Pipe network system analysis using simulated annealing

Hund-Der Yeh and Yu-Chung Lin

ABSTRACT

Water pipe network systems are designed and operated to supply fresh water from the source (or treatment facility) to the customers. Based on the energy conservation or mass balance principle, a nonlinear equation set can be formulated to describe the relationship between the nodal head and pipe flow rate. In the past, Hardy-Cross method or Newton-type method was commonly used to solve the nonlinear equation set for obtaining the solutions, such as the nodal heads and flow rates. However, these two methods have the drawbacks that a non-convergent and/or a poor result may occur if improper initial guesses for the nodal heads or flow rates are made. In this article, a pipe network solver, PNSA, is developed to solve the nodal head and flow rate for a given network. The nonlinear equation set is first formulated as an unconstrained optimization problem, and then uses simulated annealing to solve the solutions of a pipe network system. The results obtained from PNSA with initial guesses generated by a random number generator show good match with those obtained from modified Powell's hybrid method and sparse matrix method.

Key words | algorithms, evolutionary computation, network analysis, pipe networks, water distribution system

Hund-Der Yeh (corresponding author)
Yu-Chung Lin
Institute of Environment Engineering,
National Chiao Tung University,
Hsinchu,
Taiwan
Fax: 886-3-5726050
E-mail: *hdyeh@mail.nctu.edu.tw*

NOTATION

The following symbols are used in this paper:

AC	the number of the accepted trial solutions divided by the total number of generated trial solutions
C_{ij}	Hazen-Williams friction coefficient
D_{ij}	pipe diameter
ΔH_{ij}	frictional head loss
K_{ij}	major loss coefficient
k	Boltzman's constant
L_{ij}	pipe length
MC_i	mass accumulation at node
N	the total number of generated trial solutions
$P(E)$	the occurrence probability when the system and temperature are E and T, respectively.
P_{SA}	the acceptance probability that when the trial solution is not better than the current best solution
Q_{ij}	flow rate
QI_i	the demand of the source at node
RD	random number generated from a uniform (0, 1) distribution
Te	temperature

doi: 10.2166/aqua.2008.052

INTRODUCTION

Water pipe networks are the hydraulic infrastructures which are part of water supply system composed of a system of pipes, reservoirs, and hydraulic devices such as pumps and pressure reducing valves. The water flow in a pipe network is described by a set of nonlinear equations based on energy conservation or mass balance principles. The energy conservation principle specifies that the frictional head loss around each loop within the pipe network must equal zero and the mass balance principle requires

that the sum of the flows entering and leaving each node should be zero (Gagnon & Jacoby 1975).

In the past, Hardy-Cross or modified Hardy-Cross methods were often employed to analyze the pipe network system. Wood & Rayes (1981) mentioned two problems which might occur when employing these two methods to analyze the pipe network. First, convergence is slow for a large network or a network containing short lengths of large diameter mains and convergence is not assured if there are dead end mains. Second, for poor initial guess those methods may diverge from the true solution or converge slowly.

The pipe network can also be formulated as a nonlinear equation set. An iterative procedure, such as Newton–Raphson method (Martin & Peters 1963), linear theory (Wood & Charles 1972), or partial pivoting method (Demuren & Ideriah 1986) is then used to solve the nonlinear equation set. Chin *et al.* (1978) employed linearization method to transform the nonlinear equation set into a set of linear equations and then handled the linear term in the mass balance equation using the sparse matrix method. Their results showed that the sparse matrix method was an effective approach to analyze the network system and could save much computing time. EPANET2 (Rossman 2000), based on a hybrid loop-node approach, is a popular software developed in the last 10 years. The pipe network is solved initially by the gradient solution method with an initial estimate of flows in each pipe, and then the new nodal heads are found by solving the sparse matrix method iteratively.

Mays (1989) considered that Newton–Raphson method was a proper approach to solve the equation set formulated from the mass balance principle. The Newton–Raphson method converges more quickly than the linear method for a small system and requires less storage for the input of a large network. Although the convergent speed is fast, yet good initial guess is required to warrant correct solution. Cai *et al.* (2001) pointed out that if an improper initial guess was given, the gradient-type method generally converged to a local solution the nearest to the starting point. In addition, the speed and reliability of the method decreased with the increasing problem complexity.

This study develops an approach called PNSA based on the simulated annealing (SA) for analyzing the pipe network system. SA is classified as a random search algorithm which allows, at least in theory or in probability, for obtaining the global or nearby global optimum of a function in any given domain. The nonlinear equation set derived based on the mass balance principle is reformulated as an objective function. Then PNSA is used to find the nodal head when minimizing the objective function value (OFV). PNSA has two advantages over the traditional approaches such as Hardy-Cross type methods and the Newton–Raphson method. The first is that PNSA does not require preparing a proper initial guess for analyzing a pipe network. All initial guesses are generated by the random number generator in our case studies. The second is that the obtained results by PNSA are independent of the values of SA parameters, e.g., initial temperature, temperature reduction factor, and the total number of generated trial solutions by the processes of trial and error. Two examples are adopted from the literature to test the applicability and performance of PNSA. The software PNSA can be easily coupled with a forecast model for handling the problem such as trihalomethane species (Lin & Yeh 2005) in a pipe network system or expended to solve the capacity extension problem for an existing pipe network.

