boundary condition is

$$\Omega(R + \delta) = 0.$$  

(204)

Such a boundary condition can be applied here but, in addition, an assumption must be made about the operation of the dynamo at $\sigma_0 = R + \delta$. For a highly conducting compact accretor, such as a white dwarf, the magnetic diffusivity $\eta$ will become small as the surface is approached. Since the dynamo number $D = K^3$ is proportional to $\Omega/\eta^2$ while $\Omega$ and $\eta$ both tend to zero at $\sigma_0 = R + \delta$, it is reasonable to take $D$ as remaining finite at this radius. Consequently, $\dot{m}\sigma_\Lambda^2$ will also be finite.

The wind angular momentum equation (143) gives

$$\dot{m}\sigma_\Lambda^2 = -\frac{(1 + \frac{\lambda}{2}\tan^2 i_s)\frac{\lambda}{2}\sigma_0 B_0 B_{cs}}{\tan i_s \mu_0 c_k},$$

so using (128) for $B_0$ yields

$$\dot{m}\sigma_\Lambda^2 = \frac{|f|}{N_a} \frac{(1 + \frac{\lambda}{2}\tan^2 i_s)\frac{\lambda}{2}\sigma_0^2 B_{cs}^2}{\tan^2 i_s \mu_0 c_k}.$$  

(205)

Substituting (129) for $B_{cs}$ then leads to

$$\dot{m}\sigma_\Lambda^2 = \frac{3\sqrt{n} N_a^2 A_l}{8\pi} \frac{\lambda}{K^3} |f| \frac{1}{n} M (1 + \frac{\lambda}{2}\tan^2 i_s)^{\frac{1}{2}} \tan i_s.$$  

(206)

Since $\tan i_s$ is essentially independent of $\sigma_0$, it follows that the disc angular momentum equation (85) can be written as

$$\frac{d}{d\sigma_0} \left[ \frac{M}{2\pi} \sigma_0^2 \Omega + \sigma_0^2 \Omega' \nu \Sigma - \frac{4\tan i_s}{(1 + \frac{\lambda}{2}\tan^2 i_s)^2} \dot{m}\sigma_\Lambda^2 \sigma_0^2 \Omega \right] = 0.$$  

(207)

Integration gives the total conserved angular momentum flux as

$$\frac{M}{2\pi} \sigma_0^2 \Omega + \sigma_0^2 \Omega' \nu \Sigma - \frac{4\tan i_s}{(1 + \frac{\lambda}{2}\tan^2 i_s)^2} \dot{m}\sigma_\Lambda^2 \sigma_0^2 \Omega = L,$$  

(208)

being the sum of material, viscous and magnetic terms. Applying the boundary condition (204) and noting, for a sharp turn-over in $\Omega$, that $\Omega(R + \delta) = \Omega_K(R + \delta) = \Omega_K(R)$ since $\delta/R \ll 1$, (208) yields

$$L = \left[ \frac{M}{2\pi} - \frac{4\tan i_s}{(1 + \frac{\lambda}{2}\tan^2 i_s)^2} \dot{m}\sigma_\Lambda^2 \right] (GMR)^{\frac{1}{2}}.$$  

(209)

**Figure 5.** The variation of $\tan i_s$ when an inner boundary condition is used.
Equations (208) and (209) lead to
\[
\nu \Sigma = \left[ \frac{M}{3\pi} - \frac{8}{3} \frac{\tan \iota_s}{(1 + \frac{2}{3} \tan^2 \iota_s)^{\frac{1}{2}}} \sigma^2 \frac{n}{A} \right] \left[ 1 - \left( \frac{R}{\sigma_0} \right)^{\frac{1}{2}} \right].
\]  
(210)

Substituting (206) for \( \sigma^2 \frac{n}{A} \) in (210) gives
\[
\nu \Sigma = \frac{M}{3\pi} \left[ 1 - 3 \sqrt{\frac{2}{3}} \frac{N_\alpha^4}{e^2} A \frac{I_1}{R} \tan^2 \iota_s \right] \left[ 1 - \left( \frac{R}{\sigma_0} \right)^{\frac{1}{2}} \right].
\]  
(211)

An equation determining \( \tan \iota_s \) follows by equating this to (123) for \( \nu \Sigma \), giving
\[
\tan^2 \iota_s + \tilde{C} \tan \iota_s \frac{20}{\sqrt{\frac{4}{3} \frac{\sigma^2}{N_\alpha^4} A I_1}} = 0,
\]  
(212)

where
\[
\tilde{C} = \frac{1.2 \times 10^{-3}}{49} \frac{18}{N_\alpha^4} \frac{1}{e^2} \frac{K^4}{I_1} \frac{q}{A} \frac{1}{M_0^2} \frac{5}{R_1} \frac{1}{1 - (R/\sigma_0)^{\frac{1}{2}}},
\]  
(213)

using the previous normalizations for \( M \) and \( M_0 \), together with \( R_1 = R/5.5 \times 10^9 \text{ m} \). Since \( (\tan \iota_s)^2 = 1 \), to a good approximation for launching angles satisfying \( 4\pi \sigma_0^2 \sin \iota_s/M_0 \approx 1 \), (212) is essentially a quadratic equation with solution
\[
\tan \iota_s = \frac{1}{2} \left[ \left( \tilde{C}^2 + \frac{4}{3} \frac{\sigma^2}{N_\alpha^4} A I_1 \right)^{\frac{1}{2}} - \tilde{C} \right].
\]  
(214)