METHODOLOGY

Simulated annealing

SA was modified from the descent method. Rayward-Smith *et al.* (1996) pointed out that the drawback of the descent method was that the obtained solution might end up with a local optimum. In the SA, the Metropolis mechanism, also called the Boltzman's mechanism, is employed to control which ascent move could be accepted. Such a mechanism has a property of using descent strategy but allowing random ascent move to avoid possible trap in a local optimum, preventing the SA from having the same problem as the descent method. In addition, Romeo & Sangiovanni-Vincentelli (1991) also used homogeneous Markov chain and inhomogeneous Markov chain theory to prove that SA can converge to globally optimal solutions.

SA was successfully applied in a wide range of optimization applications such as capacity extension for pipe network system (Cunha & Sousa 1999; Monem & Namdarian 2005), parameter calibration and identification problems (Huang & Yeh 2007; Yeh & Chen 2007; Yeh *et al.* 2007), groundwater management problems (Doutherty & Marryott 1991; Marryott *et al.* 1993), groundwater remediation system problems (Marryott 1996; Rizzo & Dougherty 1996; Shieh & Peralta 2005), etc. Cunha & Sousa (1999) used SA to minimize the capacity extension cost in a water distribution network. Their solution sets obtained from SA and nonlinear programming (NLP) techniques for several medium size networks showed that SA did provide a better solution in general, in comparison with that obtained from the NLP technique. Shieh & Peralta (2005) used a hybrid genetic algorithm-simulated annealing to optimize an in-situ bioremediation system design. They mentioned that straightforward formulation, i.e., no requirement for computing derivatives, and easy implementation to a ground water simulation model were the main advantages as employed SA to solve such an optimization problem.

As an iterative improvement method, SA requires an initial point x to evaluate the OFV and an initial temperature at the beginning. To ensure the obtained result is a nearby global optimum, the solution domain has to be widely explored by SA at the beginning. The choice of the initial temperature is generally case by case. Kirkpatrick *et al.* (1984) gave a guideline for the choice of the initial temperature that the acceptance probability occurred if the Metropolis criterion for the trial solution, being worse than the current solution, was greater than 80% initially. This criterion has the merit of avoiding the situation that the current solution is trapped in a local optimum at early search. The second step is to update the current optimal solution if the trial solution generated from the initial solution within the solution domain is better than current optimal solution or if the trial solution satisfies the Metropolis's criterion, otherwise, continue generating the trial solution. In the minimization problem, if $f(x')$, the OFV of the trial solution x', is smaller than $f(x)$, then the trial solution takes the place of current optimal solution. If $f(x')$ is not smaller than $f(x)$, then one has to test Metropolis's criterion. For solving minimization problem, Metropolis's criterion is given as (Pham & Karaboga 2000):

$$P_{SA} = \begin{Bmatrix} 1 & ,if \quad f(x') \le f(x) \\ \exp\left[\frac{f(x)-f(x')}{Te}\right] & ,if \quad f(x') > f(x) \end{Bmatrix} \tag{1}$$

where P_{SA} is the acceptance probability of the trial solution and Te, a control parameter, is the current temperature. A random number ranging between zero and one is generated from a uniform distribution. If the number is smaller than P_{SA}, the trial solution x' is accepted and called an ascent move. Otherwise, other trial solutions are continually generated from the current solution. After a total of N trial solutions are generated by the N processes of trial and error, the current temperature is decreased by a constant, called the temperature reduction factor (R_t), and the prior steps are repeated continually. The acceptance probability is very high at the beginning of SA in expectation of exploring the entire possible solution domain completely and is decreased as the temperature goes down. Comparatively, at later period of SA, the acceptance probability is much lower in obtaining the near-by global optimal solution. The algorithm is terminated when the stopping criterion is satisfied. Note that Aarts & Korst (1989, p. 60) suggested a formula to estimate the initial temperature, Te.

Usually, after N specified processes of trial and error at each temperature, the temperature is decreased by R_t even if no improvement of the optimum takes place. The temperature should be cooled properly to guarantee the obtained solution being the nearby global optimal solutions (Pham & Karaboga 2000). Kirkpatrick *et al.* (1984) suggested that the R_t should range between 0.80 and 1. The algorithm is terminated when SA obtains the optimal solution or the obtained solution satisfies the stopping criterion.

Pipe network system

When analyzing a pipe network, the pipes are assumed to be completely filled with the fluid and under the conditions of steady state and incompressible flow. The pipe friction and fluid viscosity produce the major energy loss; on the other hand, the fluid flows in a pipe and the inlet, outlet, bend, and other devices produce the minor energy loss (McGhee 1991). In fact, the minor energy loss is significantly smaller than the major energy loss and thus neglected in this study.

For any element within the pipe network system, the Hazen-Williams equation is often applied to establish the relationship between the pipe flow rate and head loss. In the Hazen-Williams equation the major loss coefficient (K_{ij}) in a pipe is defined as (McGhee 1991)

$$K_{ij} = \frac{10.66667 \cdot L_{ij}}{C_{ij}^{1.851852} \cdot D_{ij}^{4.870370}} \tag{2}$$

where subscript ij represents the variable defined from node i to node j, L_{ij} is the length (m) of the pipe, C_{ij} is the Hazen-Williams friction coefficient, and D_{ij} is the pipe diameter (m). Using the Hazen-Williams equation, the flow rate in each pipe can thus be expressed as

$$Q_{ij} = \left(\frac{\Delta H_{ij}}{K_{ij}}\right)^{0.54} \tag{3}$$

where Q_{ij} is the pipe flow rate (m^3 sec^{-1}) and ΔH_{ij} is the frictional head loss in pipe.

In the past, a given pipe network was commonly formulated as a nonlinear equation set based on the energy conservation principle or mass balance principle. Then a gradient-type method, Hardy-Cross method, or modified Hardy-Cross method was used to solve the nonlinear equation set for obtaining the flow rate and the nodal head. The mass balance pronounces that the inflow must equal outflow at any node in a pipe network system. Thus, the mass conservation at node i can be written as

$$\mathrm{MC}_i = \sum_{j=1}^{\mathrm{nn}} Q_{ij} + QI_i \tag{4}$$

where the flow rate is positive if flow out of node i and negative for flow to the node i; j is the adjacent node to node i; nn is the number of total adjacent nodes to node i; QI_i, the demand or the source at node i, is positive for inflow and negative for outflow. While employing the mass balance principle to analyze the pipe network system, the nodal heads are considered as the unknown variables and a nonlinear equation set can be constructed based on Equations (4) and (5). In the past, the modified Hardy-Cross method was often used to solve the problem by adjusting the nodal head at each node iteratively until the sum of the flows entering and leaving each node was zero (Wood & Rayes 1981; McGhee 1991).

The energy conservation law describes that the algebra of the energy loss for any loop in each pipe should equal zero. A nonlinear equation set in terms of the flow rate as an unknown variable is formulated based Equation (4) and the energy conservation law. Then the Hardy-Cross method was used to solve the nonlinear equation set by iteratively correcting the flow rate until the frictional head loss around each loop was equal to zero (Wood & Rayes 1981; McGhee 1991).

PNSA algorithm

The objective function defined to minimize the sum of the net flow rate at each node of the pipe network is expressed as

$$\text{Minimize} \sum_{i}^{\mathrm{nd}} |\mathrm{MC}_i| \tag{5}$$

where nd is the total number of nodes needed to estimate the nodal heads in the network system. The SA is then used to find the optimal solution by minimizing the objective function.

The algorithm of PNSA includes ten steps and its flowchart is shown in Figure 1. In the first step, a random number generator is used to generate the initial solution within the solution domain, which is confined by the specified upper and lower bounds. The initial solution is given as

$$x_i^0 = \mathrm{LB}_i + \mathrm{RD} \times (\mathrm{UB}_i - \mathrm{LB}_i) \tag{6}$$

where the x_i^0 is the initial nodal head at node i, LB_i and UB_i are respectively the lower and upper bound values for node I, RD is a random number generated from a uniform (0, 1) distribution. A solution domain for an unknown variable is specified to ensure that the generated solution is physically feasible. The second step is to calculate the flow rate in each pipeline based on Equations (3) and (4) and the initial trial solutions, i.e., the trial nodal heads. The third step is to calculate the OFV by Equation (5) based on the estimated flow rate. Then the fourth step is to generate a new trial

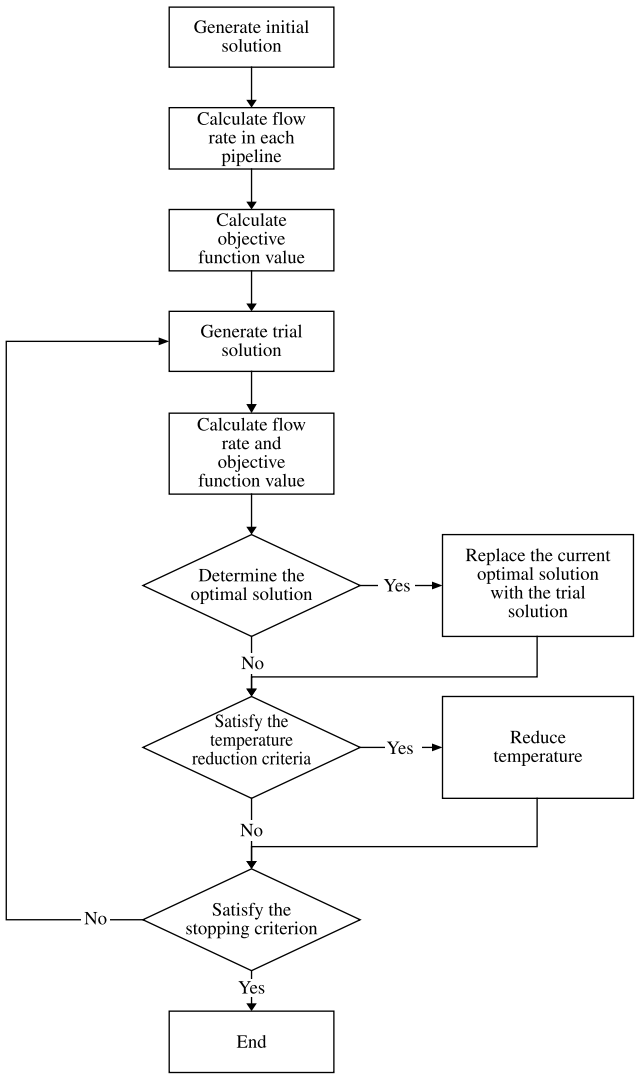

Figure 1 | The PNSA flow chart.