It is seen that applying a vanishing \( \Omega' \) condition close to the stellar surface does not change the form of the disc solution given by (131)–(136). However, the equation determining \( \tan \iota_s \) is changed since, although the solutions (153) and (214) are identical in form, the variations of \( C \) and \( \tilde{C} \) are different. In the bulk of the disc, away from the stellar surface, the difference is negligible, with \( C \) and \( \tilde{C} \) being weakly dependent on \( \sigma_0 \) and \( \tan \iota_s \) being essentially independent of position. The inner boundary condition only significantly affects the disc structure as the edge of the stellar boundary layer is approached and \( \tilde{C} \) starts to increase, with a consequent decrease in \( \tan \iota_s \). Fig. 5 shows the variation of \( \tan \iota_s \) with \( \sigma_0/R \), calculated from (214). It is seen that the launching angle is essentially constant through most of the disc, only decreasing sharply close to the stellar surface. The quantity \( \tilde{m} \sigma^2 \alpha \) given by (206) then becomes dependent on \( \sigma_0 \), so the last term in the integral (208) is not strictly valid. However, this should give the qualitatively correct behaviour of \( \iota_s \), which indicates that the wind mass flux will increase as the stellar surface is approached, since the sonic point height \( z_{\text{sonic}} \) is reduced. Hence the boundary condition (204) may lead to a jet forming in the very inner part of the disc.

It should be noted that the disc solution reduces to the standard viscous disc in the limit of vanishing magnetic field. Equations (135) and (136), together with \( B_{\text{in}} = B_2 \tan \iota_s \), show that \( B = 0 \) when \( A = 0 \). Equation (51) then yields
\[
K = K_{\text{max}} = \left( \frac{9n^2}{2} \right)^{\frac{1}{2}} \frac{1}{\tilde{C}^2}.
\]  
(215)

Multiplying (212) by \( A \) and using \( A = 0 \), together with (215) for \( K \), leads to
\[
\tan \iota_s = \frac{44}{18} \frac{e^2}{N_\alpha^4} \frac{1}{e^2} \frac{1}{M_0^2} \frac{1}{M_1^{\frac{3}{2}}} \left[ 1 - \left( \frac{R}{\sigma_0} \right)^{\frac{1}{2}} \right]^{\frac{3}{2}} \frac{1}{\frac{1}{\sigma_0^2}}.
\]  
(216)

using the previous normalizations. Substituting this in (131), together with \( N_\alpha \) from (106) and \( K \) from (215), yields
\[
h = 1.5 \times 10^9 \frac{19}{e^4} \frac{1}{M_0^2} \frac{M_1^{\frac{3}{2}}}{M_1^{\frac{1}{2}}} \left[ 1 - \left( \frac{R}{\sigma_0} \right)^{\frac{1}{2}} \right]^{\frac{3}{2}} \frac{a}{\sigma_0^2}.
\]  
(217)

This is the disc height for the standard viscous model of Shakura & Sunyaev (1973). Substitution of (216) for \( \tan \iota_s \) in (132)–(134) leads to the viscous forms for \( T_c, \rho_c \) and \( v_{\text{crit}} \).
8 DISCUSSION

The present model shows that wind-influenced disc solutions are possible in which the necessary magnetic field is generated by an $\alpha \omega$-dynamo operating in the disc. The magnetic Reynolds number $N_a$ must be close to a minimum value to prevent excessive wind mass loss which results from magnetic compression of the disc. The vertical length-scale of the disc magnetic field may adjust to accommodate this. The magnetic torque always plays a major role in the radial advection of angular momentum, but the magnetic dissipation in the disc is small relative to that arising from turbulent viscosity.

The disc has a lower opacity and lower temperature than in the absence of the magnetically influenced wind. This occurs because the magnetic torque increases the inflow speed and results in a disc of lower mass for a given accretion rate. A hot corona is not required to generate the necessary wind mass flux, ordinary photospheric temperatures being quite sufficient. The magnetically enhanced inflow speed leads to the field bending needed to give a sufficiently low gravitational–centrifugal potential barrier to generate the required mass flux through the sonic point. The application of a stellar boundary condition results in a greatly increased wind mass flux in the very inner part of the disc. The wind may become a jet in this region, in which the mass-loss rate is comparable to the accretion rate. However, the effect of this region on the main disc and wind structure is small.

The present model calculates the necessary properties of the wind, but not its detailed structure. This is a future problem, but it is anticipated that similar results will occur for a more detailed wind calculation, since the constraints imposed by the disc flow remain the same. The detailed vertical structure of the disc also needs consideration, but the method used here to find the disc surface density and temperature should be in reasonable agreement with vertical solutions for the reasons explained in Section 6.

The linear stability of the present model will be investigated in a subsequent paper.

REFERENCES

Uchida Y., Shibata K., 1985, PASJ, 37, 515

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