solution, x_i', given as

$$x_i' = x_i + (2 \times \mathrm{RD} - 1) \times \mathrm{VM}_i \tag{7}$$

where VM_i is a step length vector of the ith unknown variable. At the beginning the step length is chosen as the distance between the upper and lower bounds. Then the VM_i is automatically adjusted according to the OFV calculated from the trial solution generated at current temperature and it expects that approximately half of all trial solutions are accepted in next temperature for the ith unknown variable. The goal here is to sample the function widely. The acceptance chance (AC) is defined as the number of the accepted trial solutions from N processes of trial and error divided by N. If the AC is below 0.4, the exploration domain is too large and should be reduced. Then, the next step length VM_i' is adjusted according to the following formula

$$\mathrm{VM}_i' = \mathrm{VM}_i/(1 + (0.4 - \mathrm{AC})/0.4) \tag{8}$$

On the other hand, if the AC is higher than 0.6, the VM_i' will be

$$\mathrm{VM}_i' = \mathrm{VM}_i \times (1 + (\mathrm{AC} - 0.6)/0.4) \tag{9}$$

If the new trial solution does not fall within the solution domain, the PNSA has to randomly generate another one.

The fifth step is to re-calculate the flow rate in each pipe and the corresponding OFV. The sixth step is to determine whether the trial solution is the optimal solution or not. If the OFV of the trial solution is better than the current optimal solution or if the OFV of the trial solution satisfies Metropolis's criterion, the current optimal solution is replaced by the trial solution in the seventh step. Otherwise, the PNSA should determine to reduce the temperature or not in the eighth step. If the number of generated trial solutions reaches to N at current temperature, the temperature is reduced in the ninth step and the new temperature is

$$Te' = R_t \times Te \tag{10}$$

The tenth step is to check whether the obtained optimal solutions satisfy the stopping criterion or not. If the current optimal solution satisfies the stopping criterion, the PNSA is terminated; otherwise, the PNSA goes back to the fourth step and generates another trial solution. Notice that the stopping criterion is chosen as if the absolute differences between the two successive values of the optimal objective function are all less than 10^{-6} m^3 sec^{-1} within four iterations.

CASE STUDIES AND DISCUSSION

Two cases with different pipe network systems adopted from the literature are used to test the performance of PNSA.

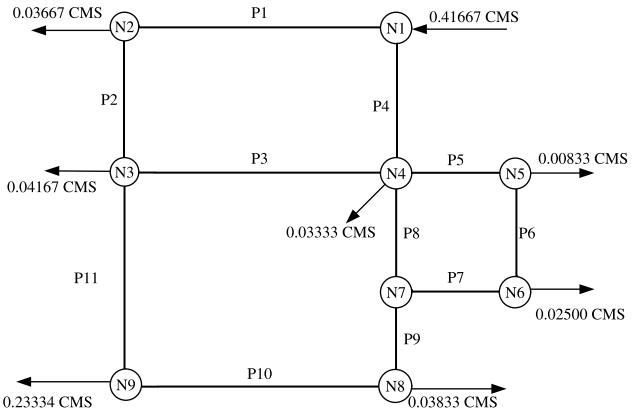

Figure 2 | Pipe network A.

Pipe network A

The first case is the one presented in McGhee (1991) with the layout of pipe network, as shown in Figure 2, and input data presented in Table 1. Pipe network A consists of 9 nodes and 11 pipes. The supply to the node N1 is at a constant rate 0.41667 (m^3 sec^{-1}) and consumptions are drawn from nodes N2, N3, N4, N5, N6, N8, and N9. The control parameters of SA for initial temperature, R_t, and N, are 10, 0.8, and 12000, respectively. The lower and upper bound values for these nine nodal heads are chosen as 0 and 50 m, respectively. Table 1 also shows the estimated flow rates obtained from PNSA which are accurate to the third decimal place as compared with those obtained from the Hardy-Cross method. In addition, the mass balance calculation for the sum of the estimated flow rate obtained from PNSA at each node gives zero for all nodes. These results demonstrate that PNSA successfully solves the nodal head and flow rate for this pipe network.

The temperature reduction should be given properly to guarantee that the obtained results are global optimal. Several key parameters of SA are chosen to examine the performance of the proposed algorithm. Three parameters in SA, the initial temperature, R_t, and N, are considered as the key parameters. Three different initial temperatures are chosen; they are 1.0, 10.0, and 100.0. Four different R_t, 0.9, 0.8, 0.7, and 0.6, are considered. Finally, three different N are selected; they are 4000, 8000, and 12000. The estimated flow rates by PNSA for various initial temperatures, R_t, or N always yield the results with the accuracy to the fourth decimal place. These results indicate that PNSA cannot only solve the nonlinear equation set but also give stable performance in obtaining optimal solutions, R_t, and N.

Pipe network B

A pipe network shown in Figure 3 was given in Chin *et al.* (1978) with data of the pipe diameter and length, roughness coefficient, and flow direction listed in Table 2. The network consists of 48 nodes, 74 pipes, and two reservoirs. The first

Table 1 | Input data and results for pipe network A

	Node		Input data			Results Flow rate (m^3 sec^{-1})	
Pipe number	From	To	Diameter (mm)	Length (m)	Roughness coefficient *C*	Hardy-Cross method	PNSA
P1	N1	N2	305.0	1000.0	100.0	0.14167	0.14095
P2	N2	N3	305.0	1000.0	100.0	0.10500	0.10428
P3	N4	N3	250.0	1100.0	100.0	0.05333	0.05364
P4	N1	N4	405.0	1250.0	100.0	0.27500	0.27572
P5	N4	N5	200.0	500.0	100.0	0.02667	0.03085
P6	N5	N6	400.0	400.0	100.0	0.01833	0.02252
P7	N7	N6	200.0	500.0	100.0	0.00667	0.00248
P8	N4	N7	355.0	400.0	100.0	0.16167	0.15790
P9	N7	N8	355.0	600.0	100.0	0.15500	0.15542
P10	N8	N9	305.0	1100.0	100.0	0.11667	0.11709
P11	N3	N9	305.0	1250.0	100.0	0.11667	0.11625

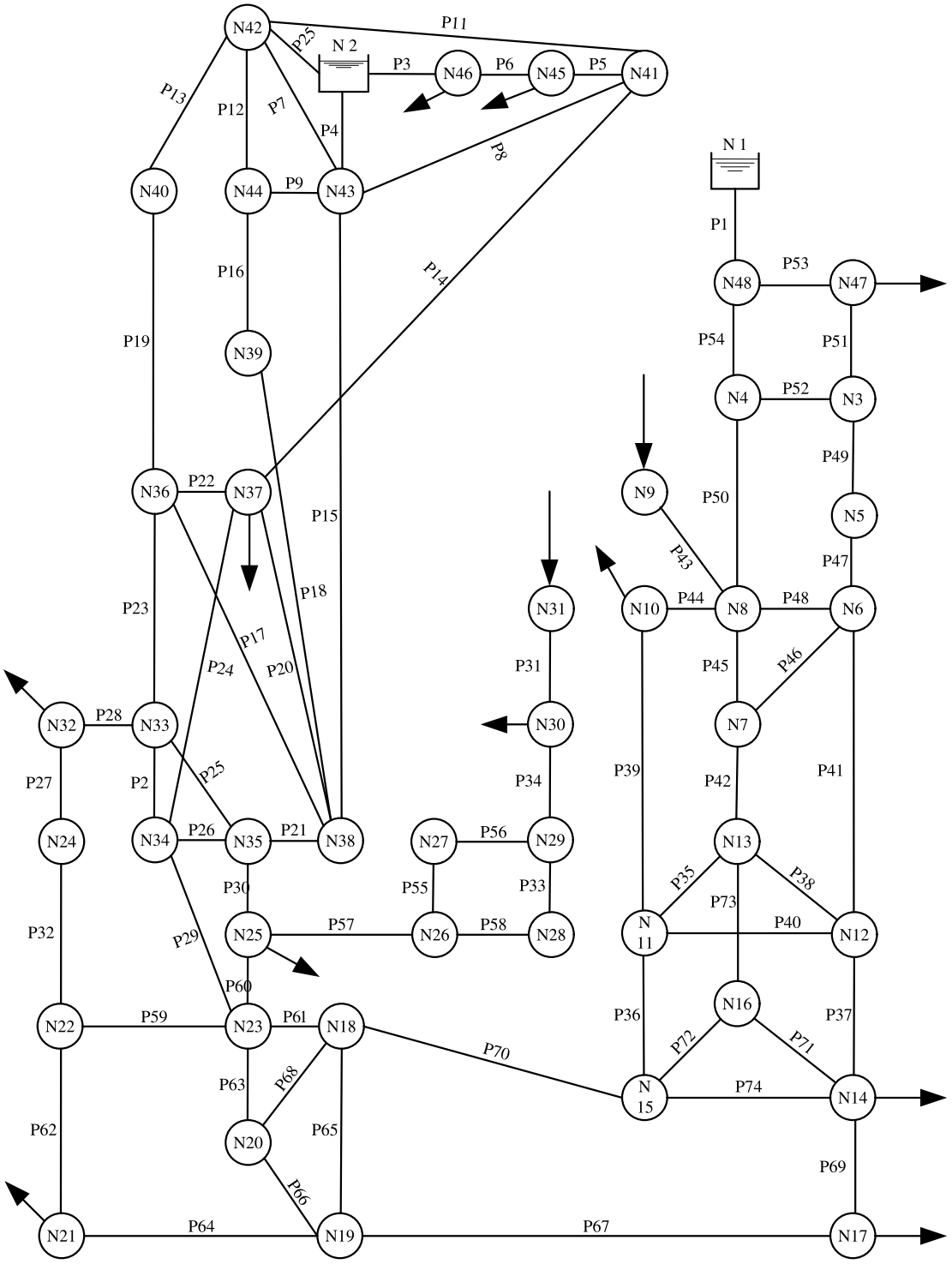

Figure 3 | Pipe network B.

Table 2 | Input data and obtained flow rates for pipe network B

	Node		Input data			Results Flow rate (m^3 sec^{-1})		
Pipe number	From	To	Diameter (m)	Length (m)	Roughness coefficient	Sparse matrix	Modified Powell's hybrid method	PNSA
P1	N48	N1	0.95	240.0	120.0	0.232524	0.250545	0.249351
P2	N34	N33	0.90	60.0	110.0	0.134618	0.134782	0.132451
P3	N2	N46	1.45	1830.0	130.0	1.223228	1.216512	1.258155
P4	N43	N2	1.15	3550.0	135.0	0.818017	0.821058	0.844972
P5	N41	N45	1.45	1220.0	130.0	1.674619	1.667900	1.709513
P6	N45	N46	1.45	640.0	130.0	1.419995	1.413271	1.454904
P7	N42	N43	0.90	60.0	110.0	0.030064	0.034912	0.036283
P8	N41	N43	0.90	60.0	110.0	0.346061	0.342034	0.349738
P9	N44	N43	1.00	50.0	110.0	0.609224	0.609822	0.626071
P10	N42	N2	0.90	3660.0	115.0	0.362625	0.361126	0.371645
P11	N41	N42	0.90	60.0	110.0	0.348081	0.344712	0.352572
P12	N42	N44	1.00	60.0	110.0	0.549771	0.549642	0.564216
P13	N40	N42	0.90	800.0	115.0	0.190967	0.191110	0.196278
P14	N37	N41	1.45	3140.0	130.0	0.980483	0.981164	1.007169
P15	N38	N43	1.15	3140.0	130.0	0.517794	0.518345	0.532125
P16	N39	N44	1.65	3140.0	135.0	1.159027	1.159464	1.190288
P17	N38	N36	0.90	60.0	110.0	0.202582	0.202754	0.208826
P18	N38	N39	1.00	60.0	110.0	1.159027	1.159464	1.190266
P19	N36	N40	0.80	2300.0	115.0	0.190969	0.191110	0.196183
P20	N38	N37	0.90	60.0	110.0	0.196925	0.197103	0.203383
P21	N35	N38	1.15	4050.0	130.0	1.277154	1.277952	1.309726
P22	N36	N37	0.90	60.0	110.0	0.040664	0.040646	0.040396
P23	N33	N36	0.80	4050.0	115.0	0.434243	0.434510	0.445327
P24	N34	N37	1.15	4050.0	130.0	1.275718	1.276509	1.308227
P25	N33	N35	0.90	60.0	110.0	0.134792	0.134792	0.132581
P26	N34	N35	0.90	60.0	110.0	0.005173	0.001128	0.004388
P27	N25	N32	0.80	2150.0	110.0	0.257474	0.257527	0.261993
P28	N32	N33	0.80	180.0	110.0	0.164881	0.164935	0.178927
P29	N23	N34	1.45	2980.0	135.0	1.409338	1.410164	1.436065
P30	N25	N35	1.45	2980.0	135.0	1.413087	1.413873	1.441208
P31	N31	N30	1.65	12000.0	135.0	1.620361	1.620370	1.625835
P32	N22	N24	0.95	670.0	110.0	0.257477	0.257527	0.261102
P33	N29	N28	1.00	60.0	110.0	0.803820	0.803819	0.807541
P34	N30	N29	1.65	13400.0	135.0	1.607627	1.607638	1.617508
P35	N13	N11	0.90	80.0	110.0	0.012752	0.000813	0
P36	N11	N15	0.95	4290.0	120.0	0.577391	0.577784	0.578488
P37	N12	N14	0.90	4290.0	115.0	0.210978	0.211666	0.211345
P38	N13	N12	0.05	60.0	110.0	0.005422	0.005636	0.005554
P39	N10	N11	0.95	2590.0	120.0	0.572823	0.572961	0.572898
P40	N11	N12	0.05	60.0	110.0	0.005422	0.005636	0.005554

Table 2 | (*continued*)

	Node		Input data			Results Flow rate (m^3 sec^{-1})		
Pipe number	From	To	Diameter (m)	Length (m)	Roughness coefficient	Sparse matrix	Modified Powell's hybrid method	PNSA
P41	N6	N12	0.90	2960.0	115.0	0.221821	0.222939	0.222451
P42	N7	N13	1.15	2960.0	135.0	1.035311	1.035636	1.035542
P43	N9	N8	1.15	2280.0	130.0	1.620356	1.620370	1.620369
P44	N8	N10	0.95	370.0	120.0	0.595979	0.596109	0.596044
P45	N8	N7	1.00	90.0	130.0	1.030185	1.030309	1.030276
P46	N6	N7	0.05	60.0	110.0	0.005126	0.005353	0.005266
P47	N5	N6	0.90	1610.0	115.0	0.232044	0.233621	0.232956
P48	N6	N8	0.05	60.0	110.0	0.005100	0.005328	0.005241
P49	N3	N5	0.95	1350.0	115.0	0.232038	0.233621	0.232947
P50	N4	N8	0.05	2960.0	120.0	0.000690	0.000721	0.00071
P51	N47	N3	0.95	6530.0	120.0	0.118437	0.119276	0.118725
P52	N3	N4	0.90	60.0	110.0	0.113346	0.114344	0.113912
P53	N48	N47	0.95	230.0	120.0	0.134665	0.135479	0.134864
P54	N48	N4	0.95	7200.0	120.0	0.114367	0.115065	0.114532
P55	N27	N26	1.00	60.0	110.0	0.803813	0.803819	0.815841
P56	N29	N27	1.15	3200.0	135.0	0.803820	0.803819	0.814356
P57	N26	N25	1.45	4300.0	135.0	1.607627	1.607639	1.650525
P58	N28	N26	1.15	3200.0	135.0	0.803820	0.803819	0.814804
P59	N22	N23	0.80	80.0	115.0	0.329009	0.329031	0.328673
P60	N23	N25	0.75	90.0	130.0	0.090365	0.089599	0.107864
P61	N18	N23	0.95	2050.0	120.0	0.594455	0.678889	0.681486
P62	N21	N22	0.80	2380.0	115.0	0.071534	0.071503	0.070808
P63	N20	N23	1.15	3050.0	135.0	1.053501	0.970706	0.974675
P64	N19	N21	0.05	670.0	115.0	0.002541	0.002570	0.002558
P65	N18	N19	0.05	60.0	110.0	0.008639	0.008777	0.008731
P66	N19	N20	0.05	60.0	110.0	0.008665	0.008799	0.008753
P67	N17	N19	0.80	1830.0	115.0	0.019911	0.020148	0.020036
P68	N18	N20	0.90	60.0	110.0	1.044846	0.961906	0.962846
P69	N14	N17	0.80	1950.0	115.0	0.181895	0.182185	0.182053
P70	N15	N18	0.95	3780.0	120.0	1.630597	1.632018	1.635133
P71	N16	N14	0.05	60.0	110.0	0.005845	0.006045	0.005969
P72	N16	N15	0.90	60.0	120.0	1.047405	1.048158	1.049356
P73	N13	N16	1.15	4290.0	135.0	1.041541	1.042113	1.043387
P74	N14	N15	0.05	60.0	110.0	0.005875	0.006074	0.005999

reservoir located at node N1 has a constant head of 138.9 m, and the second reservoir located at node N2 has a constant head of 91.4 m. The supply rates to nodes N9 and N31 are respectively 1.620370 and 1.620370 m^3 sec^{-1} and consumption rates drawn from nodes N10, N14, N17, N21, N25, N30, N32, N37, N45, N46, and N47 are 0.023148, 0.017361, 0.162037, 0.074074, 0.104166, 0.012731, 0.092592, 0.138888, 0.254629, 0.196759, and 0.016203 m^3 sec^{-1} respectively. The control parameters, initial temperature, R_t, and N, in SA are 5, 0.8, and 12000,

Table 3 | The obtained nodal heads for pipe network B

Node number	Obtained nodal head (m) Sparse matrix	PNSA	Difference (m)
N1	138.9000	138.9000	0.0000
N2	91.4000	91.4000	0.0000
N3	138.6094	138.6097	0.0003
N4	138.6125	138.6129	0.0004
N5	138.4173	138.4196	0.0023
N6	138.1192	138.1246	0.0054
N7	124.7038	124.2287	−0.4751
N8	124.8291	124.3522	−0.4769
N9	128.5591	128.0149	−0.5442
N10	124.5503	124.0779	−0.4724
N11	122.7335	122.2935	−0.4400
N12	137.6146	137.6266	0.0120
N13	122.7334	122.2935	−0.4399
N14	136.9479	136.9701	0.0222
N15	119.6778	119.2843	−0.3935
N16	119.8446	119.4493	−0.3953
N17	136.5395	136.5684	0.0289
N18	101.2657	101.1230	−0.1427
N19	136.5332	136.5620	0.0288
N20	101.0706	100.9577	−0.1129
N21	98.8352	99.0408	0.2056
N22	98.9237	99.1261	0.2024
N23	98.9737	99.1753	0.2016
N24	98.7983	98.9995	0.2012
N25	98.9793	99.1830	0.2037
N26	101.2657	101.3383	0.0726
N27	101.1429	101.4111	0.2682
N28	102.4042	102.6804	0.2762
N29	102.4761	102.7518	0.2757
N30	105.9543	106.2001	0.2458
N31	109.1145	109.3175	0.2030
N32	97.8678	98.0558	0.1880
N33	97.8338	98.0168	0.1830
N34	97.8382	98.0210	0.1828
N35	97.8382	98.0210	0.1828
N36	93.5811	93.6439	0.0628
N37	93.5806	93.6434	0.0628
N38	93.5717	93.6341	0.0624
N39	93.4302	93.4876	0.0574
N40	93.0539	93.0997	0.0458
N41	92.9252	92.9672	0.0420
N42	92.9507	92.9929	0.0422
N43	92.9504	92.9926	0.0422
N44	92.9863	93.0297	0.0434
N45	92.2397	92.2674	0.0277
N46	91.9750	91.9950	0.0200
N47	138.8569	138.8537	−0.0032
N48	138.8684	138.8646	−0.0038

respectively. Note that the initial guesses are generated by a random number generator. The lower and upper bounds for these 46 unknown nodal heads are 80 and 140 m, respectively. Table 2 also lists the calculated flow rates obtained from the sparse matrix (Chin *et al.* 1978), modified Powell's hybrid method (Lee 1988), and PNSA. The obtained flow rates from PNSA show a fairly good match with those obtained from the sparse matrix and modified Powell's hybrid method as indicated in Table 2. Table 3 lists the nodal heads obtained from the sparse matrix and PNSA. The maximum difference of the nodal head obtained by the sparse matrix and PNSA, which is 0.5442 m, occurred at node N9 and the average difference for the nodal heads estimated by those two methods is −0.0041 m. The computer time in analyzing this pipe network was about 33 minutes performed on a personal computer with 3.2 G *Pentium IV* CPU and 1 GB RAM.

CONCLUDING REMARKS

The energy conservation principle or mass balance principle is usually employed to construct a nonlinear equation set representing the relationship between the flow rate and energy loss for a pipe network system. In the past, the Newton-type method or Hardy-Cross method was commonly applied to solve the nonlinear equation set. To obtain accurate solutions, those methods generally require good initial guesses for the solutions which are, however, difficult to make for a large-scale network system. In addition, the problem of non-convergence may occur if the initial guess values are far way from the true solutions. In this study, we reformulate the nonlinear equation set as an objective function and use SA to find the optimal solution by minimizing the objective function. The SA has the merit of arbitrarily choosing the initial guess generated by the random number generator and, still, can obtain good results. Two pipe networks are adopted from the literature and analyzed by the proposed approach PNSA. The results obtained from PNSA show good match with those analyzed by the traditional approaches such as the Newton-type method and sparse matrix method. Additionally, all the results obtained from PNSA with different initial temperatures, reduction factors, or total numbers of generated trial

solutions have accuracy to the fourth decimal place. Those results indicate PNSA is very robust and with stable performance in analyzing the pipe network systems.

ACKNOWLEDGEMENTS

This study was partly supported by the Taiwan National Science Council under Grant No. NSC95-2221-E-009-109. The authors thank the two anonymous reviewers for their constructive comments and suggested revisions.

REFERENCES

Aarts, E. & Korst, J. 1989 *Simulated Annealing and Boltzmann Machines: A Stochastic Approach to Combinatorial Optimization and Neural Computing*. John Wiley & Sons, Great Britain.

Cai, X., McKinney, D. C. & Lasdon, L. S. 2001 Solving nonlinear water management models using a combined genetic algorithm and linear programming approach. *Adv. Water Res.* **24**(6), 667–676.

Chin, K. K., Gay, R. K. L., Chua, S. H., Cham, C. H. & Ho, S. Y. 1978 Solution of water networks by sparse matrix methods. *Int. J. Numerical Methods Eng.* **12**, 1261–1277.

Cunha, M. D. C. & Sousa, J. 1999 Water distribution network design optimization: simulated annealing approach. *J. Water Resour. Plan Manage.* **125**(4), 215–221.

Demuren, A. O. & Ideriah, F. J. K. 1986 Pipe network analysis by partial pivoting method. *J. Hydraul. Eng.* **112**(50), 327–334.

Doutherty, D. E. & Marryott, R. A. 1991 Optimal groundwater management: I. Simulated annealing. *Water Resour. Res.* **27**(10), 2493–2508.

Gagnon, C. R. & Jacoby, S. L. S. 1975 Computer simulation of water distribution networks. *Transportation Eng. J. ASCE* **101**(TE3), 553–567.

Huang, Y. C. & Yeh, H. D. 2007 The use of sensitivity analysis in on-line aquifer parameter estimation. *J. Hydrol.* **335**(3–4), 406–418. doi: 10.1016/j.jhydrol.2006.12.007

Kirkpatrick, S., Gelatt, C. D. Jr & Vecchi, M. P. 1984 Optimization by simulated annealing. *Science* **220**(4598), 671–680.

Lee, H. D. 1988 *A Study of Pipe Network Analysis using Modified Powell's Hybrid Method*. M.S. thesis, Institute of Environmental Engineering, National Chiao Tung University, Hsinchu, Taiwan.

Lin, Y. C. & Yeh, H. D. 2005 THM species forecast using optimization method: genetic algorithm and simulated annealing. *J. Comput. Civ. Eng.* **19**(3), 248–257.

Marryott, R. A. 1996 Optimal ground-water remediation design using multiple control technologies. *Ground Water* **34**(3), 425–433.

Marryott, R. A., Dougherty, D. E. & Stollar, R. L. 1993 Optimal groundwater management: 2. application of simulated annealing to a field-scale contaminant site. *Water Resour. Res.* **29**(4), 847–860.

Martin, D. W. & Peter, G. 1963 The application of Newton's method to network analysis by digital computer. *J. Inst. Water Eng.* **17**, 115–129.

Mays, L. W. 1989 *Reliability Analysis of Water Distribution Systems*. ASCE, New York, USA.

McGhee, T. J. 1991 *Water Supply and Sewerage*, 6th edn. McGraw-Hill, NJ, USA.

Monem, M. J. & Namdarian, R. 2005 Application of simulated annealing (SA) techniques for optimal water distribution in irrigation canals. *Irrigation Drainage* **54**(4), 365–373.

Pham, D. T. & Karaboga, D. 2000 *Intelligent Optimisation Techniques*. Springer, Great Britain.

Rayward-Smith, V. J., Osman, I. H., Reeves, C. R. & Smith, G. D. 1996 *Modern Heuristic Search Methods*. John Wiley & Sons, NY, USA.

Rizzo, D. M. & Dougherty, D. E. 1996 Design optimization for multiple period groundwater remediation. *Water Resour. Res.* **32**(8), 2549–2561.

Romeo, F. & Sangiovanni-Vincentelli, A. 1991 A theoretical framework for simulated annealing. *Algorithmica* **6**(3), 302–345.

Rossman, L. A. 2000 *EPANET2 Users Manual*. U.S. Environmental Protection Agency, Cincinnati, OH, USA.

Shieh, H. J. & Peralta, R. C. 2005 Optimal *in situ* bioremediation design by hybrid genetic algorithm-simulated annealing. *J. Water Resour. Plan Manage.* **131**(1), 67–78.

Wood, D. J. & Charles, C. O. A. 1972 Hydraulic network analysis using linear theory. *J. Hydraul. Div.* **98**(HY7), 1157–1170.

Wood, D. J. & Rayes, A. G. 1981 Reliability of algorithms for pipe network analysis. *J. Hydraul. Div.* **107**(HY10), 1145–1161.

Yeh, H. D. & Chen, Y. J. 2007 Determination of skin and aquifer parameters for a slug test with wellbore-skin effect. *J. Hydrol.* **342**, 283–294, doi:10.1016/j.jhydrol.2007.05.029

Yeh, H. D., Chang, T. H. & Lin, Y. C. 2007 Groundwater contaminant source identification by a hybrid heuristic approach. *Water Resour. Res.* **43**, W09420. doi: 10.1029/2005WR004731

First received 18 April 2007; accepted in revised form 13 December 2007